The Theory and Practice of

Modern Framed Structures

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Modern Framed Structures

Designed for the Use of Schools and for Engineers in Professional Practice

BY

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IN THREE PARTS

PART III. DESIGN

NINTH EDITION

TOTAL ISSUE, TWENTY-SIX THOUSAND

REWRITTEN BY

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UNIVERSITY OF WISCONSIN

NEW YORK

JOHN WILEY & SONS, INC.

LONDON: CHAPMAN & HALL, LIMITED

Printed in U. S. A.

6/29

PRESS OF
BRAUNWORTH & CO., INC.
BOOK MANUFACTURERS
BROOKLYN, NEW YORK

PREFACE

THE present volume is the third of a series of three volumes constituting a complete re-writing of "Modern Framed Structures," published first in 1893. Owing to the increased development and specialization of many lines of structural engineering it has been thought best in preparing the present edition to omit certain topics treated in the earlier work. These relate especially to building construction, elevated tanks, swing bridges, and trestles. The present volume includes, first, a discussion of certain topics of fundamental importance in bridge design, and, second, the detailed analysis and design of a few of the more common structures likely to be of immediate interest to the student and the young engineer.

The fundamental topics are covered in the first seven chapters. Chapter I contains a brief discussion of styles of structures and determining conditions. Chapter II treats of the fundamental factors relating to the selection of working stresses, including the subjects of impact and secondary stresses. Chapter III treats of the design of compression members. In this chapter it has seemed desirable to review the fundamental theories of the strength of columns so as to establish a basis for a discussion of working formulas and the results of tests. The question of column shear and the effect of eccentricity of load and of secondary stress has been discussed in some detail in order to get a clear notion of the behavior of the column as a part of the truss. Chapter IV deals with combined stresses and secondary stresses. In the discussion of the latter subject the results of calculations and tests are given so as to show, in a general way, the range of secondary stresses in the ordinary types of trusses. Detailed methods of calculation are given in Part II. Chapter V discusses riveted joints, including the principles of stress transmission, friction of joints and the effect of eccentric connections on rivets and connected members. Chapter VI relates to plate-girders, the first part of it covering a detailed analysis of stresses in which special attention is given to the

distribution of stresses in the web and the principles of proper web splicing. In the latter part is given a detailed design of a plate-girder bridge. Chapter VII treats of the general features of truss design.

The principles discussed in the preceding chapters are illustrated by the detailed design of several structures. In addition to the plate-girder designed in Chapter VI, a pin-connected railway bridge is fully worked out in Chapter VIII, riveted trusses in Chapters IX and X, and a steel roof truss and steel building frame in Chapter XI.

Appendix A contains the general specifications for steel railroad bridges of the American Engineering Railway Association prepared by its Committee on Iron and Steel Structures; Appendix B, a few tables of the most frequently used standards; and Appendix C, a treatment of the mechanics of unsymmetrical bending. The last named subject is perhaps out of place in a book of this kind, but it is here inserted because of a lack of adequate treatment in the commonly used text-books on the strength of materials.

The work is intended, primarily, as a text-book on bridge design, but it is hoped that some of the more general features will be of some interest and value to engineers in practice.

F. E. T.
W. S. K.

MADISON, WIS., April, 1916.

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THEORY AND PRACTICE

IN THE DESIGNING OF

MODERN FRAMED STRUCTURES

PART III

DESIGN

CHAPTER I

STYLES OF STRUCTURES AND DETERMINING CONDITIONS

1. **General Considerations.**—The selection of the proper structure to use to meet given conditions is a problem involving so many elements that it is impossible to establish general rules which will not require modification when applied to almost any particular problem. The determining factors are so variable that experience in the location and selection of the proper structure is a safer guide than any rigid formula. There are, however, certain general principles and approved rules which are worthy of attention and which may be used without any very great error by those who lack the needed experience.

The two important problems which confront a constructing engineer at the beginning, in the building of a new bridge, are, 1st, the best location for the bridge; and 2d, the proper structure to use. The items of first cost and cost of maintenance must be considered, together with the probable life of the bridge and its safety in the case of a derailed train. In general, it may be assumed that the item of first cost is the only one which can be varied for any special case, as the

other items depend upon the details of the construction which would be similar for various locations of the bridge or for different structures. Economy in first cost will then be assumed as the chief consideration.

Some attention should be given also to the question of artistic appearance. In American practice too little consideration is commonly given to this matter although conditions in this respect are gradually improving. No structure of any size, and especially no large and monumental bridge, should be designed without carefully considering its appearance. In the case of structures in populous districts, such as city bridges, very careful attention should be given to this feature and the public can well afford considerable extra expense to attain the desired results. In other cases less attention can be given this element, but in most cases the requirements of good appearance can be reasonably met with little or no increase in cost.

In the study of the cost of a bridge, attention is especially directed to the division of the structure into the two main elements, the *substructure* and the *superstructure*. Generally speaking, if a design is modified by changing the number of spans or the type of structure so as to decrease the cost of the superstructure the cost of the substructure is likely to be increased, and conversely. The first cost will be a minimum when the combined cost of the substructure and the superstructure is a minimum, and after the requirements of navigation are met this is the general result to be sought.

2. Classification of Bridges.—Bridges are classified with respect to the position of the roadway into:

a. Deck bridges;

b. Through bridges.

With respect to the method of construction they are classified into:

c. Plate-girder bridges, including rolled I-beams;

d. Riveted truss bridges;

e. Pin-connected bridges.

With respect to the method of transmitting the load to the substructure they are classified into:

f. Simply supported beams and truss bridges;

g. Continuous girders;

h. Arch bridges;

i. Cantilever bridges;
j. Suspension bridges;
k. Combinations of the cantilever with the arch or suspension system.

3. Deck and Through Bridges.—Whether a bridge should be a deck or a through bridge will depend upon the necessary position of the grade line and the clearance required underneath the structure. Where there is sufficient head-room the deck bridge is generally to be preferred as it gives a cheaper structure, considering both steel and masonry. For a bridge of several spans there is considerable saving in the masonry. For single spans there is very little saving, if any, in using the deck type, except for the shorter spans, say under 180 feet long. For these short spans the trusses of a deck bridge may be placed closer together than would be permissible for a through bridge on account of the clearance necessary between the trusses of the latter. This would reduce the length of the floor-beams and save some metal. The steel floor system may also be dispensed with in the shorter deck spans by supporting the cross-ties directly on the top chord and a further reduction made in the required amount of steel.

4. Plate-Girder Bridges.—In railroad practice plate girders are generally used for spans up to about 100 to 120 ft. There is a minimum opportunity for error in their design and calculation, small chance for defects due to faulty workmanship, and when once put in place they require little attention except the necessary application of paint to prevent rusting. The plate girder is also comparatively free from secondary stresses.

Plate girders cost less to manufacture in the shop than any other kind of construction and can be erected cheaply and quickly. For lengths over 70 ft. the riveted truss may be cheaper in cost on account of the latter requiring less metal in its construction, but good practice requires the use of the plate-girder construction for all spans up to the limiting practical length of such construction, which is about 110 ft. Plate-girder spans have been built as long as 125 ft., but the difficulty of transporting the heavy, long, and high girders of spans over 110 ft. renders this length about the practical limit. The difficulty of obtaining the component shapes and plates of proper sizes

for longer spans results in the use of a large extra percentage of material for splices, etc., which increases cost without a compensating increase of strength. It is always preferable to have plate girders riveted up completely in the shop by power, leaving the bracing between the girders as the only parts requiring hand-riveting after the girders are in place on the piers.

5. **Riveted Truss Bridges** are generally used for spans from about 100 to 150 or 175 ft. On some railroads they are the standard type for spans up to 200 ft., and a number of riveted trusses have been built in the United States and Canada since 1910 of spans exceeding 250 ft. The riveted truss is used exclusively in European practice, pin connections being practically unknown.

6. **Pin-Connected Trusses.**—The general practice of American engineers is now to use pin-connected trusses for spans exceeding 150 to 200 ft. In the early days of bridge building in the United States pin-connected trusses were used for much shorter spans than this, but the light and slender sections employed as a result of this practice did not produce satisfactory structures from the standpoint of rigidity and durability. The substitution of riveted trusses in recent years for these shorter spans is undoubtedly a distinct improvement in design, but to use riveted joints exclusively for ordinary truss spans of 200 ft. and more in length is of doubtful advantage. The excessively deep members and wide joint plates used in some of these designs are very objectionable as they give rise to very high secondary stresses. For long spans the eye-bar is a very satisfactory form of tension member. It is more economical than the riveted member, is relatively free from secondary stress and is quickly erected; and where speed of erection is important the pin-connected type is very advantageous. On the other hand, the connections of the lateral system are not quite as convenient or satisfactory in a pin-connected truss as in a riveted truss.

7. **Types of Trusses.**—The most common types of trusses in use are the Warren, the Pratt and the subdivided Pratt or Petit truss. The Warren or triangular truss usually requires less material in its construction than the Pratt, and for short spans, particularly deck spans, is commonly used. It is also often employed when it is desirable to avoid adjustable members in the truss, because the symmetry

of the truss is maintained and a better-looking structure secured than if the Pratt were used with stiffened ties in place of counter-ties. In the Warren truss some of the web members are subjected to a reversal of stress under live load, and good practice requires that such members have riveted connections. Pin-bearings in such cases are inferior on account of the wear incident to the continual reversal of stress. A great objection to this truss for through spans is that the floor-beams must either be suspended from the pins or vertical posts introduced at each panel point to which the beams are riveted above the pin, the former requirement making the design of an efficient lateral system difficult, and the latter adding material with very little compensation. This truss is rarely used for long spans.

The Pratt truss, including in this type all single-intersection trusses with vertical intermediate posts, is by far the truss most generally used. It may be termed the standard truss in American practice, and in its detail is simpler than any other form. In economy of material and cost it is second only to the Warren for the shorter spans, while for the longer, inclined-chord and Petit truss spans it requires less material and is less expensive. The recent long spans built in this country are generally Petit trusses.

Double-intersection trusses are now seldom constructed. They are objectionable on account of the fact that the stresses are statically indeterminate, but for short spans a greater objection lies in the unequal deflections of successive joints under variable live load which causes very high secondary stresses. The double Warren truss should not be used for short spans except with verticals at each panel point. For very long spans these objections do not apply to so great an extent and the double system may be advantageously used.

Recent studies of secondary stresses and deflections show that trusses with subdivided panels, like the Petit and double triangular with sub-verticals, are subjected to large secondary stresses unless special attention is given to the lengths of the short verticals.* Investigations in connection with the Quebec bridge showed that the K-truss there adopted is remarkably uniform in its joint deflections

* See deformation diagrams in paper by Ralph Modjeski on "Design of Large Bridges," Jour. Frank. Inst., Sept., 1913, p. 239.

and was adopted in part for that reason. This truss thus combines the advantage of the subdivided truss in having short panels with an economical inclination of diagonals, but without its disadvantage of irregular joint deflections. The examples given in Chapter IV, Article 76, show also the merits of the K-truss in this respect.

8. Proportions of Trusses.—The economic depth of a truss is, in general, such as to make the weight of the web members approximately equal to that of the chord members. As a general rule the centre depth should be from $^1/_5$ to $^1/_7$ of the span length, the ratio decreasing somewhat with increase of span, and being somewhat larger for curved chords than for parallel chords of the same span. The width between trusses is determined by the question of lateral strength and rigidity. The general rule is a width of $^1/_{18}$ to $^1/_{20}$ of the span length, with a minimum clear width of 14 to 16 ft. for through spans, and about $^1/_{15}$ of the span length for deck spans with a minimum of 10 ft.

The panel length is made such as to give an inclination for the diagonals of approximately 45 degrees or a little steeper. For long spans (above 250 or 300 ft.) this requirement leads to some form of subdivision by double system or otherwise, so as not to make the floor system excessively heavy.

9. Weights of Simple Span Bridges.—The weight of steel *per lineal foot* for spans up to about 300 ft. can be closely represented by a formula of the form

$$w = a\,l + b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which l is the span length and a and b are constants which depend upon the type of bridge, the load for which it is designed and the unit stress used. The term $a\,l$ represents principally the weight of the trusses and laterals, or those portions whose weight per lineal foot increases approximately in proportion to span length. The term b represents mainly the steel floor whose weight per foot depends on panel length and width of bridge and is approximately constant.

For very long spans the dead load exercises a greater influence on the weight and the term representing truss weight increases faster than the span length, the exponent of l in the term $a\,l$ being greater than unity. With increasing span length this exponent increases,

becoming infinite at the theoretical maximum span length at which the truss is capable of supporting only its own weight.

For the various common types of bridges up to 300-ft. span, and for Cooper's E-50 loading, and Am. Ry. Eng. Assn. specifications the steel weights *per lineal foot* of single track railroad bridges are closely represented by the following formulas:

Deck plate girders:

$$w = 12\tfrac{1}{2}l + 100 \quad \ldots\ldots\ldots\ldots \quad (2)$$

Through plate girders with beams and stringers:

$$w = 14l + 450 \quad \ldots\ldots\ldots\ldots \quad (3)$$

Through pin-connected trusses:

$$w = 8l + 700 \quad \ldots\ldots\ldots\ldots \quad (4)$$

where l = length centre to centre of bearings. In (4) add 5,000 lbs. to the total steel weight if end floor-beams are used.

An increase or decrease of live load will change the weight of steel by a percentage which, for a moderate range, may be estimated at from $^1/_2$ to $^2/_3$ the percentage of change of load. Thus a design for E-40 or E-60 loading will change the steel weight by from 10 to 13 per cent. The relative effect is generally less on plate girders than on trusses.

10. The Economical Span Length.—When the span lengths are fixed by local conditions the problem becomes equivalent to that of a series of bridges of one span each, the only question being the kind of bridge and the style (*i. e.*, deck or through). When the span lengths may be varied, a very close approximation to the correct length of span to use for economy in total cost can be obtained as follows:

Let A = cost of the two end abutments;

B = cost of the floor and that part of the steel weight which remains constant;

C = cost of pier, assumed as constant;

L = total length of bridge in feet;

x = number of spans;

l = length of one span;
p = price of steel per pound;
y = total cost of bridge;
al = weight per foot of variable portion of bridge, eq. (1).

Then the total cost of the bridge will be

$$y = A + B + (x - 1)C + al \times L \times p \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From this we find y to be a minimum when $\frac{L}{x} = \sqrt{\frac{C}{ap}}$.

$$\text{If } a = 8 \text{ and } p = 5 \text{ cts. per lb.,} \frac{L}{x} = \sqrt{\frac{C}{.40}} = 1.58\sqrt{C} \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{If } a = 8 \text{ and } p = 4 \text{ cts. per lb.,} \frac{L}{x} = \sqrt{\frac{C}{.32}} = 1.76\sqrt{C} \quad . \quad . \quad . \quad . \quad (7)$$

As an example of the application of this method, the economical lengths of span, in feet, for piers of various costs, are given in the following table, assuming the steel in place to cost four and five cents per pound respectively, and the truss weight to be given by formula (4) of Art. 9.

Cost of One Pier	Economical Length of Span in Feet		Cost of One Pier	Economical Length of Span in Feet	
	Steel, 4 Cts. per Lb.	Steel 5 Cts. per Lb.		Steel, 4 Cts. per Lb.	Steel, 5 Cts. per Lb.
$5,000	125	112	$15,000	216	193
7,500	152	137	17,500	233	209
10,000	176	158	20,000	250	223
12,500	197	177	25,000	279	250

As the length of the bridge is fixed, such a length of span may be readily selected which will make the total cost of the bridge a minimum.

The assumptions made in deriving the formula for the economical length of span are not liable to be in error enough to affect the choice of the proper length of span to use if the total length of the bridge

is fixed, as this length can rarely be divided into a whole number of spans of the exact economical length.

11. **Long Span Bridges.**—For very long spans the weight of the structure itself becomes of great importance, and for any particular type of structure and material there is a definite limit of span length beyond which the structure cannot be made to support its own weight. The practicable span limit is of course much less than this theoretical maximum.

12. *The Simply Supported Truss* built of carbon steel is, under ordinary circumstances, the most economical type of structure for spans up to about 600 ft. By the use of nickel steel the span length can be increased to about 750 ft. The design of the new Ohio River bridge at Metropolis, Ill., makes use of 720-ft. spans.

13. *Cantilever Bridges.*—Where it is impossible or undesirable to construct false work in the stream, the cantilever method of erection is usually employed, which, in the case of long spans, generally leads to the use of the cantilever type of bridge, as in the case of several of the long-span bridges over the Mississippi and Ohio Rivers. The arch type may also be conveniently erected in this way, as at Niagara Falls. Simple truss spans are also sometimes erected on the cantilever principle. For spans up to 600 or 700 ft. the cantilever bridge is as expensive as the simple truss span, and, owing to its relatively large deflections, is a less satisfactory structure. For very long spans the cantilever type becomes more economical than the simple truss, and by the use of nickel steel may be built up to a span length of about 2,000 ft. The Forth Bridge of carbon steel has a span length of 1,700 ft., and the Quebec Bridge of nickel steel a span length of 1,800 ft. In the latter case the dead load per foot near the centre is about 17,000 lbs., and near the piers it is 76,000 lbs. The total live load amounts to about 11,000 lbs. per ft. The wind pressure is equivalent to about 25 per cent of the total live load.

As shown by the above figures the weight of a cantilever is concentrated largely in the region near the supports. As compared to a simple span, where the weight is nearly uniform, the cantilever is thus enabled to carry its dead load much more economically and so can be used for a much greater span length than the simple truss.

14. *Arch Bridges.*—Where foundation conditions are such that abutments can be cheaply provided to take the thrust and a deck bridge is feasible the arch type of structure is suitable and economical. Such are the conditions in the Niagara gorge and similar situations elsewhere. The arch bridge consists substantially of a braced compression member, the tension member being supplied by the abutment thrusts. Except for the effect of the moving loads, an arch bridge with natural abutments would be a very economical structure, but with heavy moving train loads the bracing required to stiffen the arch ribs greatly reduces the economy. The arch bridge is a suitable type for very long spans where the conditions are favorable, designs having been made for spans up to 3,000 ft. length. The longest arch span built is the Hell Gate Bridge in New York, which is a two-hinged arch of 977½ ft. span length. Under ordinary foundation conditions the arch bridge will generally be more expensive than some other type of structure.

Of the various types of arch bridges the two-hinged spandrel braced arch is the favorite type for railroad bridges. It is less flexible than the three-hinged arch and is a convenient form to erect. The arch with fixed ends is more rigid than the other type, but it is difficult to adjust to the assumed conditions of stress and requires very rigid and large abutments. It is seldom used.

From the artistic standpoint the arch type is the most handsome of all and for this reason it is often used even where strict economy would favor the simple span.

15. *Suspension Bridges.*—For the very longest spans the suspension type must be used. The principal element is the cable or tension member, the anchorages taking the place of the compression member of a truss. As in the arch bridge, the cable must be stiffened to provide for moving loads. The suspension type has the advantage over the arch type, however, in the fact that by using high carbon wire a working stress of 55,000 to 60,000 lbs. per sq. in. may be used, and, further, the stiffening truss may be made less rigid than in the arch type. An objection to the suspension bridge for railroad traffic is its relative flexibility, so that except for the longest spans the cantilever or arch is preferable. The maximum practicable span length for a wire suspension bridge has been estimated at about

4,335 ft., with a unit stress in the cable of 60,000 lbs. per sq. in.* The limit of length of such a cable with no load except its own weight is 15,160 ft. with a deflection of one eighth of the span length. An eye-bar chain of nickel steel stressed to 30,000 lbs. per sq. in. would have a possible length of 7,010 ft.†

* Report of Board of U. S. Engineers as to Maximum Span Practicable for Suspension Bridges, 1894.

† See paper by Ralph Modjeski on "Design of Large Bridges," Jour. Frank. Inst., Sept., 1913, p. 239.

CHAPTER II

WORKING STRESSES—TENSION MEMBERS

16. General Considerations Pertaining to the Working Stress.—In the determination of the working stress to be used in designing it is necessary to consider several factors, both with respect to the nature of the load to be carried and the character of the material to be used. In the case of structural steel, which is the only material here considered, the fundamental requisite is that no stress to which the material may be subjected by any possible combination of loads shall exceed its elastic-limit strength. On the other hand, it is desirable for the sake of economy to work as close to this limit as practicable. In mild steel the margin between the elastic limit and the ultimate strength is large, and so long as no part of the structure is stressed beyond the former limit the factor of safety against absolute failure is ample. In fact, an elevation of the stress at rare intervals beyond the elastic limit would not necessarily injure the material or endanger the structure, but the permanent distortions which would be caused thereby would result in a new and uncertain distribution of stress and would in general be inadmissible.

While the elastic limit may, therefore, be taken as the limit toward which the actual stress may approach with safety, yet for many reasons it is impracticable to use such elastic limit for the working stress in calculating sectional areas. Such working stress must be considerably less than the elastic limit in order to make provision for the following:

(a) Variations and imperfections in the material and workmanship on account of which the strength of the structural member may be proportionately less than that of tested specimens;

(b) Corrosion or other deterioration of the material;

(c) Secondary and other stresses not taken account of in the calculations;

(d) Some increase in live load beyond that specified without endangering the structure;

(e) The dynamic effect of moving loads whereby the actual stresses produced in the structure are considerably greater than the calculated static stresses;

(f) The possible effect of repetition of stress on the elastic strength of the material, in accordance with the theory of the fatigue of metals;

(g) In the case of unlikely but possible combination of circumstances, the working stress is made to depend somewhat, also, upon the probability of the assumed stress occurring.

Considerations (a), (b) and (c) are not usually specifically allowed for, but provision is made in a general way by using a working stress lower than would otherwise be required. Corrosion is sometimes separately considered in structures specially exposed, and the working stress modified accordingly, or a certain thickness of metal added to the calculated sections. Secondary stresses are seldom calculated; the aim of the designer is to avoid them as much as practicable, but they are always present to a considerable extent and must be allowed for in selecting the working stress. In well-proportioned structures secondary stresses will ordinarily amount to from 30 to 40 per cent of the primary stresses as a maximum value. In some designs, however, they will run as high as 80 to 100 per cent. (See Chapter IV for fuller discussion.) The relative effect of dead and live load, including items (d), (e) and (f), will be considered in detail. Item (g) refers to such rare combinations as, for example, a maximum live load at maximum speed accompanied by maximum wind pressure; or a multiple-track structure with maximum loads on all tracks. For such combinations an increase in unit stress is allowed; 25 per cent in the ordinary case of maximum combined dead, live and wind stress on single-track structures.

17. The Elastic Limit.—For the standard structural steel the average commercial elastic limit (yield point) in tension is about 32,000 lbs. per sq. in. A minimum of 30,000 lbs. per sq. in. is allowed in specimen tests and 29,000 lbs. per sq. in. for full-sized eyebars.* A value of 30,000 lbs. per sq. in. may, therefore, be taken as about the elastic-limit value to consider in fixing the tensile

* See Specifications, Appendix A.

working stress. The e.astic limit in compression on short specimens is practically the same as in tension, although tests usually show slightly higher results.

It is true that the limit of proportionality of stress and deformation (true elastic limit) is somewhat below the yield point; in fact, by extremely precise methods of measurement, the stress-deformation line is found to curve at stresses very much below the yield point. The permanent deformations for these low stresses are, however, so very small that they are of little significance except as they may be affected by many repetitions of stress, a subject discussed in Art. 22. For the purposes of this discussion the yield point may then be taken as the elastic limit for structural steel.

18. The Dead-Load Stress.—The dead-load stress is a static stress and of fixed amount. If all secondary stresses were calculated so that the true maximum fibre stress could be known, it would be perfectly safe to allow a working stress for dead load alone practically equal to the minimum elastic limit as above indicated. In fact a slight overstepping of the elastic limit, if due to bending from secondary stress, could do no more than to give a slight set to the member, as in cold straightening. Secondary stresses are controlled and limited by the small angular changes in the form of a truss when loaded, which cause a certain amount of bending deformation in the members. When the necessary amount of bending has occurred the stress ceases to increase, no matter whether the fibre stress has exceeded the elastic limit or not. However, as any considerable permanent set, or inelastic deflection, is objectionable as disturbing the distribution of stress on rivets and connections, the elastic limit should be considered as the limit of fibre stress.

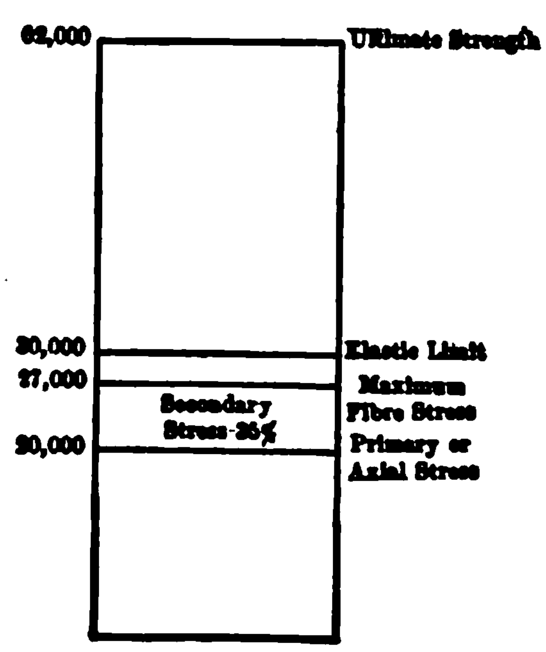

FIG. 1.

The dead-load working stress for structural bridge steel for tension members is usually placed at from 16,000 to 20,000 lbs. per sq. in., secondary stresses being neglected. The higher value is employed only when the live-load stresses are fully provided for in the live-load working stress. Taking the val-

ue of 20,000 lbs. per sq. in. and the elastic limit at 30,000 lbs. the margin would be utilized about as follows (Fig. 1):

Primary static stress	20,000	lbs. per sq. in.
Secondary stress at 35 per cent	7,000	" " " "
Total fibre stress	27,000	" " " "
Additional margin for inequalities, etc.	3,000	" " " "
Elastic limit	30,000	" " " "

It would appear, therefore, that 20,000 lbs. per sq. in. is about the maximum permissible working stress under the conditions assumed.

It should be understood that this discussion relates to *elastic* strength and not to *ultimate* strength, or the factor of safety against ultimate failure. Before failure can occur the stress at some point would have to reach the ultimate strength of the material—about 60,000 lbs. per sq. in. As a matter of fact, when the elastic limit is exceeded the relative amount of secondary stress becomes less, the deformations serving to equalize the fibre stress and to equalize the stress on different elements of the same member. Hence, at failure the axial or primary stress would be at least 85 to 90 % of the breaking strength of the material. With a working stress of 20,000 lbs. per sq. in. for the primary stress the factor of safety against ultimate failure is at least 2½, which is ample for static fixed loads. This indicates that so far as dead load is concerned the structure is entirely safe with a working stress of about two-thirds the elastic limit, even though the secondary stresses exceed somewhat the proportions here assumed. An excess stress on extreme fibres at certain points will affect only its elastic strength.

In compression members the relations between elastic and ultimate strength are very different, as shown in Chapter III.

19. The Live-Load Stress.—The live or variable load differs in its characteristics from the dead load in three ways, as noted under (d), (e) and (f), Art. 16. It is subject to future increase, it causes stresses whose true values are considerably larger than the calculated static values, and it is repeatedly applied and removed. Differentiation between dead- and live-load stress in determining

sectional areas is made by three general methods: (1) by increasing the calculated live-load stress by a certain estimated amount to cover "impact" and then applying the same, or nearly the same, unit stress as for dead load; (2) by using a comparatively low unit stress for live load and applying it directly to the calculated static stress, the effect of impact, repetition of stress, etc., being covered by the unit stress; and (3) by the use of a "fatigue" formula applied to the combined static dead- and live-load stresses and (usually) neglecting the effect of impact as such. The various elements involved will be first discussed, after which these methods of treatment will be considered.

Secondary stresses have the same relative magnitude as in the case of dead load and need to be provided for in the same general manner.

20. *Future Increase in Load.*—The most rational method of providing for future increase in load is to estimate what such load will be and use it in the stress calculations. However, owing to the impossibility of correctly estimating future conditions, either in the amount of such load or in its distribution, it is desirable to have a considerable margin in the unit stress itself. What this should be depends much on the individual conditions. For very large costly structures a larger margin for growth should be allowed than for ordinary structures. In the former case it is customary to assume a much heavier load than for ordinary structures, thus insuring a longer life. On account of the effect of dead load, the assumption of a relatively heavy live load and high unit stress gives a better balanced design, so far as future increase in load is concerned, than a light load and low unit stress; and where the dead-load stress is of opposite sign to the live load, as in counter stresses, it is necessary to consider the actual possible future live load in order to get the benefit of the full capacity of other members. (See provision in the Specifications, Appendix A, Art. 23.)

21. *Impact and Vibration.*—When a train moves rapidly across a bridge it causes certain shocks and vibrations by reason of which the actual stresses produced in the structure are considerably greater than the calculated static stresses. If these additional stresses could be calculated they should, of course, be added to the static stresses and the sums taken as the actual live-load stresses. In lieu of calcula-

tion, the best that can be done is to make an estimate of these stresses from results of experiments that have been made on structures in actual service.

The most recent and extensive experiments on impact are those conducted by the American Railway Engineering Association, the results of which are given in Bulletin No. 125, 1910. The principal conclusions from these tests are discussed in Part II of this work, to which the reader is referred for further details. The formula for impact stress there suggested is

$$I = \frac{L}{1 + \dfrac{l^2}{20{,}000}} \quad \dots\dots\dots\dots \quad (1)$$

in which I = stress due to impact or dynamite effect;
L = static live-load stress;
l = length of span in feet.

The impact stress determined from eq. (1) is to be added to the static stress to get the total live-load stress. The value of l to be used is the span length for all members of the truss proper. For the floor system and suspenders the value of l is to be taken as the length of the structure affecting the stress in question,—for stringers, one panel, and for floor-beams and suspenders, two panels. In the tests in question the impact on other web members was observed to be about the same as on the chord members of the same bridge.

In order to allow a little larger impact for spans from 100 to 200 ft. in length, and to bring the curve closer to the extreme experimental values, the authors propose to modify eq. (1) by making the constant in the denominator 30,000 instead of 20,000, giving the formula

$$I = \frac{L}{1 + \dfrac{l^2}{30{,}000}} \quad \dots\dots\dots\dots \quad (2)$$

This is shown in Fig. 2.

The impact formula which has been most widely used is that given in the American Railway Engineering Association specifications:

$$I = L\frac{300}{l + 300} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

In this formula l is the length of the structure which must be covered to produce the maximum stress in the given member, so that the impact stresses for web members of a given bridge are higher than for chord members. This formula was adopted many years ago by Mr. C. C. Schneider for the Pencoyd Bridge Co., and, later, by the American Bridge Co., and has come into quite general use. Experiments indicate, however, that it gives too large values for long spans and its use may be considered to cover to some extent secondary stresses or future increase of load.

Another formula based on experimental data is that proposed by Mr. H. B. Seaman.* It is

$$I = L\frac{125 - {}^1/_8\sqrt{2{,}000\,l - l^2}}{100} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

in which l has the same significance as in eq. (3).

Fig. 2 shows the three formulas, eqs. (2), (3) and (4), and the results of all important experiments on American bridges.† It is apparent that so far as experimental data go the formula of eq. (3) gives too high values for long spans.

22. *The Effect of Repeated Stresses.—Fatigue of Metals.*—Experiments by Wöhler, Bauschinger, and others have shown that under repeated applications of stress, rupture will take place at a much lower load than for a single such application. That is, the ultimate strength under repeated loads is much less than the ultimate strength of the material as determined by a single application. It is also shown that the ultimate strength under repeated or varying stress depends upon the range over which such stress varies and on the number of repetitions. Thus, if all the stress is removed each time, a stress of about 43,000 lbs. per sq. in., if repeated about one million times, is sufficient to rupture a steel bar whose ultimate strength from a single test is about 64,000 lbs. per sq. in. A stress of 38,000 lbs. per sq. in. will cause rupture if repeated about five million times. If, however, the

* Trans. Am. Soc. C. E., Vol. 75, 1912, p. 341.

† Compiled by Mr. Bowen in Trans. Am. Soc. C. E., Vol. 75, 1912, p. 354.

stress is not all removed, a larger number of repetitions, or a higher maximum value, is required to cause rupture. In the case of stresses alternating from tension to compression the strength under repeated loads is less than where the stress is of one kind only, the least strength being for a stress alternating between equal limits in tension and compression. Thus the material in question would be ruptured by several million applications of a stress alternating from about 20,000 lbs. compression to 20,000 lbs. tension, the range of stress in this case

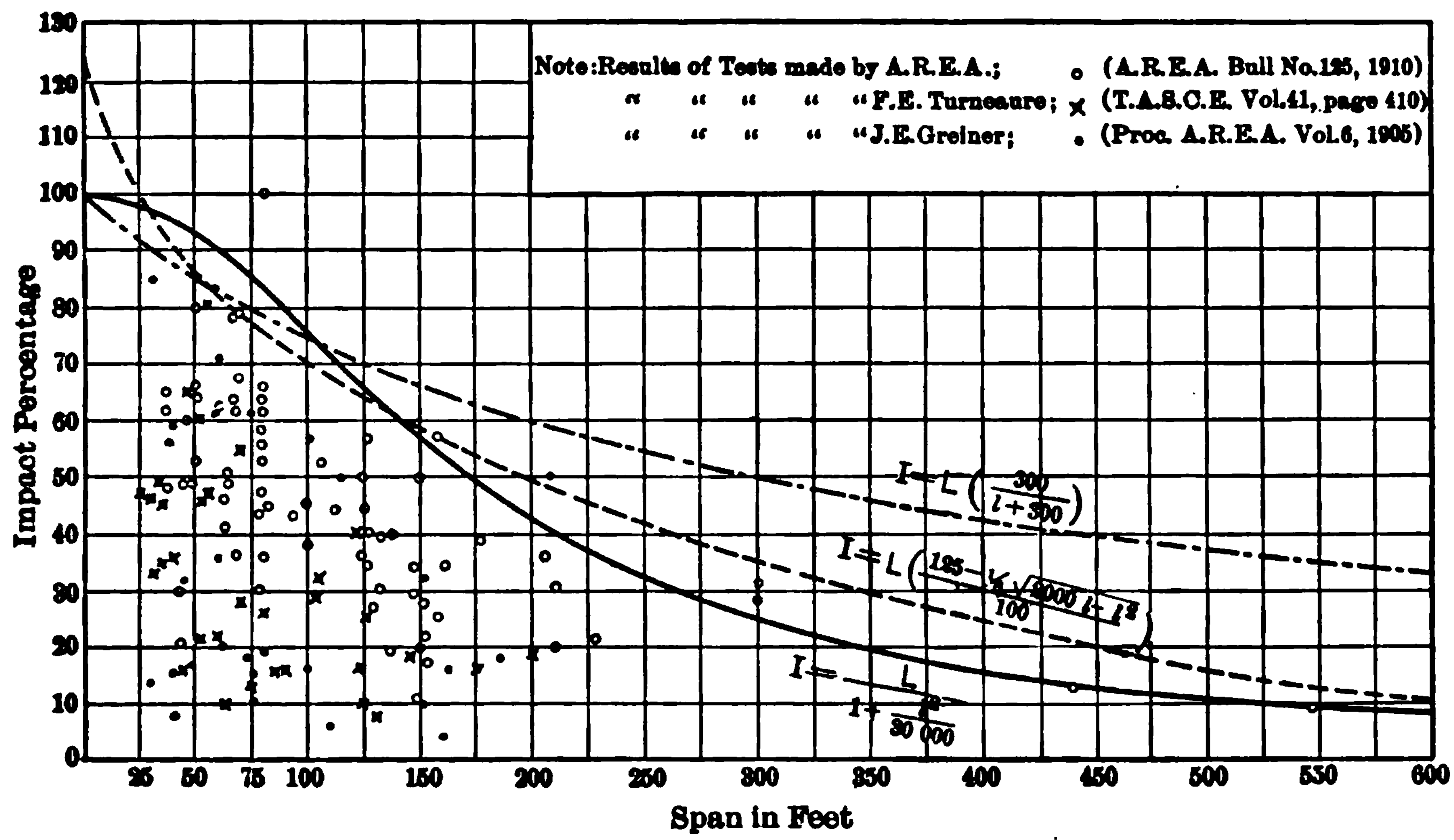

FIG. 2. Impact Tests.

being 40,000 lbs. per sq. in. More recent tests indicate that under still larger numbers of repetitions (running into the billions) the breaking load is much lower than the above figures.* It may be said, however, that for repetitions extending only to a few millions the ultimate strength for loads varying from zero to the maximum is somewhat above the ordinary elastic limit; and that for stresses alternating between equal values in tension and compression it is approximately two-thirds of the elastic limit.

Formulas have been deduced from the experiments mentioned which express the ultimate strength of steel in terms of the maximum

*See paper by H. F. Moore and F. B. Seely in Proc. Am. Soc. Test, Material, 1915, Vol. 15, Part II., p. 437.

and minimum stresses to which it is subjected. The most commonly used is that of Prof. Launhardt for stresses of one kind only and that of Prof. Weyrauch for alternating stresses. Inserting the usual numerical coefficients, these formulas are:

For stresses of one kind $$m = p\left(1 + \frac{\text{min. stress}}{\text{max. stress}}\right) \quad . \quad . \quad . \quad (5)$$

and

For stresses of opposite kinds $$m' = p\left(1 - \frac{\text{min. stress}}{2\ \text{max. stress}}\right) \quad . \quad . \quad (6)$$

in which

m and m' = maximum or ultimate strength, and

p = ultimate strength under a repeated stress with lower limit zero. In the second formula the "minimum stress" is the lesser of the two maximum stresses.

If the working stresses be now determined by applying a fixed factor of safety of about 3 to the ultimate strength as given by the above equations, we derive approximately

$$s = 10{,}000\left(1 + \frac{\text{min.}}{\text{max.}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$s' = 10{,}000\left(1 - \frac{\text{min.}}{2\ \text{max.}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which s and s' are working stresses. For a purely static (dead) load $s = 20{,}000$, and for a purely live or repeated load $s = 10{,}000$ lbs. per sq. in. The element of impact is here supposed to be already included so that the stress in question is the true maximum live-load stress.

A serious objection to the use of these formulas is that while they may correctly represent the *ultimate* strength of the material under the different conditions, and, therefore, eqs. (7) and (8) represent a factor of safety of about 3 with respect to *ultimate* strength, these equations do not give proper relative margins or factors of safety with respect to the *elastic* strength. A working stress of 20,000 lbs. per sq. in. for dead load may be considered about as high as practicable, but a stress of 10,000 lbs. for live load (including impact) is unneces-

sarily low, as 16,000 lbs. has long been in common use. As a matter of fact, when we come to stresses well within the elastic limit, the element of repetition has little or no effect, at least for such numbers of repetitions to which a bridge structure is subjected, and if any allowance at all should be made for repetition effect it should be only such as to make certain that a repeated stress (including impact and secondary stress) shall remain within the elastic limit. This is evidently more important than in the case of dead-load stresses. Instead, therefore, of reducing the stress one-half below that allowed for dead load as is done in eq. (7), a reduction of 10 or 15 per cent would appear to be sufficient if any allowance at all is to be made.

23. Use of Various Formulas in Determining Sectional Areas.—We will now consider the various methods mentioned in Art. 19 for taking account of the effects of live load discussed in the preceding articles.

24. (*a*) *Use of Impact Formula.*—The most rational and commonly used method at the present time is to first add to the static live-load stress the effect of impact, as nearly as can be estimated by the use of some empirical formula, and then to treat the results substantially the same as a dead-load stress, secondary stress being allowed for in the same way as in the case of dead load. This neglects the effect of repetition of stress, and in view of the facts heretofore mentioned it would seem desirable to be more conservative with respect to live-load than dead-load stress by applying to the total live load (including impact) a somewhat lower working stress than for dead load. This lower working stress may also, at the same time, provide for some increase in live load beyond that specified. The result of such method of procedure may be illustrated by Fig. 3. It is assumed that in this case the impact is 60 per cent and that the unit stress applicable to live load and impact is 16,000 lbs. per sq. in. With 35 per cent secondary stress there remains a margin of 8,400 lbs. within the elastic limit, instead of 3,000 lbs. in the case of dead load, Fig. 1.

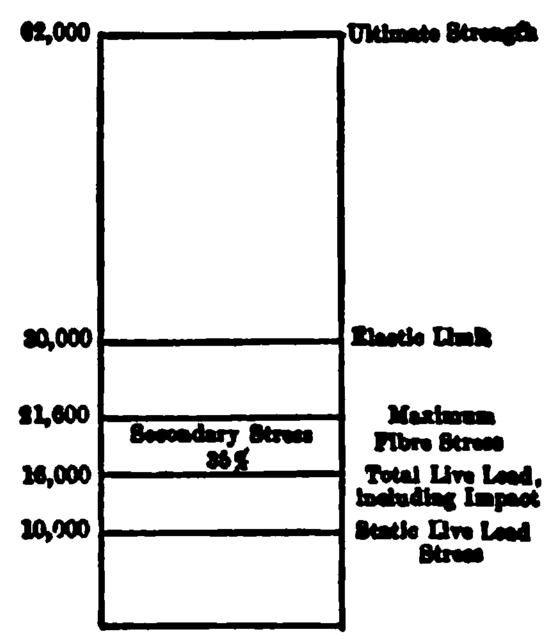

FIG. 3.

Considering differences in conditions it would seem that 16,000 lbs. for live load and impact correspond very well with 20,000 lbs. for dead load, and that both values are safe and reasonable for tension members with properly designed details. The specifications in Appendix A uses the same value, 16,000 lbs., for both dead and live load, but this allows a relatively large margin for dead load, and when a bridge becomes overloaded so that higher unit stresses are allowed temporarily (the A. R. E. A. rules of practice permit a maximum total of 26,000 lbs. per sq. in., including impact figured by the formula of the specifications), the excess strength secured in using 16,000 lbs. for dead load is not properly distributed to take care of the increased live-load stresses. To make use for emergency purposes of excess material requires such material to be distributed in proportion to the live-load stresses, and instead of using 16,000 lbs. for both dead and live load it would be better to use 18,000 or 20,000 lbs. for dead load and then 14,000 or 15,000 lbs. for live load.

25. (*b*) *Use of Low Unit Stress for Static Live Load.*—The second method of design is to use a fixed stress for dead load and a much lower fixed value for static live load, taking care of impact, etc., by the selection of a proper working stress. The application of this method, using a single unit stress for live load, provides in effect for a constant percentage of impact. If, for example, 16,000 lbs. be taken for the desired unit stress for the total live load, the use of 10,000 lbs. for static load implies a uniform impact ratio of 60 per cent. Values of 20,000 lbs. for dead load and 10,000 lbs. for live load are often used, as in Cooper's well-known specifications. This allows 100 per cent for impact and other live-load effects. Experiments show, however, that impact on long spans is very much less than on short spans and, so far as this element is concerned at least, the unit stress applied to static stresses should be varied. So far as the element of future increase in load is concerned, and also that of repetition effect, this method is rational, but inasmuch as the impact effect is now approximately known and is relatively large it would appear more rational to provide for this directly, and then, if desired, use somewhat different working stresses for dead load and for live load plus impact, as described under (*a*).

26. (*c*) *Use of Fatigue Formula.*—In using formulas (7) and (8),

based on the theory of fatigue, the maximum stresses are usually obtained by combining the dead-load stresses with the *static* live-load stresses, all impact effect being neglected. It is assumed in so doing that in the use of the fatigue formula a sufficient provision has been made for all the differences between live- and dead-load effects. This method of treatment is not rational, as the real live-load stress is not the calculated static stress, but something considerably greater, and the best estimate that can be made of this should be used for the live-load stress. The possible effect of repetition, or fatigue, is a separate thing and should be so considered. It is so treated in some recently proposed specifications.

There are many engineers, however, who question the propriety of giving any consideration to the matter of fatigue where the stresses are of one kind only. With working stresses safely below the elastic limit it is held that the range of stress and the number of repetitions are immaterial, as the experiments themselves show indefinite resistance so long as the maximum stress is below this limit, or at least that repetitions to the extent likely to occur in bridge members have no effect. There is much truth in this argument, and the more common practice is to allow liberally for impact by some such formula as given in Art. 21 and to neglect the fatigue element entirely except for alternating stresses. If considered at all, it should not be in the place of impact, but in addition to it.

Other formulas are used more or less which may be considered either as impact or fatigue formulas. Such is the formula used by the C. & N. W. Ry. Co. and others. This is $I = L\dfrac{L}{L + D}$, in which D = dead-load stress and L = live-load stress. The impact is thus made dependent upon the ratio of live to dead load. For purely live load this gives 100 per cent impact, as in formulas (2) and (3) for $l = 0$. For $D = L$, which would occur for a span length of about 400 ft., I = 50 per cent, which is much higher than the impact shown in Fig. 2.

27. *Comparative Results.*—Fig. 4 shows relative results obtained by the application of various formulas and unit stresses. The ordinates represent the net unit stress which, if applied to static live- and

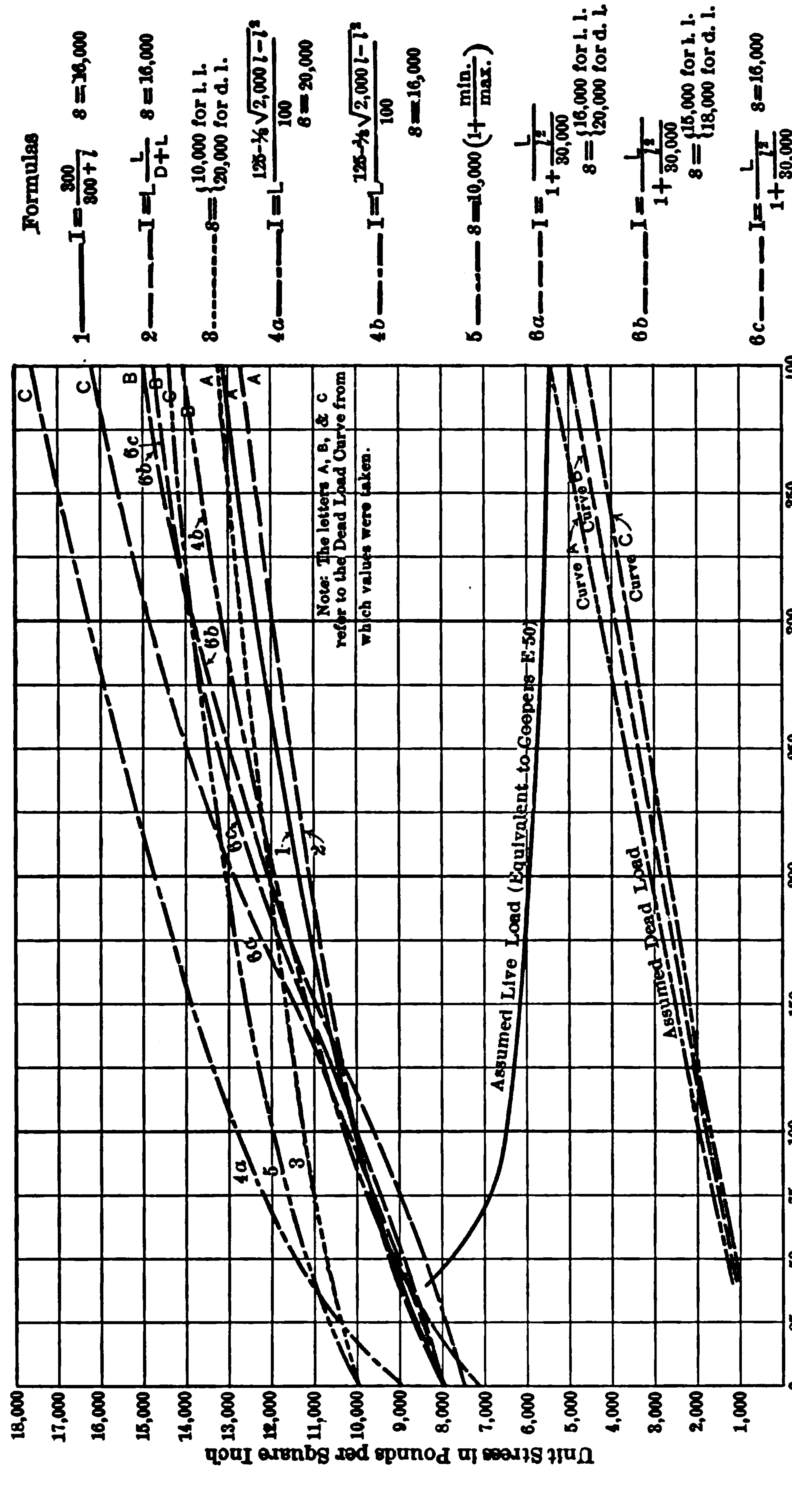

FIG. 4.—Equivalent Unit Stress for Total Load Resulting from the Use of Various Impact Formulas and Working Stresses.

dead-load stress, would give the same sectional area in each case as the use of the various formulas and unit stresses noted. These values are determined from an estimate of the actual weights of single-track railroad bridges of various spans, using a uniform load equivalent to Cooper's class E-50. Chord stresses only are considered.

Thus, for example, for a bridge 250 feet long, the dead load is about 3,500 lbs. per ft. and the equivalent live load is 5,800 lbs. per ft. The ratio of live- to dead-load stress is then equal to 5,800/3,500. Using the impact formula of the specifications, $I = L\dfrac{300}{300 + l}$, we find the impact load to be $\dfrac{300}{550} = 54.7$ per cent of the live load = $0.547 \times 5{,}800 = 3{,}170$ lbs. per ft.; and total dead, live and impact = 12,470 lbs. Then, with a unit stress of 16,000 lbs. per sq. in., the sectional areas of the chords would be proportional to 12,470/16,000, and the equivalent unit stress applicable to static live and dead load would be $(5{,}800 + 3{,}500) \div \dfrac{12{,}470}{16{,}000} = 11{,}900$ lbs. per sq. in., which is the value plotted. The ordinates to the respective curves thus represent the relative net unit stresses allowed by the formulas, and are on a comparable basis. The sectional areas of the chord members are inversely proportional to these net unit stresses, since the static live- and dead-load stresses are the same in all. Curves for the following formulas and unit stresses are plotted:

1. Am. Ry. Eng. Assn. $I = L\dfrac{300}{300 + l}$; $s = 16{,}000$.

2. C. & N. W. Ry. and others. $I = L\dfrac{L}{D + L}$; $s = 16{,}000$.

3. Different unit stresses, after Cooper. $s = \begin{cases} 10{,}000 \text{ for } l.\ l. \\ 20{,}000 \text{ for } d.\ l. \end{cases}$

4. Seaman's Specifications.* $I = L\dfrac{125 - \frac{1}{8}\sqrt{2{,}000\, l - l^2}}{100}$; $s = 20{,}000$.

* See Trans. Am. Soc. C. E., Vol. 75, p. 313.

5. Fatigue formula. No impact; $s = 10{,}000\left(1 + \frac{\text{min.}}{\text{max.}}\right)$.

6. Proposed by authors. $I = \dfrac{L}{1 + \dfrac{l^2}{30{,}000}}$.

(a) $s = \begin{cases} 16{,}000 \text{ for } l.\,l. \\ 20{,}000 \text{ for } d.\,l. \end{cases}$

(b) $s = \begin{cases} 15{,}000 \text{ for } l.\,l. \\ 18{,}000 \text{ for } d.\,l. \end{cases}$

From this diagram we find, for example, that the equivalent net unit stresses for chord members, derived from the various formulas for a 200-ft. span, are approximately as follows:

Formula	Unit Stress
1	11,400 lbs. per sq. in.
2	11,200 " " " "
3	12,000 " " " "
4	14,900 " " " "
5	13,100 " " " "
6 (a)	13,000 " " " "
6 (b)	12,000 " " " "

28. Alternating Stresses.—When the total stress (live plus dead load) in a member alternates from tension to compression, the unit stress used is much less than for stresses of one kind only. The fatigue experiments already referred to show that the limit of strength for alternating stresses, for a few million repetitions, is about two-thirds the ordinary elastic limit. It therefore follows that to insure a proper factor of safety against failure the working stress for stresses alternating equally from tension to compression must be kept to about one-half or two-thirds of that for stress of one kind only. For stresses alternating from a large stress of one kind to a smaller stress of the other a higher value may be employed. A common method of design formerly employed was to design for each maximum stress and add the areas; but more recent specifications require the addition of only 50 to 75 per cent of the lower maximum to each stress, taking the greater of the two areas thus resulting. The Weyrauch formula

mentioned in Art. 22 is nearly equivalent to the method of adding areas above mentioned.

29. Tension Members may be divided into four kinds, according to their method of construction:

1. Eye-bars.
2. Square or round rods.
3. Single shapes.
4. Compound riveted sections.

1. *Eye-bars* are used for the main tension members of pin-connected trusses. They are made by forging or upsetting the eye or head of the bar in a die, and subsequently reheating and annealing the finished bars previous to boring the pin-holes (Fig. 5).

In Table No. VI, Appendix B, are given standard sizes and proportions of steel eye-bars which fairly represent present practice. It

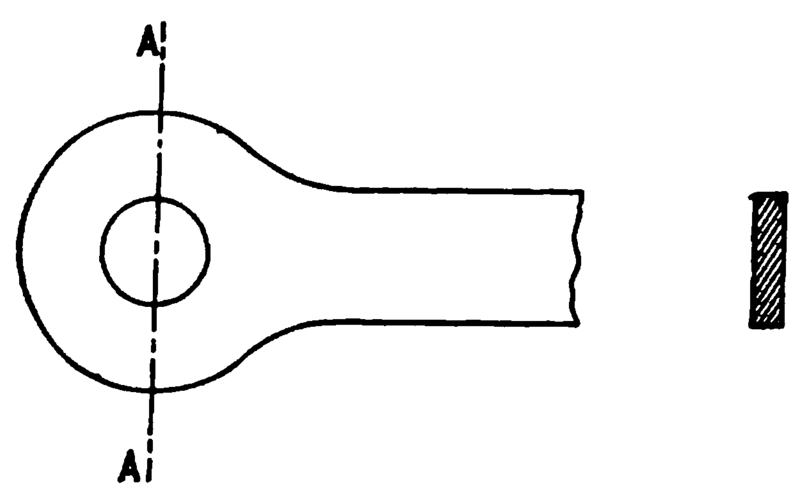

FIG. 5.

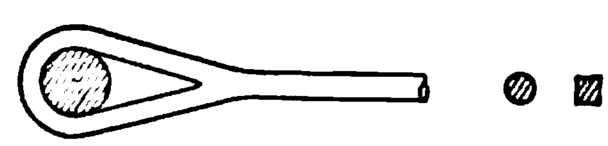

FIG. 6.

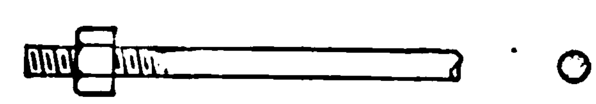

FIG. 7.

will be noted that there is a specified limiting minimum thickness of bar. This has been found to be advisable on account of the method of manufacture, and because thin bars will buckle in the head when under strain. This minimum thickness increases with the width of the bar and the diameter of the eye. Bars can readily be made thicker than the maximum dimension given, but a maximum of 2 ins. is seldom exceeded, as thicker bars are not likely to be adequately worked in rolling and forging to give a good quality metal. Thick bars also tend to cause large bending moments on pins, thus requiring large pins and eyes. The tendency in designing is, therefore, to avoid the use of relatively thick bars where other conditions, such as convenient number of bars, space for packing, etc., permit.

It is always better to use an eye the diameter of which is about

two and one-quarter times the width of the bar. In extreme cases the diameter of the eye may be made two and one-half times the width of the bar, but it is never desirable to exceed this, as the cost and difficulty of manufacture increase rapidly if larger eyes are used. Tests of full-sized eye-bars do not, as a rule, give as high results as the usual specimen test. This is due to the effect of thickness and of annealing. (See Specifications, Arts. 87, 162.) The distribution of stress in the metal around the eye is very unequal and the necessary dimensions can only be determined by tests. On the section *A-A*, through the eye, it has been found that the area should be about 35 per cent in excess of the body of the bar.

In principle the eye-bar is a very economical, convenient and reliable form of tension member. Its manufacture is fully completed in the shop, where, with modern conditions of manufacture, the

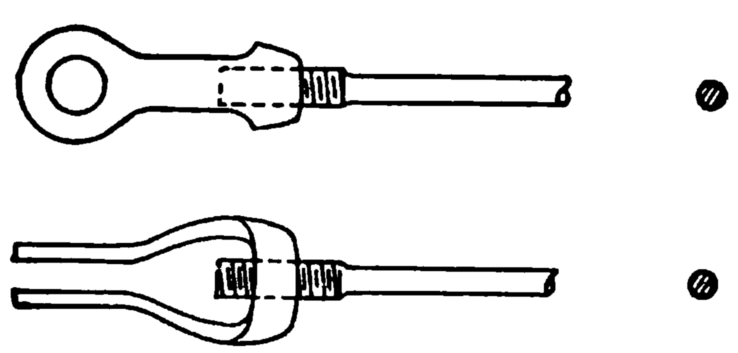

FIG. 8.

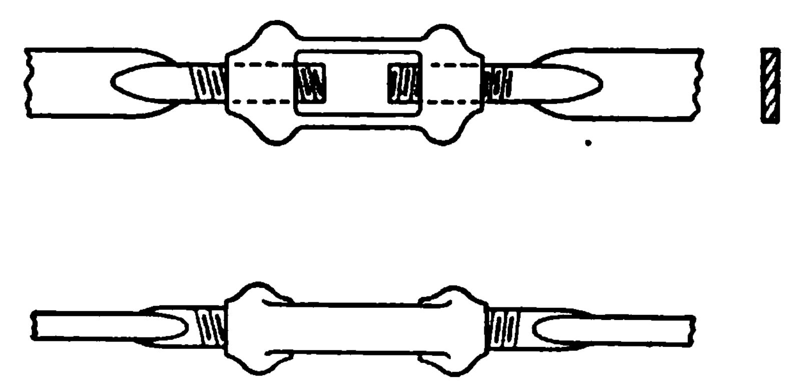

FIG. 9.

product is uniform and reliable. It is convenient to transport and is economical and easy to erect. Furthermore, in the structure, it is subjected to very low secondary stresses, there are no rivets to work loose, and it is less subject to corrosion than thinner riveted sections. In the case of overstress it will also show a much greater

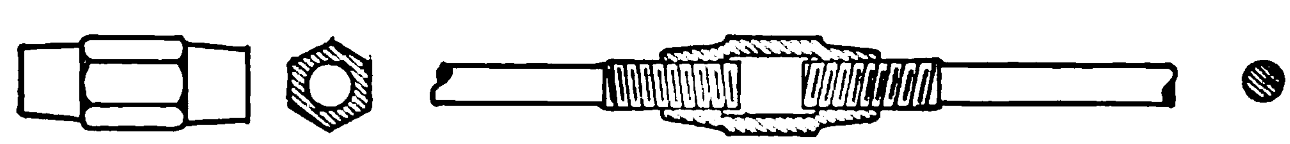

FIG. 10.

elongation than riveted members and in this sense possesses a greater resilience to shock. On the other hand eye-bars of small section are relatively slender and are very much more vibratory under moving loads than riveted shapes and built-up members, and are therefore not so satisfactory for small structures.

Eye-bars are made adjustable by making one end a screw end and using a turnbuckle (Fig. 9), or sleeve-nut (Fig. 10), connection. Screw ends must have a considerable excess at root of threads to insure fracture in body of bar. (See dimensions in Table of eye-bars.) An advantage of the turnbuckle is the fact that the position of the ends of the rods can be readily inspected. Adjustable eye-bars are needed only where used as counters. Many specifications prohibit adjustable members, requiring the use of shapes or built-up members capable of taking compression as well as tension. (See Appendix A, Art. 36.)

2. *Square or Round Rods* were formerly much used for small tension members, such as the lateral diagonals of trusses and trestle towers. Where attached to pins a loop eye is formed (Fig. 6) and a turnbuckle or sleeve-nut used for adjustment. They are also made with screw ends and nuts (Fig. 7), or clevises (Fig. 9), for attachment to joint plates. Rods are not generally allowed in bridge design under modern specifications, riveted shapes being preferred on account of their greater rigidity against vibration. An advantage of the adjustable rod is that it can be adjusted to a considerable initial tension and in the case of long, light members is likely to be quite as rigid as one or two small angles. Rods are well suited for lateral bracing in structures sustaining dead load only.

3. *Plates and Single Shapes.*—Plates are sometimes used for small tension members with riveted connections, but their flexibility, and hence tendency to vibration, render them generally less desirable than angles or channels. For short members and for structures not subject to vibration the plate is a satisfactory form. Single angles are somewhat stiffer than plates, but the end connections are not so simple. Even if connected by both legs there is likely to be considerable eccentricity of stress and bending in the angle. (See Art. 92.)

4. *Compound Riveted Sections,* or built-up members, have to be used for large members of riveted structures, and are occasionally used for pin-connected members. They are made up of angles, or channels, or angles and plates, arranged so as to be symmetrical with respect to the axis of the truss (Fig. 11). Unlike compression members it is not necessary to secure a large moment of inertia so that the metal should be concentrated mostly in plates parallel to the plane

of the truss, Figs. (*d*) and (*e*), so as to make the transmission of stress at the joint connections as direct as practicable. The two halves of a member need not be connected by lacing but only by tie-plates at

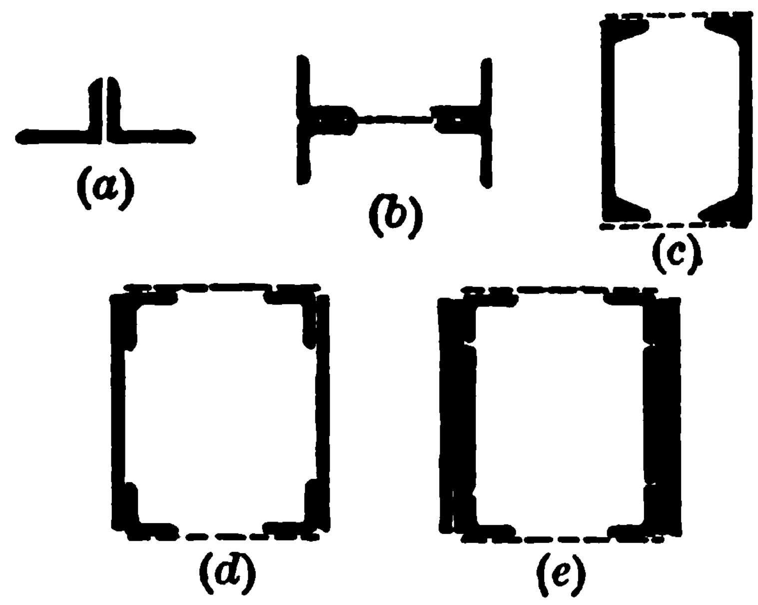

FIG. 11.

occasional intervals to keep the segments in line and prevent injury in shipment. At the ends the tie-plates should be of considerable length in order to resist any bending moments in the segments due to eccentric connections in the plane of the truss. (See Art. 92.)

CHAPTER III

COMPRESSION MEMBERS

30. Factors of Safety in Tension and Compression Members Compared.—As explained in Art. 16 there are many allowances that must be made in the determination of the sectional areas of members which are not capable of calculation. The calculated primary stress being used as a basis, these allowances are provided for in the *margin* or *factor of safety*, which the adopted working stress secures. If, for example, the elastic limit (yield point) of the material is 36,000 lbs. per sq. in., and the working stress is taken as 16,000 lbs. per sq. in., the margin of safety with respect to the yield point is 20,000 lbs. per sq. in., and the factor of safety is $36{,}000/16{,}000 = 2\frac{1}{4}$.

As the possible future increase of live load is one of the elements for which allowance must be made in the margin of safety, it is necessary to inquire how this margin is affected by an increase of load beyond that assumed in the design. If an increase of load causes a *proportionate* increase of stress intensity then the factor of safety with respect to load is as assumed, namely, $2\frac{1}{4}$ in the above illustration; if, however, the stress intensity increases faster than the load, the factor of safety is less than $2\frac{1}{4}$, and if it increases at a less rate than the load, the factor is greater than $2\frac{1}{4}$.

In the case of tension members, and beams with compression flanges well supported laterally, the fibre stresses are closely proportional to load; an increase of 50 per cent in the load will cause an increase of 50 per cent in the stress. In the case of long columns this is not true. As soon as bending begins, the maximum fibre stress increases at a faster rate than the load, the rate of increase becoming greater and greater until the column fails. An increase of load of 50 per cent may, for example, double the stress, or produce an even greater relative increase. In the determination of a proper working stress for long columns, therefore, it is necessary to proceed in a

different manner than for members where stress and load are proportional.

The principles above explained may be illustrated diagrammatically. Suppose we have two members alike in all respects, but one is to be stressed in tension and the other in compression. Both are

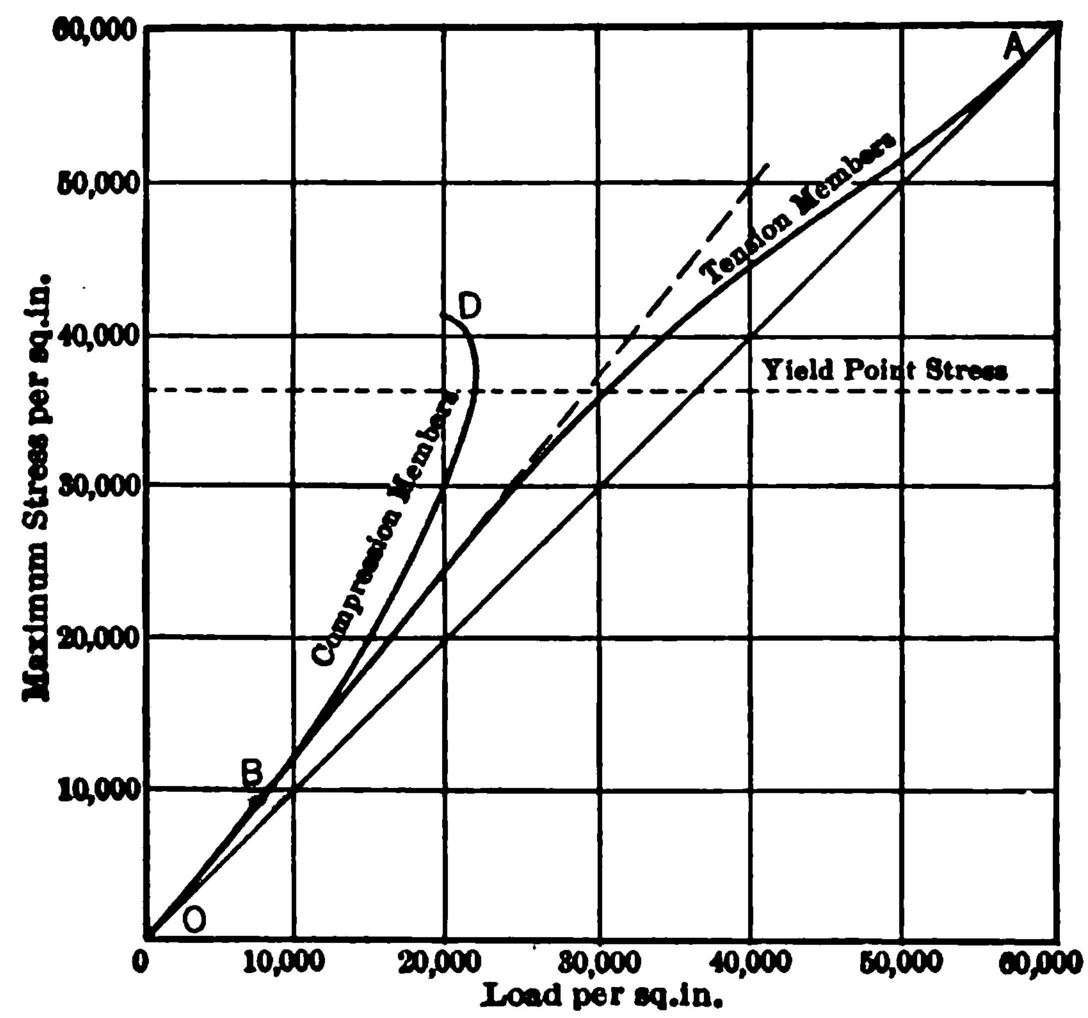

FIG. 1.

subject to the usual small defects unavoidable in such construction, such as slight bends, irregularities in character of material and workmanship, internal stresses, etc., but each to the same extent. We may also assume that the applied load is somewhat eccentric in both cases. Let us now follow the behavior of the two members as the applied load is increased. In Fig. 1 let abscissæ represent load, expressed as average load per square inch, and ordinates represent maximum intensity of stress.

If the members were ideally straight and perfect and the load centrally applied, the straight line $O A$, at $45°$ inclination, would represent the relation of stress and load. In the assumed case, however, the results are different. By reason of the various imperfections already mentioned and the eccentricity of load there will be some bending, and the fibre stress on one side of the member at any

given section will be greater than on the other. At the beginning of the application of load this bending effect is the same in both members, and the line representing maximum fibre stress will be some line such as $O B$. A continued increase of load will, however, produce very different effects in the two members. In the tension member an increase of load will tend to straighten the member and the stress line will tend to approach the line $O A$, especially after the yield point is reached. At the ultimate load, the flow of metal will have practically equalized the stress on the cross-section so that the maximum is practically the same as the average. In the compression member the effect is very different. As soon as the deflection becomes appreciable, the effect is to *increase* the difference between maximum and average stress, and the stress line $O B D$ will curve upward more and more rapidly until the yield point is reached, after which failure soon ensues. The average load per square inch causing this failure (22,000 lbs. per sq. in.) will be considerably below the yield-point stress of the material.

It will be seen, therefore, that with any given load of, say, 15,000 lbs. per sq. in. on each member, the factor of safety in the two cases will be different. In the tension member the yield point is reached with a load of about 30,000 lbs. per sq. in., and we have a factor of safety of two with respect to yield point. In the compression member the yield point (practically the ultimate strength) is reached with a load of 22,000 lbs. per sq. in., and the factor of safety is $22{,}000/15{,}000 = 1.47$. To secure a factor of two in the latter case requires a reduction of load to about 11,000 lbs. per sq. in.

From the above discussion it is evident that in the design of the two classes of members very different procedures must be followed. In the tension member the uncalculated items and imperfections have a decreasing relative effect as the load increases, and as these items are not greatly different in different kinds of members, a fixed unit stress may be applied to the calculated stresses and the resulting margin of safety will be fairly definite and constant. In the compression member the effect of these items depends largely upon the properties of the individual column, especially its length and moment of inertia, and it is not proportional to load. It is, therefore, necessary to have recourse to actual tests on columns and, so far as

practicable, determine the working stress for each member on the basis of its probable ultimate strength.

31. Strength Formulas for Ideal Straight Columns.—In a study of the strength of compression members it is useful to approach the subject by the analysis of certain cases of the perfect or ideal column, loaded in a definite and exact manner. The cases which will be considered are:

(A) *The Centrally Loaded Column;* and

(B) *The Eccentrically Loaded Column*, with loads applied with a known amount of eccentricity, the same at each end.

32. (A) The Centrally Loaded Ideal Column.—(a) *Failure by Direct Crushing.*—If the column is short, perfectly straight and homogeneous, and the load is applied exactly at its centre of gravity, all fibres will be equally stressed and will reach their elastic limit and yield point at the same time. While very short blocks will continue to show some increased resistance after the yield point is reached and the material begins to flow, compression pieces of a few diameters in length will have an ultimate strength substantially equal to the yield point in compression, which is practically equal to the yield point in tension. The yield point may therefore be taken as the ultimate strength of the ideal column whose length is less than that specified in the next article.

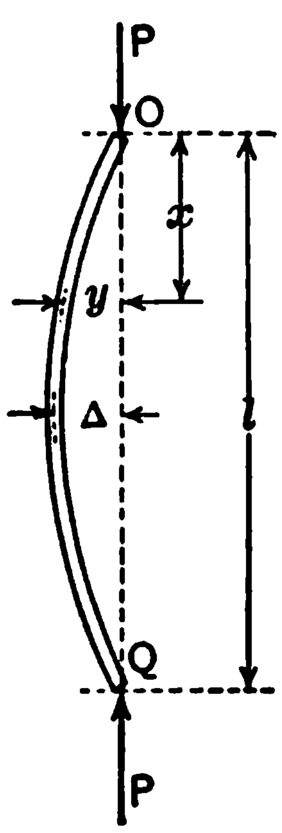

FIG. 2.

33. (b) *Failure by Elastic Bending, Euler's Formula.*—If the length of the ideal column here considered exceeds a certain limit, it is found that, as the load increases, a condition of unstable equilibrium is reached while the stresses are still below the elastic limit. If loaded beyond this point the slightest deflection of the column from a straight line, from any cause, will disturb the equilibrium, the column will then continue to bend under its load and will promptly fail. For a smaller load than this critical load, any such disturbance will not cause failure; if the column is deflected out of line by an external force it will spring back into place.

Let OQ, Fig. 2, represent a column of length l, free to turn at its ends and supporting a load P, as shown, which load is just sufficient to

hold the column in equilibrium in a bent position. The maximum deflection is Δ and the deflection at distance x from O is y. Assume that the deflection is relatively small so that, as is the common theory of beams, we may write $dx = dl$. Furthermore, let

A = sectional area of column;
r = least radius of gyration; and
p = average load per sq. in., $= P/A$.

The bending moment M at any point is equal to Py and we have

$$EI\frac{d^2y}{dx^2} = -M = -Py \qquad (1)$$

and

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}y \qquad (2)$$

The general integral of (2) is

$$y = C_1 \sin q\,x + C_2 \cos q\,x \qquad (3)$$

in which $q = \sqrt{\frac{P}{EI}}$, and C_1 and C_2 are integration constants. For $x = 0$, $y = 0$, hence $C_2 = 0$, and therefore

$$y = C_1 \sin q\,x \qquad (4)$$

which is the general equation of the curved column. This is a sinusoidal curve, the value of y varying from zero for $q\,x = 0\,\pi$, $2\,\pi$, $3\,\pi$, etc., to a maximum value of $\pm\, C_1$ for $q\,x = \frac{1}{2}\,\pi$, $3/2\,\pi$, etc. The value of C_1 is therefore equal to Δ, the maximum deflection, and we have, in terms of Δ,

$$y = \Delta \sin q\,x = \Delta \sin\sqrt{\frac{P}{EI}}\cdot x \qquad (5)$$

For single curvature, as in the figure, $q\,x$ varies from 0 to π, hence

for
$$x = l,\ q\,l = \pi, \text{ or } \sqrt{\frac{P}{EI}}\cdot l = \pi \qquad (6)$$

Writing $I = A\,r^2$ we derive from (6)

$$p = \frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2} \qquad (7)$$

which is *Euler's formula* for long columns.

Euler's formula gives the relation between p, l, and r for which equilibrium is established between load and deflection. The value of the deflection, Δ, is indeterminate, which means that the relations of eq. (7) hold good for any value of Δ, so long as $d\,x = d\,l$ as assumed. This is readily understood when we note that, in general, the deflection of a beam is proportional to the bending moment. In this case the moment is equal to $P\,\Delta$, so that deflection and moment are proportional and may correspond to each other for any value of Δ without a change in value of P. It follows, therefore, that when the load P, given by eq. (7), is reached, the column will be in unstable equilibrium in any position and will promptly fail if the load is increased by any amount.

If a more exact analysis is made by taking into account the inclination of the column axis and the change in length due to stress, in getting the relation of $d\,x$ and $d\,l$, the value of Δ is no longer indeterminate. Such an analysis shows directly that a very slight increase of load above P, of eq. (7), causes a very large increase in deflection and in maximum fibre stress, so that Euler's formula may be taken as giving the correct maximum load under the ideal conditions assumed.*

Theoretically, the assumed conditions of equilibrium are satisfied for $q\,l = 2\,\pi$, $3\,\pi$, etc., corresponding to curves with one, two or more nodes (Fig. 3). For double curvature, for example (Fig. 3 (*a*)), we have $q\,l = 2\,\pi$ and hence $p = \dfrac{4\,\pi^2 E}{\left(\dfrac{l}{r}\right)^2}$, which is four times the value given by eq. (7). That is, the ultimate load for double curvature is four times that for single curvature. Multiple curvature cannot of course occur in practice with free, round-ended columns, but may readily be induced by intermediate lateral support. This analysis shows to some extent the strengthening value of such intermediate support for very long columns, or, what amounts to the same thing, a reduction of the unsupported length.

34. *Effect of End Conditions.*—Equation (7) has been derived for

* See paper by Wm. Cain, on the "Theory of the Ideal Column," Trans. Am. Soc. C. E., Vol. 39, 1898, p. 96.

the case of freely pivoted ends, giving zero bending moments at these points. The same formula may be applied to other cases by using for l the actual length between points of inflection or of zero moment.

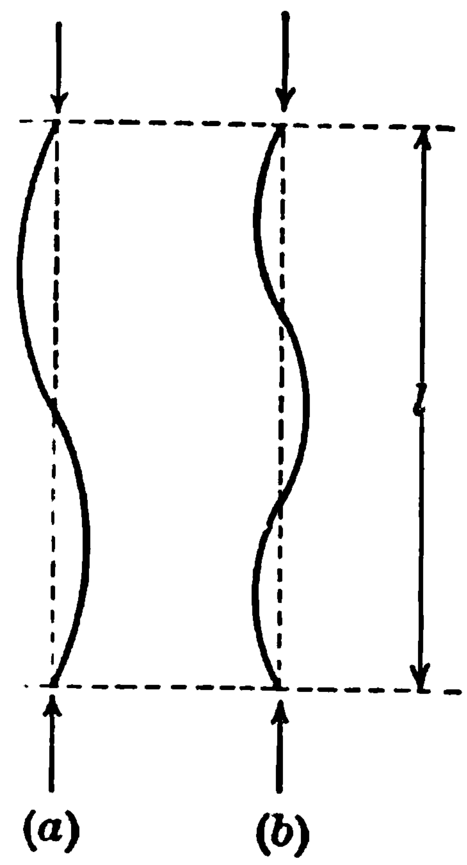

FIG. 3.

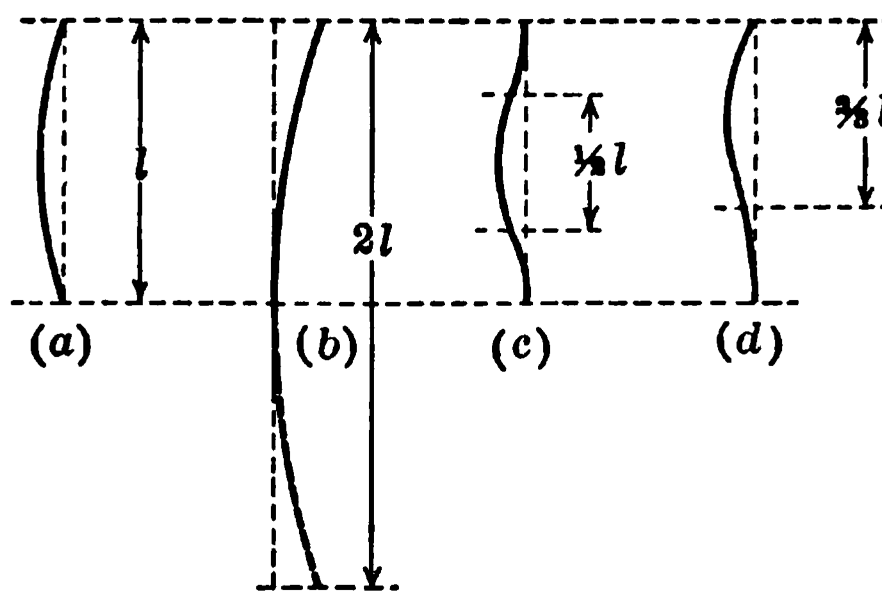

FIG. 4.

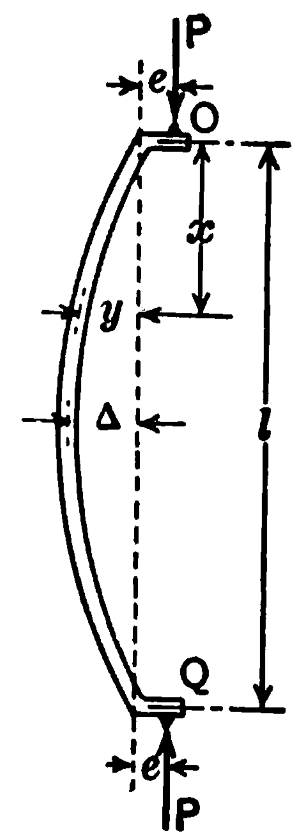

FIG. 5.

In Fig. 4 are shown four cases of end conditions with the lengths indicated between points of zero moment. Substituting these in eq. (7) we have, for the four cases, the following theoretical formulas:

$$\left.\begin{array}{lll} (a) \text{ Pivoted Ends,} & p = \dfrac{\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \\ (b) \text{ One end fixed, one end free,} & p = \dfrac{\frac{1}{4}\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \\ (c) \text{ Both ends fixed,} & p = \dfrac{4\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \\ (d) \text{ One end fixed, one end pivoted,} & p = \dfrac{\frac{9}{4}\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

35. (*B*) **The Eccentrically Loaded Ideal Column.**—The column OQ, Fig. 5, sustains loads P, applied with an eccentricity e, the same at each end. As in Art. 33,

$$EI\frac{d^2y}{dx^2} = -M = -P(y+e) \quad \ldots\ldots \quad (9)$$

or,

$$\frac{d^2y}{dx^2} = -\frac{Py}{EI} - \frac{Pe}{EI} \quad \ldots\ldots\ldots\ldots (10)$$

The general integral is

$$y = C_1 \sin qx + C_2 \cos qx - e \quad \ldots\ldots (11)$$

in which $q = \sqrt{\frac{P}{EI}}$, as before.

From the conditions that $y = 0$ for $x = 0$ and $x = l$, it is found that $C_1 = e \tan\frac{ql}{2}$ and $C_2 = e$, whence eq. (11) becomes

$$y = e\left(\tan\frac{ql}{2}\sin qx + \cos qx - 1\right) \quad \ldots\ldots \quad (12)$$

Placing $x = \frac{l}{2}$ we find the central deflection to be

$$\Delta = e\left(\sec\frac{ql}{2} - 1\right) \quad \ldots\ldots\ldots \quad (13)$$

If f = maximum fibre stress and c = distance to extreme fibre, we have, from the usual formula for bending moment,

$$f = p + \frac{Mc}{I} = p + \frac{P(\Delta + e)c}{Ar^2} = p\left(1 + \frac{(\Delta + e)c}{r^2}\right).$$

Substituting Δ from (13) we have

$$f = p\left(1 + \frac{ec}{r^2}\sec\frac{l}{2}\sqrt{\frac{P}{EI}}\right) \quad \ldots\ldots \quad (14)$$

and hence

$$p = \frac{f}{1 + \frac{ec}{r^2}\sec\frac{l}{2}\sqrt{\frac{P}{EI}}} \quad \ldots\ldots\ldots (15)$$

Equations (14) and (15) express the relation between the average stress $p = P/A$ and the maximum fibre stress f. This relation is perfectly definite, and hence for the ideal straight column, loaded

with a given eccentricity e, the resulting maximum stress f can be calculated for any given load, or the load p determined which will cause a given stress f. Making f equal to the yield-point stress, eq. (15) becomes a formula for the ***ultimate strength*** of ***eccentrically loaded columns***, applicable to any length. For very short columns, the term $\sec \frac{l}{2}\sqrt{\frac{P}{EI}}$ becomes practically unity and then $f = p\left(1 + \frac{ec}{r^2}\right)$, the usual formula for combined stress where deflection is neglected. If the column is long, and $e = 0$, the denominator of eq. (15) becomes indeterminate when $\frac{l}{2}\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$, which is Euler's formula.

36. *Equivalent Eccentricity.*—For any particular column it is evidently possible to select such a value for eccentricity of load that all the imperfections of the column and the imperfect centering of the load may be accounted for, and the actual test result be given by eq. (15). Some effort has been made by writers to establish empirical rules for eccentricity which may be of general applicability and which will give good average results when substituted in the formulas of the preceding articles. Tests of columns may be studied in this way to some advantage, and also the various empirical column formulas. Professor O. H. Basquin has made an interesting study of this subject,* and proposes the value for the eccentric ratio $\frac{ec}{r^2}$ of $0.1 + .001\frac{l}{r}$, as corresponding fairly well with the values implied in the common formulas. Taking an average value of r/c of .70, the value of e, according to this rule, would be approximately equal to $.07\,r + .0007\,l$, thus increasing with both r and l. For very short columns ($l/r = 0$), this would give an eccentricity of $.07\,r$ and eccentric ratio $\frac{ec}{r^2} = 0.1$, which would give a reduction in strength of 10 per cent below the assumed yield point f. In view of the relatively high test results obtained for very short columns it would seem hardly necessary to assume any eccentricity for very

* Journal West. Soc. Engrs., Vol. 18, 1913, p. 457.

small values of l/r. As to the effect of length on eccentricity, it is a common practice in the fabrication of columns to allow a maximum deviation from a straight line of $^1/_{16}$ in. for each five ft. of length. This is equal to about .001 l. It will be shown in the next article that by making $\frac{ec}{r^2} = .001\frac{l}{r}$ in eq. (15) the result is a rational and fairly accurate formula for pivoted columns of wrought iron.

37. Applicability of the Foregoing Theoretical Formulas.—The formulas of the preceding articles are rational and correct within the limits mentioned. Their application in practice is, however, dif-

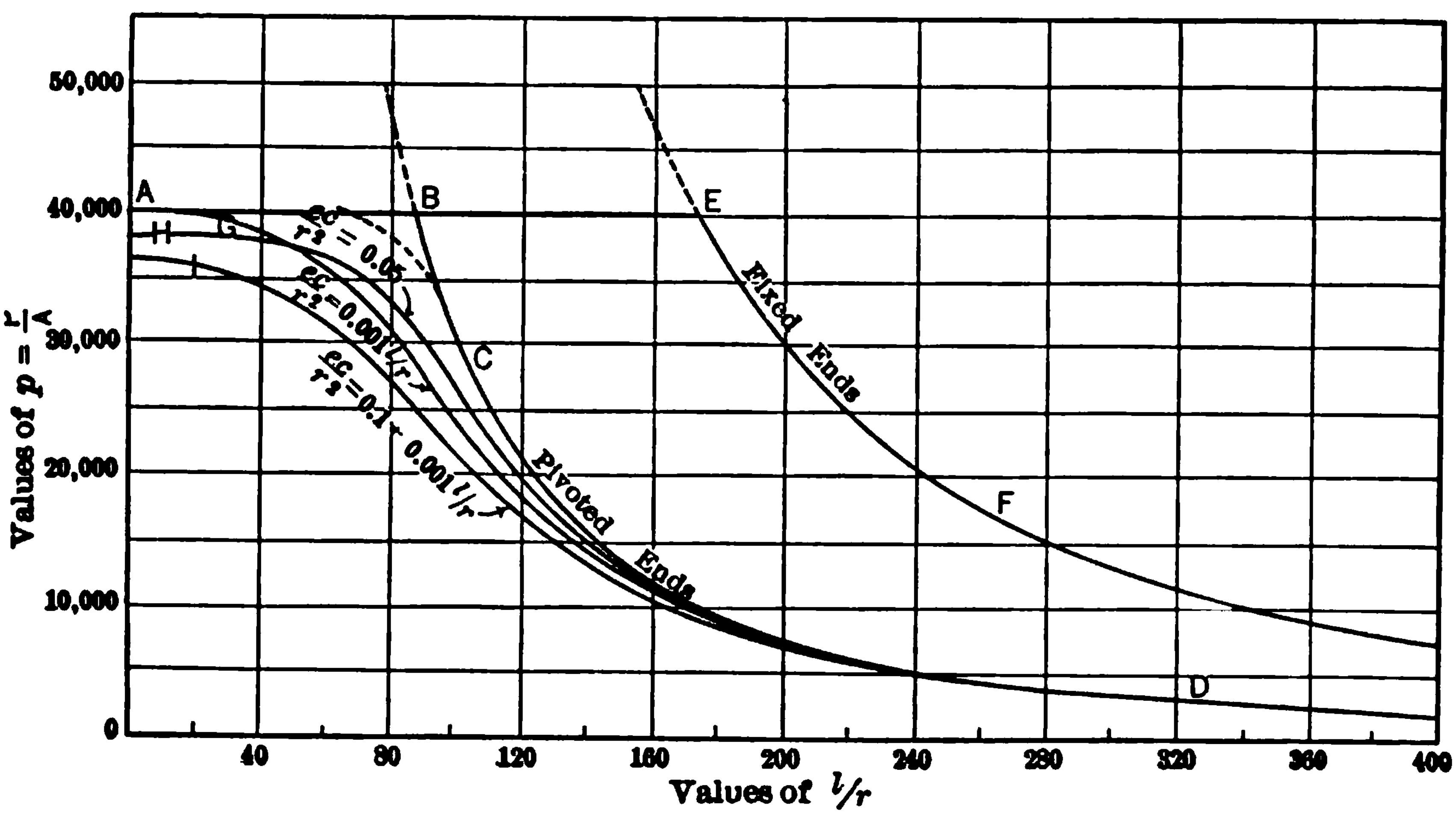

FIG. 6.

ficult by reason of the variation of the practical column from the ideal, and the difficulty of taking account of such variations. The formulas of Art. 35 are, moreover, not in convenient form for practical use. A study of these ideal cases will, however, be of considerable value in the establishment of practical working formulas.

Fig. 6 illustrates these formulas applied to columns of steel having a yield point of 40,000 lbs. per sq. in. The line $A\ B\ C\ D$ gives the theoretical strength of centrally loaded columns with pivoted ends. For values of l/r from zero to about 86 the column fails when the stress reaches the yield point; for values of l/r greater than 86 the failure is by bending according to Euler's law. $A\ B\ E\ F$ is the

corresponding line for fixed ends; the value of l/r for a given value of p is double that for pivoted ends. These results assume that the stress-strain diagram of steel in compression, Fig. 7 (a), is perfectly straight up to the yield point, beyond which it becomes suddenly horizontal, making a sharp break in the curve. Actually the limit of proportionality (true elastic limit) is reached considerably below the yield point, as in (b), so that in effect the value of E becomes smaller as this point is approached. The effect of this on the theoretical line $A\ B\ C$, Fig. 6, is to round off the line near B, into some such form as shown by the dotted line. This modification would occur under the most perfect conditions of loading.

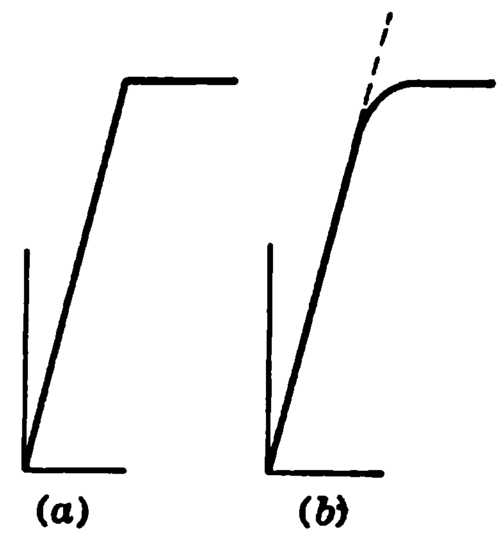

FIG. 7.

The curves G, H and I represent the values of p for the eccentrically loaded column, eq. (15), for various assumed values of ec-

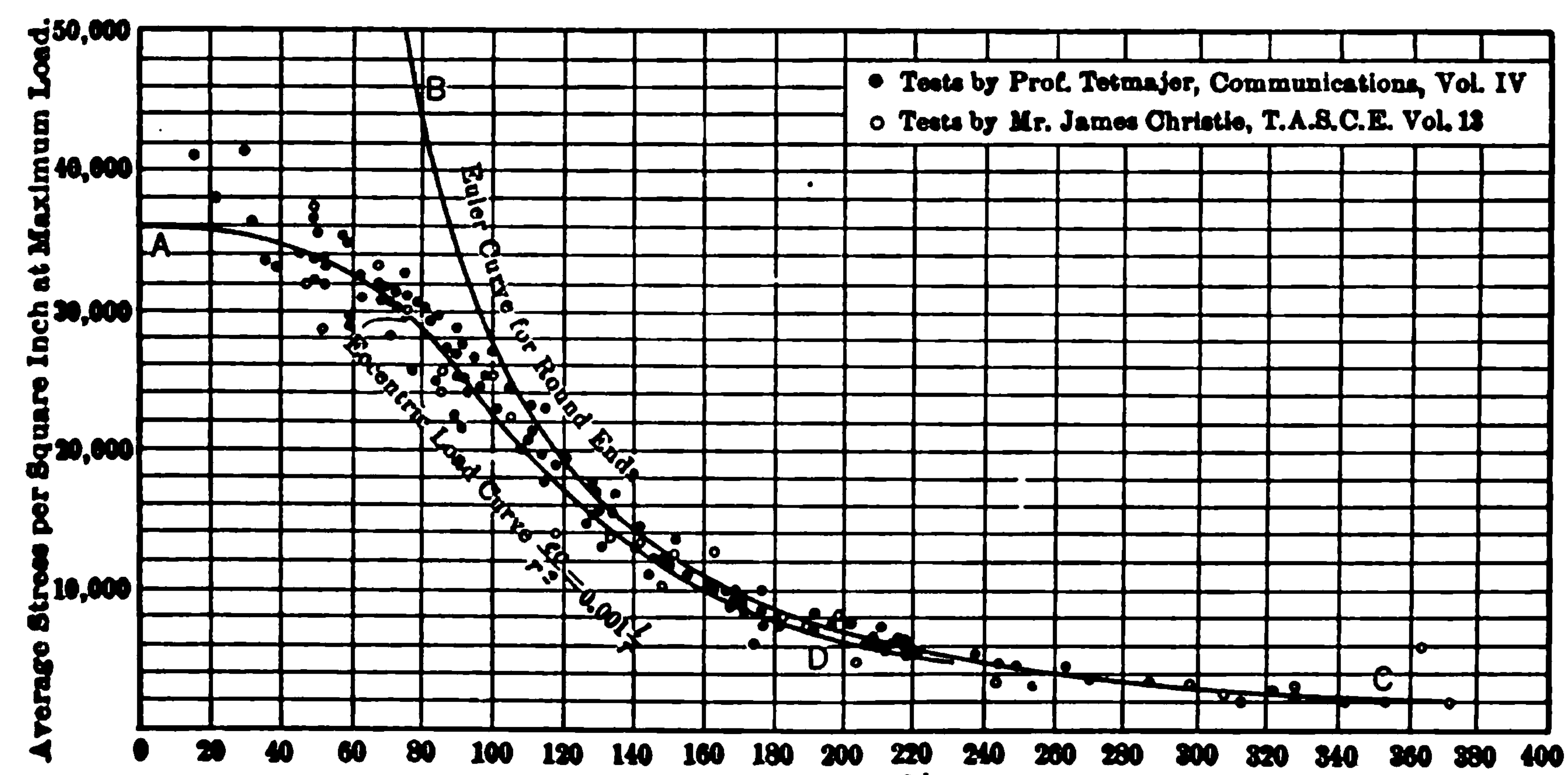

FIG. 8. Tests of Round End Columns of Wrought Iron.

centricity, or the eccentric ratio $e\,c/r^2$. These curves are particularly instructive as they show clearly the effect of eccentricity on the strength of columns in general.

In the case of very long columns, where failure occurs by bending before the elastic limit is reached (Art. 33), the results of care-

fully conducted tests agree closely with Euler's curve. For shorter lengths, the effects of accidental eccentricity, bends, and other variations from the ideal conditions are greater, and the results of tests fall considerably below the theoretical line $A\ B\ C$, corresponding more nearly with some theoretical curve for eccentric loads.

As illustrating the foregoing, Fig. 8 gives the results of tests on wrought-iron columns with pivoted (pointed) ends conducted by Professor Tetmajer* and by Mr. James Christie.†

These were carried out on single shapes, and on small angles and tee-bars riveted together, having values of l/r up to about 360. On the diagram are drawn the Euler curve $B\ C$ for pivoted ends, and the eccentric-load curve $A\ D$, for an eccentric ratio $e\ c/r^2 = 0.001\ l/r$. The value of f (yield point) is taken at 36,000 lbs. per sq. in., and E at 28,000,000 lbs. per sq. in. The close agreement between the Euler curve and the experimental results for high values of l/r is to be noted; also the fact that for lower values of l/r the average results can be well represented by a theoretical eccentric-load curve. The curve as drawn fits the observations very well.‡

The tests here mentioned show what may be expected of single shapes and compactly built, small columns; and the behavior for high values of l/r is well substantiated by other tests. In bridge practice, however, compression members are generally composite pieces of a great variety of form; the values of l/r will also seldom exceed 100 for important members, varying usually from 40 to 60 or 70, and for unimportant or lateral members not higher than about 120. The practical problem, therefore, relates to a field of values of l/r, over which even the tests above given show a wide variation, and with a type of column quite different from those represented in these tests. The practical utility of the preceding formulas in design is confined to very short columns, where a constant working stress may be used ($l/r < 30$ or 40), and very long columns, which may be met with occasionally ($l/r >$ about 150) where the Euler formula may be

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* *Communications*, Vol. IV.

† Trans. Am. Soc. C. E., Vol. 13, 1884, p. 253.

‡ See discussion of Prof. A. Marston in Trans. Am. Soc. C. E., Vol. 39, p. 109, for illustrations of other curves.

applied, and to special cases of eccentricity where the problem may profitably be studied by means of the formulas of Art. 33.

38. Conditions of the Practical Column.—As compared to the ideal conditions heretofore assumed, the actual conditions surrounding the compression member as used in structural design are greatly different. The practical column is generally made up of several component parts connected together more or less perfectly by rivets, plates and lattice bars; the material varies considerably in its elastic limit and yield point, due to variations in thickness and quality; the modulus of elasticity is not uniform, although this property is probably the least varying of any; and the workmanship is imperfect, causing considerable bends, initial stresses, and other irregularities. The connections frequently fail to transmit and distribute the stress uniformly, and the end conditions as to fixity are also uncertain. The nearest approach to pivoted ends in practice is the ordinary pin-connection, but the friction of the pin offers very considerable resistance to turning, which may increase the strength of the column, or decrease it by the effect of the bending of other members connected thereto. Columns are seldom sufficiently well anchored to give truly fixed ends. The ordinary flat ended column resting upon a base is not perfectly fixed unless the base is absolutely rigid, and then only so long as the resultant stresses are compressive at all points in the cross-section.

Finally, the column which acts as a compression member in a framed structure is subjected to very considerable bending stresses due to the action of connected members. In pin-connected members this is limited, in the plane of the truss, to the friction on the pin, plus the effect of pin eccentricity, if any; in riveted structures it depends upon the distortion of the entire structure. These bending stresses often amount to 30 per cent, or more, of the primary stresses, which is equivalent to a considerable eccentricity of the axial load. Such bending stresses exist, more or less, both in the plane of the truss and in the transverse plane, so that the maximum fibre stress is often largely in excess of the average stress without any consideration of flexure due to column action.

It is evident, therefore, that the design of safe compression members demands much more than the selection of a column formula

based on tests on rolled shapes, or compactly riveted small members.

39. **Empirical Formulas.**—Most column formulas now in use are more or less empirical in character. They are based largely on the results of experiments, but to be satisfactory they should satisfy certain limitations and general laws as shown by the theoretical formulas of Arts. 33–35. The most commonly used formulas are the *Rankine-Gordon* formula, and the *straight-line* formula developed by Mr. T. H. Johnson. The *Parabolic* formula suggested by Professor J. B. Johnson in the early edition of this work has much merit and has been used to some extent.

40. (*A*) **The Rankine-Gordon Formula.**—This formula is derived in the following manner: If the centre deflection of a centrally loaded column is Δ, the maximum fibre stress is $f = \frac{P}{A} + \frac{P\Delta c}{A r^2} = p\left(1 + \frac{\Delta c}{r^2}\right)$.

It may be assumed that the deflection at failure, including the effect of initial bends and eccentricities, is directly proportional to the square of the length and inversely proportional to the width, or the quantity c. Inserting a coefficient a, we may then write

$f = p\left(1 + a\frac{l^2}{r^2}\right)$, whence

$$p = \frac{f}{1 + a\left(\frac{l}{r}\right)^2} \quad \ldots\ldots\ldots \quad (16)$$

which is the general form of the Rankine formula. The coefficient a is determined from tests and depends upon the material and end conditions of the column. For wrought iron and pivoted ends the value of a is commonly taken as $\frac{1}{9,000}$. The corresponding value for fixed ends, based on the same considerations as explained in Art. 34, is $\frac{1/4}{9,000} = \frac{1}{36,000}$, and for one end fixed and one end hinged it is $\frac{4/9}{9,000} = \frac{4}{81,000}$. For pin or hinged ends various values are employed between those for pivoted ends and fixed ends. Values of $\frac{1}{13,500}$ and

$\frac{1}{18,000}$ are common. The value of f is taken as the compressive strength of the material in short blocks—for wrought iron generally 36,000 lbs. per sq. in.

An objection to the above theory is that the deflection Δ, corresponding to a given fibre stress f, is assumed proportional to l^2/c. This is true for *beams* subjected to transverse loads, but in the case of *columns* the fibre stress due to the bending is not f but $f - p$, and the deflection is really proportional to $(f - p)\frac{l^2}{c}$. If this relation is assumed, the Euler formula results, a formula of no value for low values of l/r. If we consider, however, that the deflection Δ in this case includes the effect of initial bends, accidental eccentricity, etc., as well as bending due to stress, the assumption made in the above formula appears more reasonable. The resulting formula, eq. (16), gives a value of $p = f$ for $l/r = 0$, as should be the case, and by proper selection of the constant a the formula can be fairly well fitted to experimental results, at least over a considerable range of values.

41. **(*B*) The Johnson Straight-Line Formula.**—In 1886 Mr. Thomas H. Johnson proposed a straight-line formula applicable to ordinary values of l/r, and based on results of tests then available.* He used Euler's curves for high values of l/r and made his straight lines tangent thereto. Although these linear equations can be made to fit the observed results for ordinary lengths satisfactorily, they give too high values of ultimate strength for the shorter lengths, provided failure be taken at the elastic limit of the material, or where the permanent set becomes appreciable. These formulas are, however, very simple in application, and working formulas derived therefrom, or of similar form, have come into very general use. As developed by Johnson these formulas are as follows:

$$\textit{Wrought iron}\quad \left\{\begin{array}{ll} \textit{round ends} & p = 42{,}000 - 203\dfrac{l}{r} \\ \textit{hinged ends} & p = 42{,}000 - 157\dfrac{l}{r} \\ \textit{flat ends} & p = 42{,}000 - 128\dfrac{l}{r} \end{array}\right\} \quad \ldots\ldots \quad (17)$$

* Trans. Am. Soc. C. E., Vol. 15, 1886, p. 517.

$$\textit{Mild steel} \quad \left\{\begin{array}{ll} \textit{round ends} & p = 52{,}500 - 284\dfrac{l}{r} \\ \textit{hinged ends} & p = 52{,}500 - 220\dfrac{l}{r} \\ \textit{flat ends} & p = 52{,}500 - 179\dfrac{l}{r} \end{array}\right\} \quad \ldots\ldots \quad (18)$$

The corresponding Euler curves adopted were:

$$\left.\begin{array}{ll} \textit{For round ends} & p = \dfrac{\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \\ \textit{For hinged ends} & p = \dfrac{\frac{5}{3}\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \\ \textit{For flat ends} & p = \dfrac{\frac{5}{2}\pi^2 E}{\left(\dfrac{l}{r}\right)^2} \end{array}\right\} \quad \ldots\ldots \quad (19)$$

As compared with the theoretical formula for fixed ends, $p = \dfrac{4\pi^2 E}{\left(\dfrac{l}{r}\right)^2}$, the test results for flat ends were much lower, leading to the adoption of a coefficient of $\frac{5}{2}$ as above given, instead of 4.

The formulas derived for mild steel, eq. (18), were based on a small number of tests of steel of relatively high elastic limit and are not applicable to the standard structural steel now in use. The latter has an elastic limit and column strength very little higher than the wrought-iron columns represented in the above formulas.

42. (*C*) **The Parabolic Formula.**—It having been well demonstrated by tests that the Euler formula is correct for very long columns, it would appear that a fairly satisfactory working formula for the usual range of values of l/r can be determined by writing the equation of a smooth curve cutting the Y-axis at a point corresponding to the compressive strength of the material and tangent to the Euler curve for some relatively large value of l/r. A formula of this kind of much

merit is that proposed by the late author, Professor J. B. Johnson. It is the equation of a parabola whose axis coincides with the Y-axis and which is tangent to Euler's curve. This is derived as follows:

Letting $x = \frac{l}{r}$, the equation of Euler's curve (Art. 33) is

$$y = \frac{k}{x^2} \quad \ldots \ldots \ldots \ldots \quad (20)$$

where k represents the numerator, $\pi^2 E$, $4\pi^2 E$, etc., depending on end conditions. The equation of the parabola is

$$y = f - bx^2 \quad \ldots \ldots \ldots \ldots \quad (21)$$

At the point of tangency the values of $\frac{dy}{dx}$ of the two curves are equal, and the values of y are equal. From these conditions we find that, for the point of tangency, $x = \sqrt{\frac{2k}{f}}$, whence $b = \frac{f^2}{4k}$, and we obtain

$$\textit{Equation tangent parabola} \quad p = f - \frac{f^2}{4k}\left(\frac{l}{r}\right)^2 \quad \ldots \ldots \ldots \quad (22)$$

$$\textit{Equation Euler's curve} \quad p = \frac{k}{\left(\frac{l}{r}\right)^2} \quad \ldots \ldots \ldots \ldots \quad (23)$$

For the point of tangency, $y = \frac{f}{2}$ and $\left(\frac{l}{r}\right)^2 = \frac{2k}{f}$, hence for values of $\frac{l}{r} > \sqrt{\frac{2k}{f}}$ use Euler's curve.

The theoretical values of k are:

$$\left.\begin{array}{ll} \textit{Pivoted ends} & k = \pi^2 E \\ \textit{One end pivoted, one end fixed} & k = \frac{9}{4}\pi^2 E \\ \textit{Both ends fixed} & k = 4\pi^2 E \end{array}\right\} \quad \ldots \ldots \ldots \quad (24)$$

For practical use, values of k were adopted by Professor Johnson corresponding closely to those used by Mr. T. H. Johnson for his Euler curves, namely,

$$\left.\begin{array}{ll} \textit{For hinged ends} & k = 16\,E \\ \textit{For flat ends} & k = 25\,E \end{array}\right\} \quad \ldots \ldots \ldots \ldots \ldots \quad (25)$$

43. Comparison of Various Formulas.—In Fig. 9 are shown diagrams of the various formulas discussed in the preceding articles for the case of pivoted ends and wrought iron. There are also shown the results of the Tetmajer tests given in Fig. 8, averaged in groups. The compressive strength of the material is taken at 36,000 lbs. per sq. in., which is the value of p for $l/r = 0$ for all but the straight-line formula. It is noted that for values of l/r from 30 to 150 the pa-

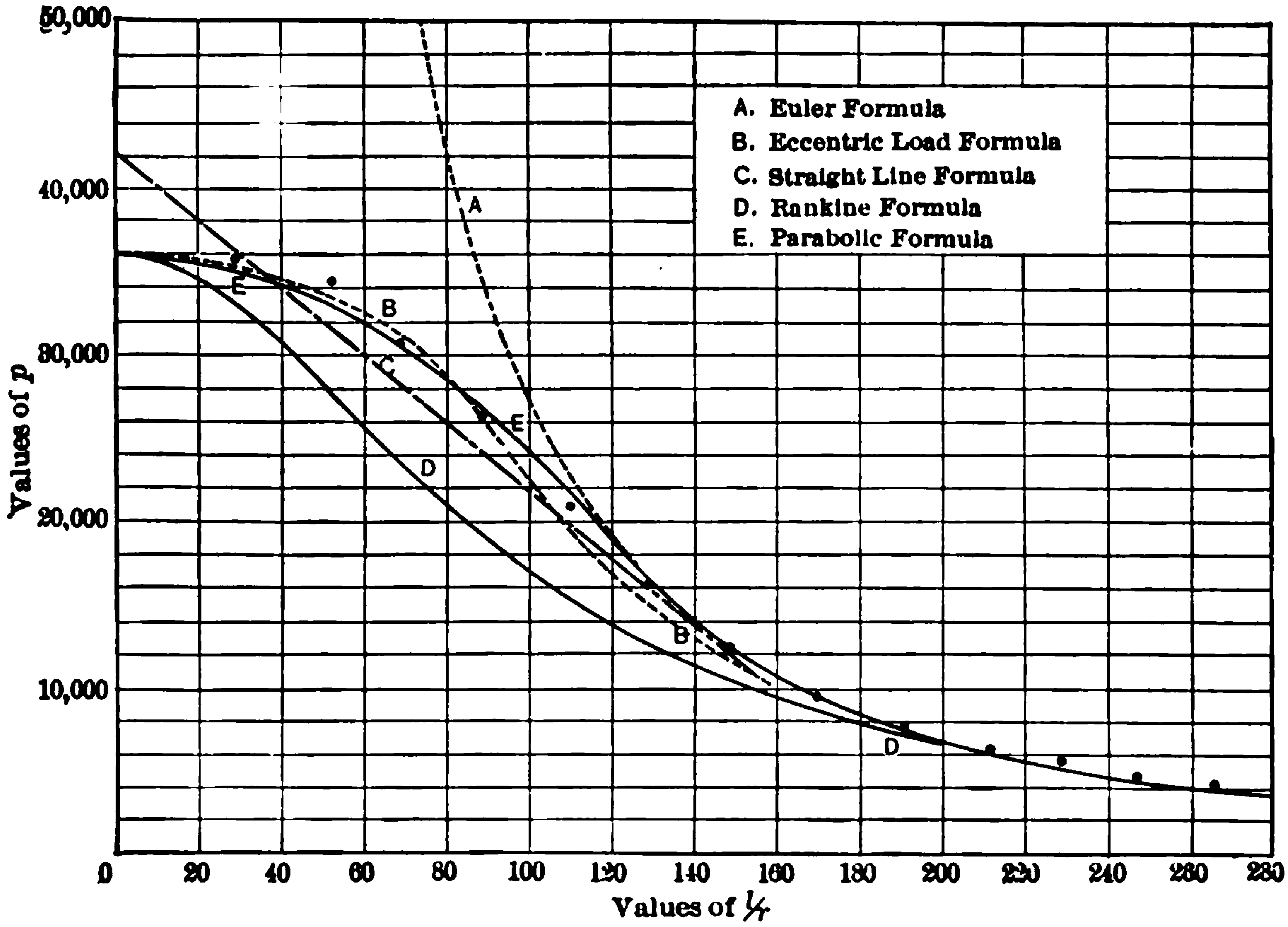


FIG. 9. Formulas for Ultimate Strength, Round End Columns.

rabola, straight-line, and theoretical eccentric-load curves are not far apart. The Rankine curve lies considerably below the others. Obviously by proper selection of the constants, any one of the formulas can be made to fit results of tests over a considerable range of values of l/r, but a formula is preferable if, without being too complex to be easy of application, it may be applied over a wide range. The Rankine formula is designed to be used for all values of l/r, but if adjusted to fit the experiments for ordinary values it gives too high results for low values of l/r. The parabolic formula commends itself in form for values of l/r up to about 150, which covers all ordinary use.

44. Working Formulas.—A working formula for columns is usually obtained by applying a factor of safety to the ultimate strength, using the same factor of all lengths and such as will give for short columns ($l/r = 0$) about the same unit stress as the adopted tensile unit stress for the given material. As the elastic limit of the material is practically the ultimate strength for short columns, the result of this practice is to give about the same factor of safety to compression members with respect to their *ultimate strength* as is given to tension members with respect to their *elastic limit*. Inasmuch as tension members possess a considerable margin of safety beyond their elastic limit, this would appear to give relatively less safety to compression members, especially for the shorter lengths. This effect is obviated in some of the more recent specifications by limiting the unit stress for the shorter lengths to certain fixed values.

The most common material used in structural design is the ordinary mild steel having an ultimate tensile strength of about 62,000 lbs. per sq. in. and an elastic limit in specimen tests of about 32,000 lbs. per sq. in. On the basis of dead-load, or equivalent dead-load stress, the unit tensile stress for this material is commonly taken at about 16,000 lbs. per sq. in. The corresponding working formulas in common use for pin-ended columns are the following:

(*a*) *The straight-line formula*, as adopted by the American Railway Engineering Association and by many railroads (see Appendix A):

$$p = 16{,}000 - 70\frac{l}{r}, \text{ but not to exceed } 14{,}000 \quad . \; . \; . \; . \; . \quad (26)$$

(*b*) *The Rankine type of formula*, as used by many railroads and designers:

$$p = \frac{16{,}000}{1 + a\left(\frac{l}{r}\right)^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (27)$$

in which a is variously taken at from $\frac{1}{9{,}000}$ to $\frac{1}{18{,}000}$, a common value being $\frac{1}{13{,}500}$.

(*c*) *Broken-line formula.*—A useful modification of the straight-

line formula, making it correspond more nearly to a curved line and thus applicable to a wider range of values of l/r, is that of the American Bridge Company for buildings. It is

$$\left.\begin{aligned} p &= 13{,}000, && \text{for } \frac{l}{r} \text{ from } 0 \text{ to } 60 \\ p &= 19{,}000 - 100\frac{l}{r}, && \text{for } \frac{l}{r} \text{ from } 60 \text{ to } 120 \\ p &= 13{,}000 - \ \ 50\frac{l}{r}, && \text{for } \frac{l}{r} \text{ from } 120 \text{ to } 200 \end{aligned}\right\} \quad . \ . \ . \quad (28)$$

(*d*) *The Johnson parabolic formula.*—Allowing something for increase in yield point for very short lengths, the constant f in the para-

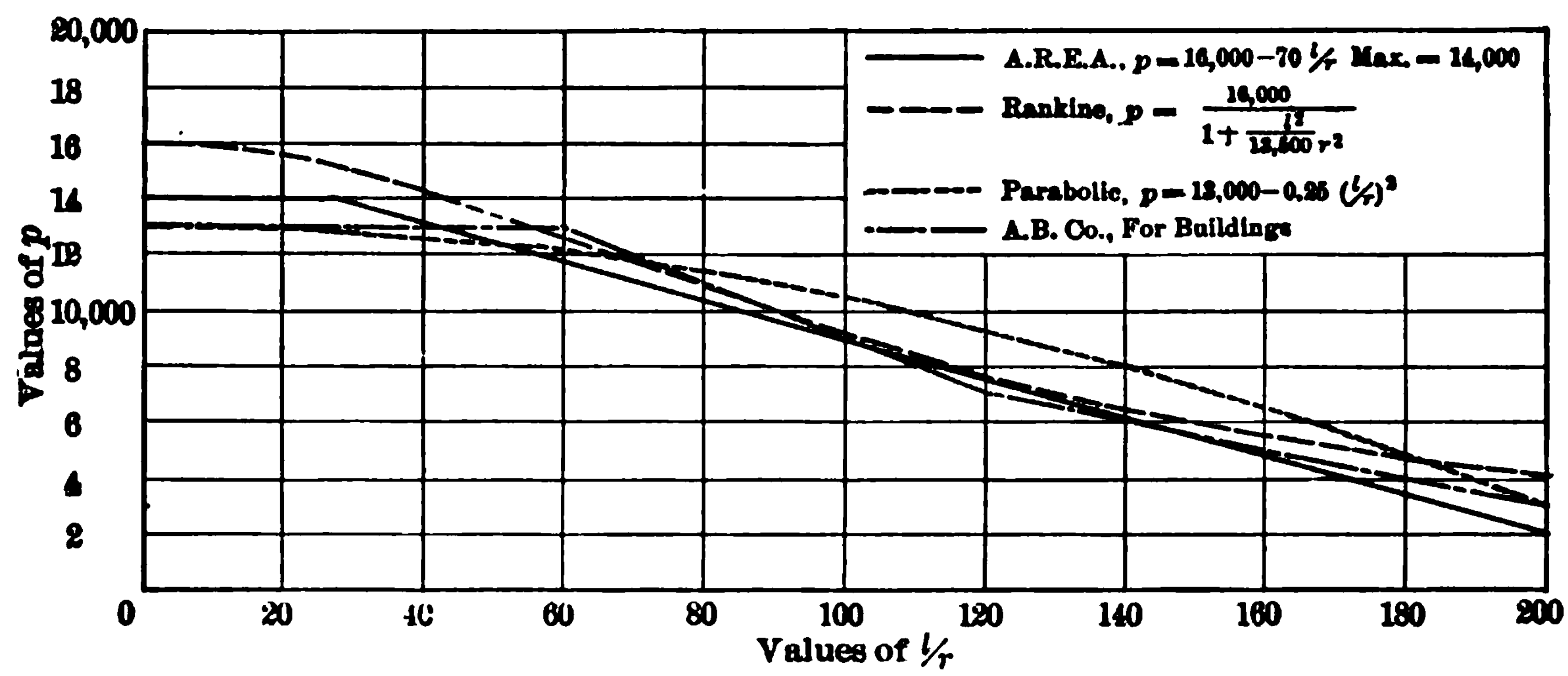


FIG. 10. Working Formulas, Pin-end Columns.

bolic formula, eq. (22), may be placed at 36,000. Applying a factor of safety of $2\frac{3}{4}$ to the formula for hinged ends, eq. (25), we have, as a working formula which will correspond to the above formulas,

$$p = 13{,}000 - 0.25\left(\frac{l}{r}\right)^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (29)$$

The preceding working formulas, with the exception of (*d*), have not been obtained from the formulas for ultimate strength in a very consistent manner by the application of a certain definite factor of safety. The aim has been rather to secure a formula that would be applicable within the usual range of values and that would be safe outside those limits, especially for high values of l/r. The relative

factors of safety obtained by the use of these formulas are shown in Art. 46.

The above working formulas are illustrated in Fig. 10. They are intended to represent pin-ended columns, and are applicable to com-

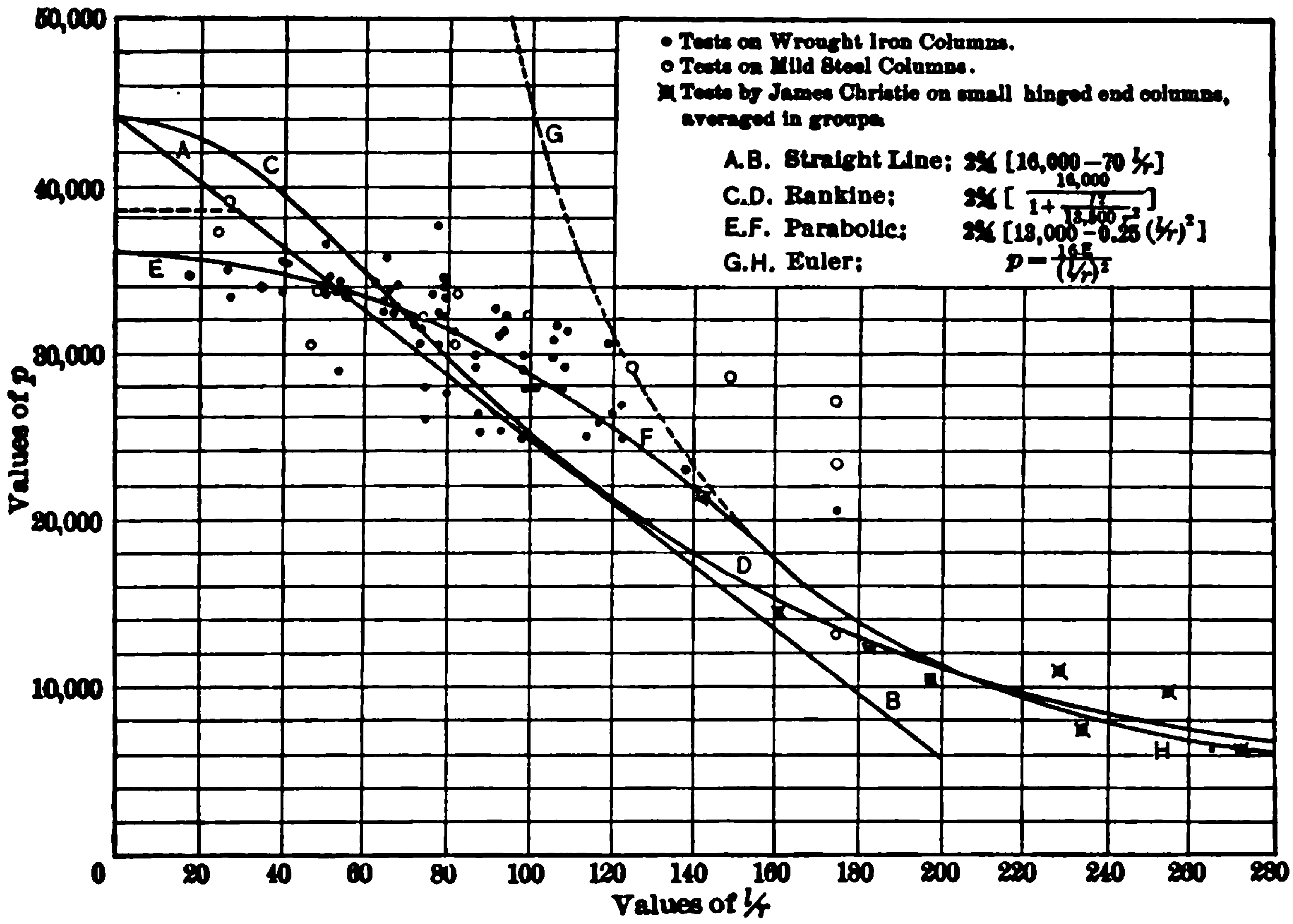


FIG. 11. Tests on Built-up Pin-end Columns.

pression members in general in bridges and buildings. For fixed-end columns, see Art. 48.

45. **Tests of Built-up Columns.**—Most of the early tests on columns were made on single rolled shapes or compactly riveted members of small shapes which would readily act as a unit. Such are those represented in Fig. 8 and others by the same investigators. While such tests are of great value in determining the strength of columns, care must be taken in applying the results to built-up columns not so compactly riveted together. In the design of built-up columns it is the aim to so design the details that the entire column will in fact act as a single rolled unit. To fully accomplish this, however, is very difficult. Where several shapes are riveted together there will be variations in quality of material, and small bends and internal

stresses due to straightening and riveting. Furthermore, stresses on rivets imply a certain amount of strain or distortion, and each element of the column between points of support acts as an unsupported column subject to local buckling. On this account it cannot be expected that the results of tests on built-up columns of a great variety of form, even with the best of design, will be as consistent as those upon single shapes. The strength will depend upon the form of the elements and on the details, as well as upon the value of l/r. Tests upon built-up columns are not numerous and are as yet quite inadequate to fully determine the value of different forms of sections and details.

Tests on Pin-ended Columns.—The most reliable and serviceable results of tests on large built-up columns thus far made are those on columns with pin ends. Owing to the difficulty of truly fixing the ends of a column, either in the testing machine or in practical construction, the working formulas generally used are based on tests on pin-ended members. Fig. 11 gives results of tests on built-up wrought-iron and mild steel columns of a cross-section greater than about 4 sq. in., tested with central loads and hinged ends.* For values of $l/r > 140$, average results of Christie's tests on hinged-end columns are also given, as being the best indication available as to the effect of high ratics of l/r on columns of this kind.

The strength of columns, depending upon the yield point of the material, is very considerably affected by the thickness of the sections, especially in the case of wrought iron. Thin sections, such as used in early tests, thus show higher results, especially for low values of l/r, than thick sections. Again, the standard mild steel now generally used is considerably softer than the medium steel and Bessemer steel

* The data used include the following from those listed in Trans. Am. Soc. C. E., Vol. 66, 1910, p. 401: Eighty tests at the Watertown Arsenal, 1881-84; two tests by Bouscaren; two tests by Buchannan on wrought iron and three on steel, and six tests by Waddell. Several tests by Buchannan are omitted on account of eccentricity of load. Tests by Dagron on Bessemer steel of 52,000 lbs. elastic limit, by Waddell on nickel steel, and by Moore and Talbot on wrought iron are omitted on account of the specially high or low values of the elastic limit. Other tests given are those on 21 steel columns of I-section made at the Watertown Arsenal, 1909, and on five large steel columns made at the same place and reported in Trans. Am. Soc. C. E., Vol. 73, 1911, p. 429.

used formerly. It results, therefore, that the yield point of the material now used in mild steel columns is about the same as that formerly used in the lighter wrought-iron sections. This is shown in the diagram, where the steel results are seen to average but little more than those on wrought iron.

46. Comparison of Working Formulas with Results of Tests.—To compare the different actual working formulas noted in Art. 44 with the results of tests, the values given by the four formulas have been multiplied by 2¾ and plotted in Fig. 11, thus assuming an implied factor of safety of 2¾ in the use of the formulas. The Euler curve, $p = \dfrac{16\,E}{\left(\dfrac{l}{r}\right)^2}$, is also given, to which the parabola is tangent.

Comparing the various formulas it appears that the parabolic formula fits the results better and is more generally applicable than any other. From the comparison in Fig. 9 it is seen also that the parabolic formula corresponds closely to a theoretical eccentric-load curve. The Rankine and the straight-line formulas give too high values for short lengths if they are made to fit the tests for ordinary lengths. This defect is remedied somewhat by limiting the maximum stress, as in the straight-line formula, to 14,000 lbs. per sq. in., giving a value of 38,500 when multiplied by 2¾ (shown by dotted line).

47. Conclusions as to Working Formulas.—For pin-ended columns of wrought iron and mild steel the following are recommended as being consistent and as giving a factor of safety of about 2½ to 2¾:

Straight-line formula:

$$p = 16{,}000 - 70\frac{l}{r}, \text{ but not to exceed } 13{,}000 \quad . \; . \; . \quad (30)$$

Rankine formula:

$$p = \frac{16{,}000}{1 + \dfrac{l^2}{13{,}500\,r^2}}, \text{ but not to exceed } 13{,}000 \quad . \; . \; . \; . \quad (31)$$

Parabolic formula:

$$p = 13{,}000 - 0.25\left(\frac{l}{r}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

Euler formula:

$$p = \frac{465,000,000}{\left(\frac{l}{r}\right)^2} \text{ for } l/r > 160 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

For higher grades of steel the parabolic formula can readily be adapted by using the general form of Art. 42 and the proper value for f.

48. Columns with Flat and Fixed Ends.—Where conditions are such as to give approximately fixed ends, this condition may be al-

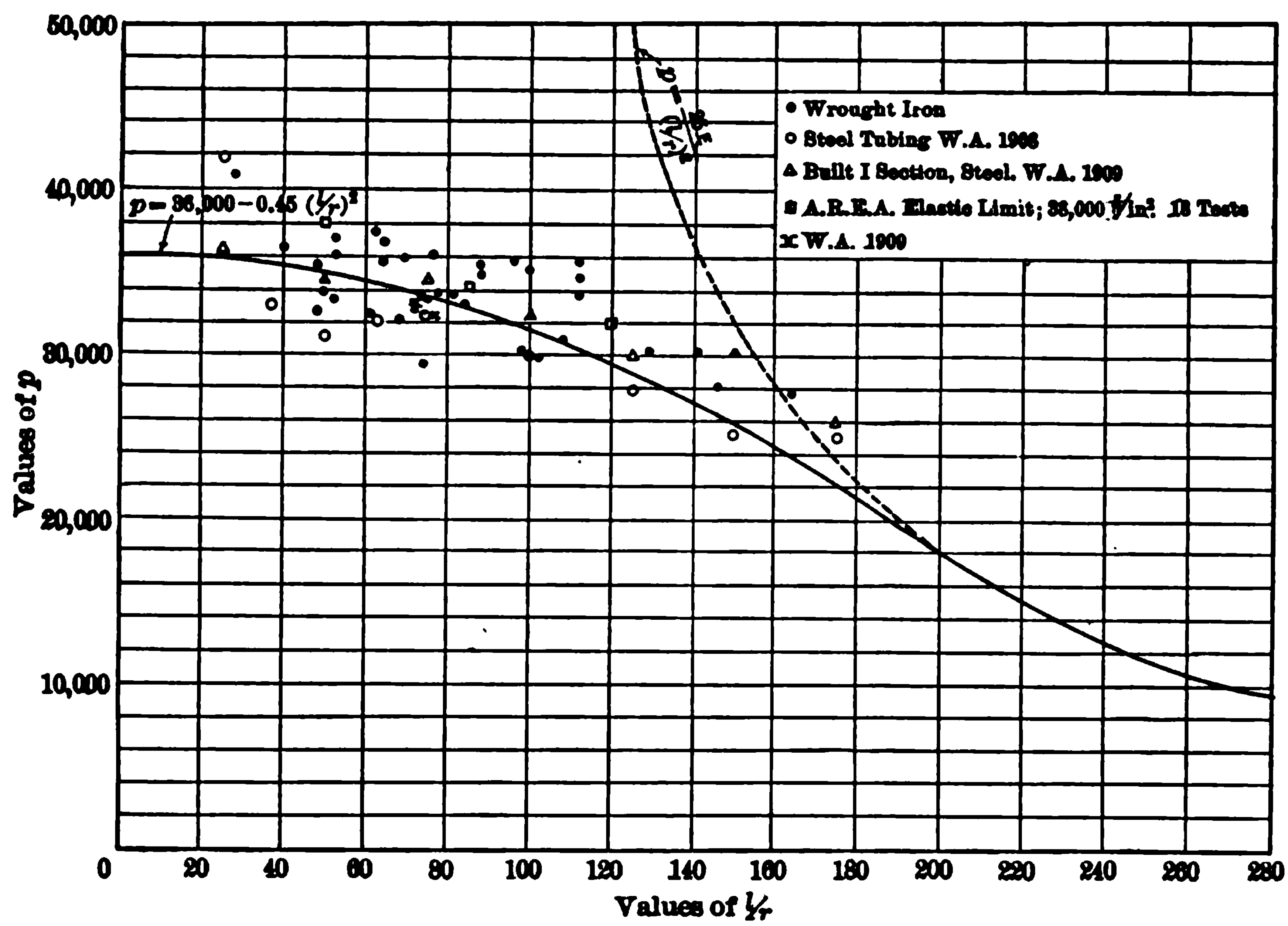


FIG. 12. Tests on Flat-End Columns.

lowed for by using a value of l in the preceding formulas somewhat less than the centre to centre length of the member. Tests on flat-ended columns indicate that about 80 per cent of their length may be taken and applied to the formula for hinged ends. It is to be noted, however, that a member of a *truss* is not "fixed" merely because it has riveted connections to other members. In fact, such riveted connections may actually result in greater bending stresses

than if the member were hinged. In other cases such riveting may tend to strengthen or *fix* the member. The effect in any case is a question of secondary stresses, which is more fully discussed in Art. 49. Unless a careful analysis shows to the contrary, all compression members in trusses should be calculated on the assumption of hinged ends.

In Fig. 12 are plotted all the important available tests of flat- and fixed-end columns of relatively large cross-section of wrought iron and mild steel.* The results are, in some cases, arranged in groups of from two to six tests each. The curves plotted are the Euler curve and the parabolic curve for flat ends as suggested by J. B. Johnson, using $f = 36{,}000$ lbs. per sq. in., namely,

$$\text{Euler curve} \quad p = \frac{25\,E}{\left(\frac{l}{r}\right)^2}$$

$$\text{Parabolic} \quad p = 36{,}000 - \frac{(36{,}000)^2}{100\,E}\left(\frac{l}{r}\right)^2$$

$$= 36{,}000 - 0.45\left(\frac{l}{r}\right)^2$$

The parabola is seen to fit the experiments fairly well and much better than the straight-line formulas for either wrought iron or steel of Art. 41.

The recent tests for the American Railway Engineering Association were very carefully made on material of an elastic limit of 38,000 lbs. per sq. in. The three average results for $l/r = 50$, 85 and 120, lie somewhat above the curve drawn, but correspond very closely to a parabola drawn for $f = 38{,}000$.

From these tests we may draw the conclusion that for ends fairly fixed the strength is well represented by the general parabolic formula of Art. 42, eqs. (22) and (25):

* From Trans. Am. Soc. C. E., Vol. 66, 1910, p. 401, the following: 5 tests by Phoenix Iron Co. on Phoenix sections, 1873; 11 tests by Bouscaren on built columns; 20 tests on built columns by the Watertown Arsenal, 1881–84; 22 by Clark Reeves & Co. on Phoenix sections; 15 by Strobel on built sections; and 34 by Watertown Arsenal on steel tubing, 1908. Also 21 by Watertown Arsenal on mild steel I-sections and 4 on channel sections, 1909; and 18 by U. S. Bureau of Standards on channel sections of the Am. Ry. Eng. Assn., 1914.

$$p = f - \frac{f^2}{100\,E}\left(\frac{l}{r}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

This is equivalent to using 80 per cent of the length and applying the formula for hinged ends, or 64 per cent of the length and using the formula for pivoted ends. The theoretical relation between fixed and pivoted ends calls for a ratio of 50 per cent instead of 64 per cent, which shows the difficulty of so truly fixing the ends as to satisfy the theoretical requirements.

49. **Effect of Secondary Stresses on the Bending Strength of Columns.**—Where the end connections of a compression member in a frame-work are of a rigid or inflexible nature, as in the case of riveted joints, bending stresses due to such rigidity (secondary stresses) will often be large and will have a considerable effect upon the strength of the strut. Whatever these secondary bending moments may be they have the same effect on the stresses in a column as a certain eccentricity of axial load. Thus if the bending moment at the end is M and the axial load is P, the combined effect is the same as would be produced by the load P applied with an eccentricity $e = M/P$.

It is convenient in discussing secondary stresses to measure them in terms of the principal or primary stresses, since the ratio of secondary stress to primary stress is fairly well defined for any particular type and proportions used. Thus it is shown in Part II that in trusses of ordinary form and proportion the secondary fibre stress is likely to reach 30 to 40 per cent of the primary unit stress. It will be useful, therefore, to find the relation between this ratio of secondary to primary stress and the corresponding eccentricity of load P which will give the same bending moment.

If p = primary stress per unit area = P/A;
f' = secondary stress on extreme fibre;
α = secondary stress ratio = f'/p;
c = distance from centre of gravity to extreme fibre;
e = eccentricity;
A = area, r = radius of gyration of section;

Then
$$f' = \frac{M\,c}{I} = \frac{M\,c}{A\,r^2}.$$

But $M = P\,e$, therefore,

$$f' = \frac{p\,A\,e\,c}{A\,r^2} = p.\frac{e\,c}{r^2} \quad \ldots\ldots\ldots \quad (35)$$

Hence

$$\alpha = \frac{f'}{p} = \frac{e\,c}{r^2} \quad \ldots\ldots\ldots\ldots \quad (36)$$

That is, the *ratio* of secondary to primary unit stress is the same as the eccentric ratio $\frac{e\,c}{r^2}$ for the axial load which will produce the same bending moment.

According to circumstances, the secondary bending moments at the two ends of a compression member may be of such sign as to cause the member to bend in a single curve, Fig. 13 (a), or they may be such as to cause the member to bend in an S-shaped curve, Fig. (b), with an intermediate point of inflection. In the first case the result is equivalent to eccentricities of load in the same direction at the two ends, and in the latter case to eccentricities in opposite directions. In the first case the central deflection of the column is considerably increased by such eccentricity and its resistance to buckling decreased, and in the second case the tendency is to force the column to deflect in double curvature, thus actually decreasing the stress and deflection near the centre and reducing the tendency to failure by buckling. So far as buckling is concerned it is more important, therefore, to investigate the effect of secondary stresses where they are equivalent to eccentricities of the same sign at the two ends. The general formulas for this case have already been developed in Art. 35.

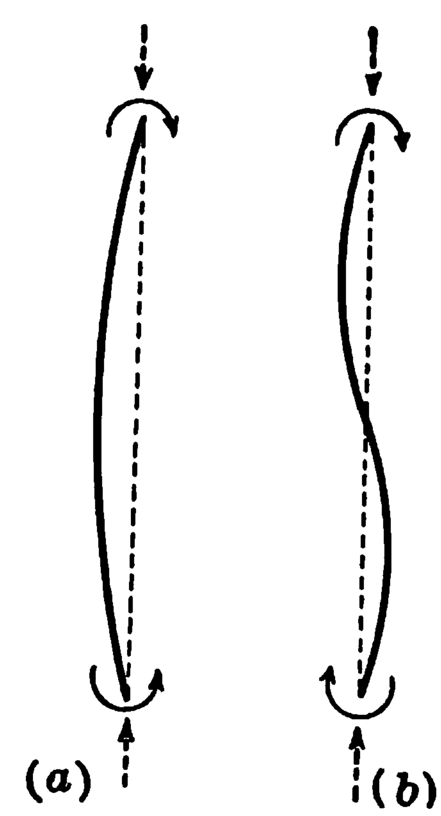


FIG. 13.

For the purposes of this article the strength of a round-ended column without secondary stress or intentional eccentricity may be represented by eq. (15) of Art. 35:

$$p = \frac{f}{1 + \frac{e\,c}{r^2}\sec\frac{l}{2}\sqrt{\frac{P}{EI}}} \quad \ldots\ldots\ldots \quad (37)$$

in which $\frac{e\,c}{r^2}$ is taken at 0.001 $\frac{l}{r}$ (see Figs. 8 and 9). The effect of *additional* eccentricities can then be readily determined by using other values for the eccentric ratio $\frac{e\,c}{r^2}$. Thus, for a column with 10 per cent secondary stress the value of $\frac{e\,c}{r^2}$ becomes $0.10 + 0.001\,\frac{l}{r}$; for 20 per cent secondary stress it is $0.20 + 0.001\,\frac{l}{r}$, etc. Calculations have been made of the ultimate strength of columns (value of p) for various values of l/r and for various ratios of secondary stress of

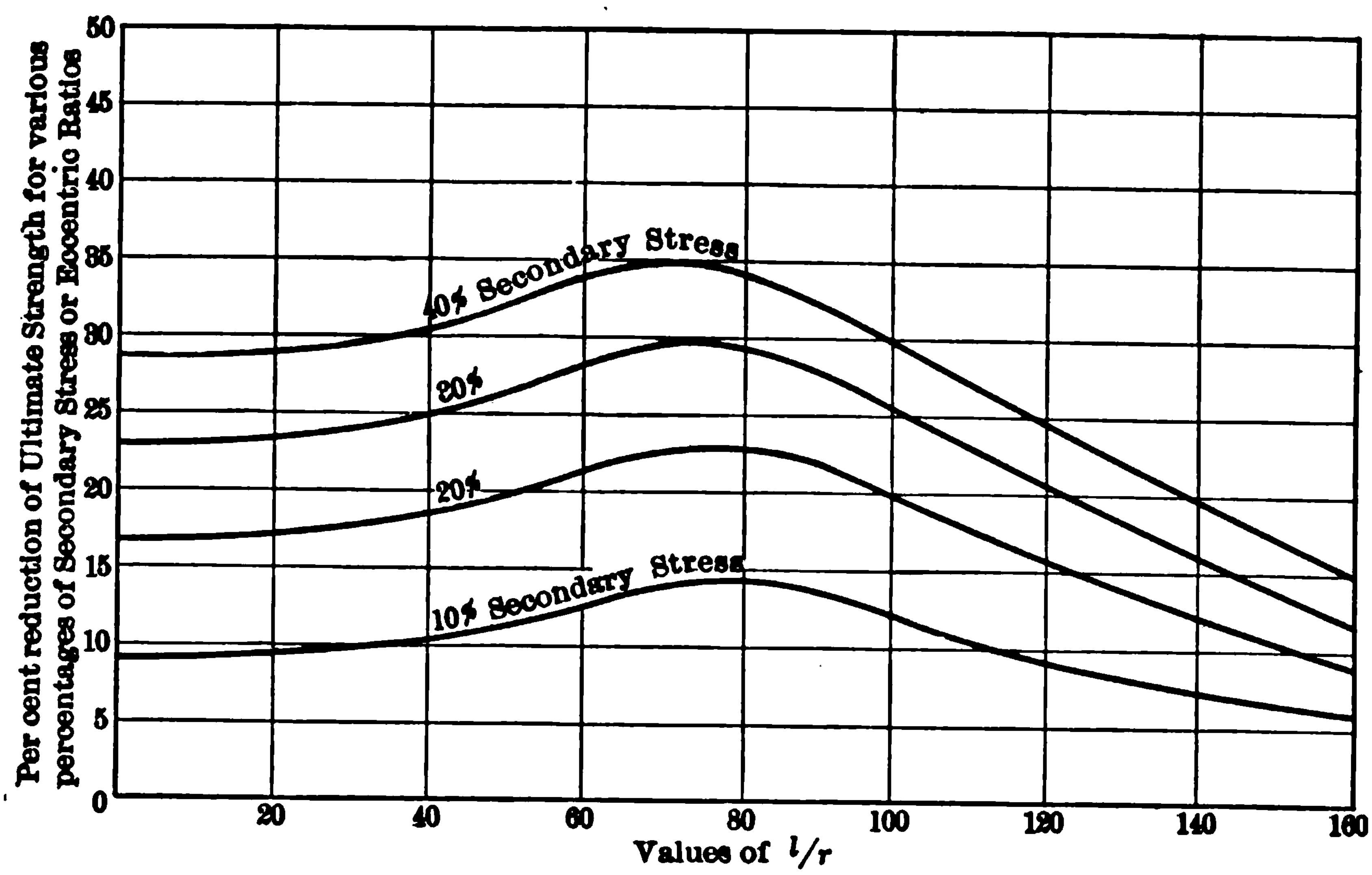


FIG. 14. Effect of Secondary Stress or Eccentric Loads on Columns.

10, 20, 30 and 40 per cent, taking $f = 36{,}000$ lbs. per sq. in. The results of these calculations are shown in Fig. 14 in terms of the per cent *reduction* in ultimate strength due to the various ratios of secondary stress, assuming the same value for the ultimate compressive fibre strength f. Thus, for $l/r = 0$, 10 per cent secondary stress gives a total fibre stress of $1.10\,p$, and hence in this case $p = f/1.10 = 0.91\,f$, a reduction in strength of 9 per cent; for 20 per cent secondary

stress and $l/r = 0$ the reduction is $16\frac{2}{3}$ per cent. For $l/r = 80$, the column strength without secondary stress is 28,600 lbs. per sq. in.; with 10 per cent secondary stress it is 24,600 lbs. per sq. in., a reduction of 14 per cent; with 20 per cent secondary stress it is about 22,100 lbs. per sq. in., or 22.7 per cent less, etc.

The curves of Fig. 14 show that the greatest *proportionate* reduction in column strength occurs for a value of l/r equal to from 70 to 80, and that for high values of l/r the proportionate reduction in strength becomes even less than for $l/r = 0$. We find, for example, that for $l/r = 75$ and $\alpha = 30$ per cent the reduction in strength is 29.5 per cent, which is the same as would occur in a very short column, or in a tension member, for $\alpha = 42$ per cent. For $l/r = 120$ the effect is somewhat less than for $l/r = 0$.

We may therefore conclude that eccentric, or secondary bending stresses, acting in the same direction at both ends, have a greater relative effect in compression members than in tension members, and that the maximum effect is for values of l/r of about 75. Bending stresses are also more serious in compression members than in tension members for the reason that as the elastic limit is approached their effect tends to be increased rather than decreased, as shown in Art. 30, so that the margin of safety beyond the elastic limit is very slight. For the same degree of safety against ultimate failure the unit fibre stress to be used in compression members should therefore be considerably less than in tension members. In drawing conclusions from this analysis it should be observed that for slender members the actual secondary stresses are not apt to be high; and, furthermore, very high values of secondary stress occur only for the case of bending moments of opposite sign at the two ends, thus bending the member in a reversed curve which brings the maximum fibre stress near the end and gives practically the same effect as in a tension member.

Another interesting result of this analysis is that very long, slender members ($l/r = 150$, for example) are less affected, relatively, by eccentricities of load than shorter columns. This accords with the results of tests on very long columns which show that their strength, while comparatively low, can be more accurately represented by formula than that of the shorter lengths. So far as strength under static loads is concerned, the longer column is, in fact, the more

reliable. The objection to the use of the slender column is by reason of its tendency to vibrate unduly under moving loads and its liability to injury in transportation and from accidents otherwise of minor importance.

50. Shear in Columns.—In the design of built-up columns the problem is to secure a column which will be economical and to so connect the several component parts that they will act together as

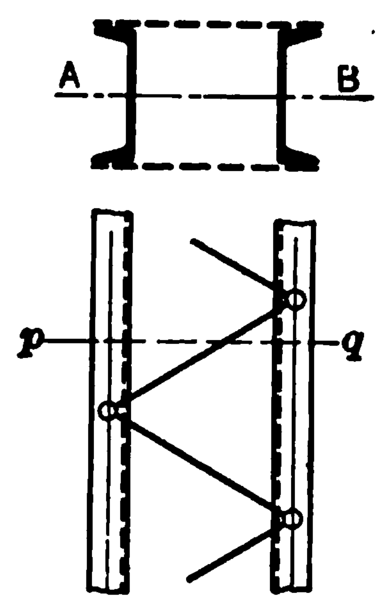


FIG. 15.

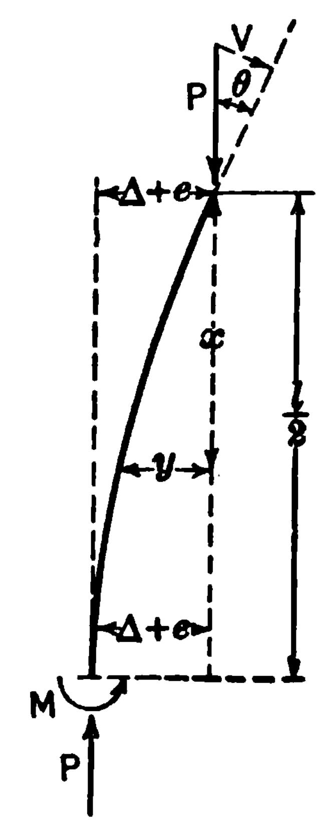


FIG. 16.

a unit. By reason of the effect of the slenderness ratio l/r on the strength of a column, the sum of the strengths of the several parts acting singly is much less than the strength of the entire column taken as a unit. To insure unit action the connections must not only have the necessary strength but must be *stiff*, as *stiffness* in a column is an important factor in its strength. In other words, the requisite resistance to the load must be developed quickly and with a slight amount of deformation.

Assuming a column to be made up of two or more component rolled or built-up forms, connected by transverse lacing or plates (Fig. 15), the problem is to determine the stresses in such transverse elements. It is assumed that the bending of the column will occur in the plane A B, parallel to the planes of such connecting elements. The principal stress in these connecting details at any section will then be due to the shearing stress in the column at the section in question.

There are two quite distinctly different conditions under which this shearing stress may be relatively large. First, the case of the comparatively long column, Fig. 13 (*a*), bent in single curvature, so that before failure occurs the angle of deflection is considerable; and second, the relatively short column, Fig. 13 (*b*), bent in double curvature by reason of the load being applied with opposite eccentricities at the two ends, or by reason of secondary bending moments producing the same effect.

51. (1) *Shear Arising from Large Deflections (Single Curvature).*— In analyzing this case we may consider the load applied centrally, or with eccentricities of like direction, so that the column is bent in single curvature. Fig. 16 represents one-half of the column. For the present purpose it will be sufficiently accurate, and on the safe side, to assume that any eccentricity of load, e, is due to, or equivalent to, an initial bend of like amount in the column, and that the curvature of the bent column is similar to the curvature produced by the load, that is to say, is a sinusoidal curve. The equation of the curve of the loaded column is then

$$y = (\Delta + e) \sin q x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where e is the original deflection or initial bend, Δ the deflection due to the load, and $q = \sqrt{\frac{P}{EI}}$ as in Art. (35). The inclination of the axis at any point is $\tan \theta = \frac{dy}{dx} = q (\Delta + e) \cos q x$. This is a maximum at the end where $x = 0$, and is equal to

$$\begin{aligned} \tan \theta &= q (\Delta + e) \\ &= (\Delta + e) \sqrt{\frac{P}{EI}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39) \end{aligned}$$

Finally, the shear is $V = P \sin \theta$, but as θ is a small angle we may write

$$V = P \tan \theta = P (\Delta + e) \sqrt{\frac{P}{EI}} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The value of $\Delta + e$ corresponding to the *ultimate strength of* the column

is found by placing the maximum fibre stress due to combined compression and bending equal to the ultimate strength of the material (yield point f). This maximum stress is, as given in Art. 35,

$$f = \frac{P}{A} + \frac{M\,c}{I} = p\left(1 + \frac{(\Delta + e)\,c}{r^2}\right). \quad . \quad . \quad . \quad . \quad (41)$$

Solved for $\Delta + e$ we have

$$\Delta + e = \frac{f - p}{p} \cdot \frac{r^2}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Substituting in (40) above, we have the formula for shear

$$V = A\,\frac{r}{c}\left(f - p\right)\sqrt{\frac{p}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

This is to be interpreted as the maximum shear near the end of a column bent in single curvature, and loaded with its ultimate load, producing the maximum fibre stress f and the average stress p.

From eq. (43) it is seen that the average shearing stress per square inch, V/A, depends upon the form of the column as represented by r/c, and upon the values of f and p. Taking $f = 36{,}000$ and calculating p by the parabolic formula, $p = 36{,}000 - 0.70\left(\frac{l}{r}\right)^2$ (Art. 43), and assuming $r/c = 0.80$, as an ordinary maximum value, there results the shearing stresses V/A as given in Fig. 17, curve $A\,B$. This diagram shows clearly the variation of shear with l/r. The maximum value of V of eq. (43) always occurs for $p = \frac{1}{3}f$ and is equal to

$$V_{max} = A\frac{r}{c} \cdot \frac{f^{\frac{3}{2}}}{14{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

For $r/c = 0.8$ and $f = 36{,}000$, we have

$$V_{max} = 390\,A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

as shown on the diagram. This, therefore, may be taken as an approximate formula for maximum shear. The corresponding value of l/r is 185.

Another method of estimating the shear is such as that specified in the specifications of the Am. Ry. Eng. Assn. These provide that

the shear shall be calculated as equal to that which would be produced by a uniformly distributed load, applied to the column as a beam, which would cause a bending fibre stress equal to that assumed by the column formula. Thus in the formula in those specifications, $p = 16{,}000 - 70\frac{l}{r}$, the term $70\frac{l}{r}$ represents what is meant in the specifications as the effect of bending and may be taken as the bending fibre stress under *safe loads*. Considering here the *ultimate* strength and corresponding shear, the bending stress corresponding to the above formula will be about $2\frac{3}{4} \times 70\frac{l}{r} = 192\frac{l}{r}$ lbs. per sq. in. (See Art. 43.)

The bending moment due to a uniformly distributed load w per ft. is $\frac{1}{8}\, w\, l^2$, hence we have

$$\frac{1}{8}\, w\, l^2 = \frac{192\frac{l}{r} \cdot I}{c}.$$

Solving for w, we get

$$w = \frac{1{,}536\, A\, r}{c\, l} \quad \ldots\ldots\ldots\ldots \quad (46)$$

The end shear is $V = \frac{w\, l}{2}$, therefore from (46) we get

$$V = 763\, A . \frac{r}{c} \quad \ldots\ldots\ldots\ldots \quad (47)$$

a value of V corresponding to the ultimate strength of the column. Placing $r/c = 0.8$, we have, approximately,

$$V = 610\, A \quad \ldots\ldots\ldots\ldots \quad (48)$$

This gives a constant value of shear for all lengths, and about 55 per cent greater than the maximum as given by eq. (45). (See Fig. 17.) This is in part due to the high value of the constant f assumed in the straight-line formula (about 44,000) and in part to the fact that a uniformly loaded beam gives a greater shear for a given centre moment than the sinusoidal column curve. The rule is therefore on the safe side for the case considered.

52. (2) *Shear in Short Columns Due to Opposite Eccentricities or Secondary Stresses.*—Suppose the load is applied with eccentricity e at each end (Fig. 17). The curvature will be double and the point of inflection will be at the centre. The deflection from the straight line at any point will be very small, and for this case the effect of curvature may be neglected. The shear at all sections then will be

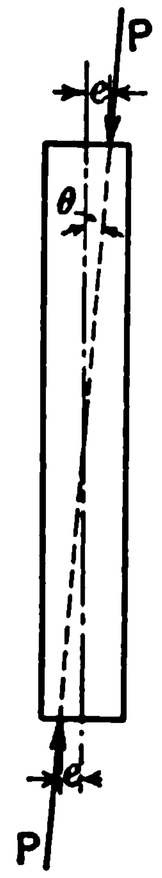


FIG. 17.

$$P \tan \theta = P \frac{2e}{l}, \text{ or}$$

$$V = pA\frac{2e}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

As a measure of the possible eccentricity e, the worst case is probably that of the continuous compression member of short panels, subjected to bending by the uneven deflections of the joints, producing large secondary stresses. Calculations by the authors show in some cases secondary stresses in the plane of the truss, due to this cause, as high as 100 per cent of the primary stress. They are seldom less than 20 per cent and commonly amount to from 30 to 40 per cent. In the transverse plane they are likely to be lower, but 30 per cent may be considered a minimum to be provided for.

Let α represent the ratio of secondary to primary stress in any case, then the equivalent eccentricity, as shown in Art. 49, is

$$e = \frac{\alpha r^2}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

The shear is then

$$V = \frac{2pA\alpha r^2}{cl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Taking $r/c = 0.8$, approximately, eq. (51) becomes

$$V = 1.6\,pA\frac{\alpha}{l/r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

The shear therefore increases in this case with α, or the secondary stress ratio, and also increases with a decrease in l/r. The shorter the column the greater will be the shearing stress, due to the causes here considered.

In Fig. 18 are shown values of V/A, or average unit shear (curves C), for various values of l/r and of α, using for p the values given by the parabolic formula $p = 36{,}000 - 0.70\,(l/r)^2$. From this diagram it is evident that the shears for this case are likely to be very high

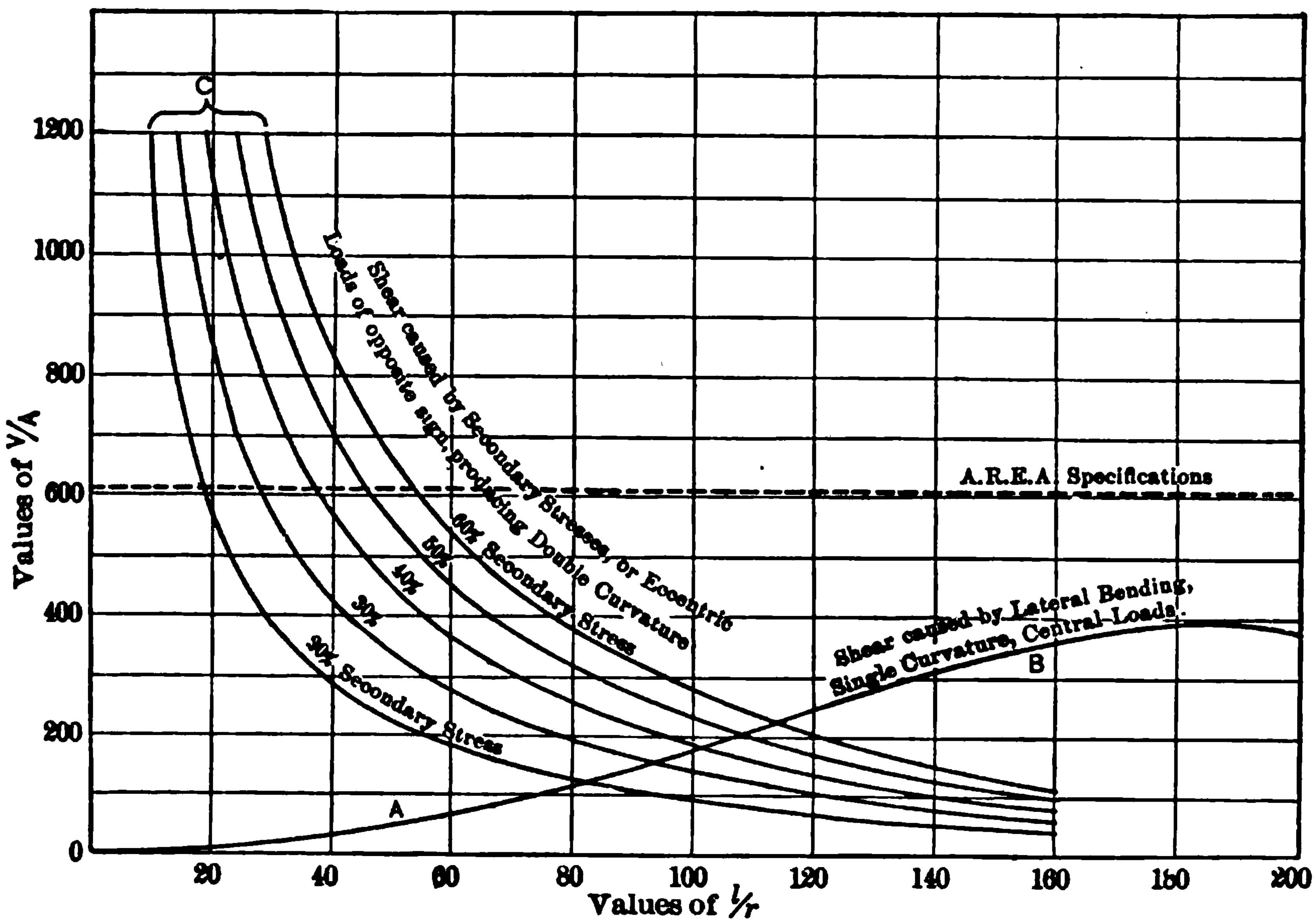


FIG. 18. Shearing Stresses in Columns at Ultimate Loads.

and much higher in short than in long columns. For a secondary stress of 50 per cent, for example, and $l/r = 25$, a not unusual condition with short panels, the average shearing unit stress $= V/A = 1{,}140$ lbs. per sq. in., or about double the value given by application of the Am. Ry. Eng. Assn. specifications.

The discussion in Chap. IV shows that secondary stresses increase in general with a decrease in the ratio of width of member to length, or in the value of l/r, the secondary stress ratio α reaching a value as high as 80 to 100 per cent for very short sub-panels where $l/r = 12$.* These values occur in a bending plane coinciding with

* See bulletin 163, p. 463, Am. Ry. Eng. Assn., for case of lower chord of $\alpha =$ 100 per cent at one end and 72 per cent at other, or average value of 86 per cent; $l/r = 12$, $r/c = 0.69$. If ultimate $p = 36{,}000$, then $V/A = 3{,}560$ lbs. per sq. in.

the plane of the truss; in a lateral direction they are much less, but the action of lateral bracing, especially if eccentric, causes very considerable secondary stresses also in this direction which may easily reach 30 to 40 per cent.

53. **Conclusions and Working Formulas.**—The preceding analysis has considered the shearing stresses corresponding to the ultimate strength of the column. Since good design requires that the details of a member should be made somewhat stronger than the body of the member, and since stiffness is especially necessary in column details in order to secure strength, it may be said in general that the details should be so proportioned that their stresses will not exceed their elastic limit under the conditions here assumed. Otherwise it is likely that the ultimate strength of the column will be measured by the strength of detail instead of by the strength of the entire combination acting as a unit, as assumed in the column analysis.

If the usual safe working stresses are applied to the connecting details, such as the lacing bars and the rivets, then the shearing stresses to be assumed may be taken at about one-half of the values given in the previous analysis. As a factor of $2\frac{1}{2}$ to $2\frac{3}{4}$ is used in the column formulas, this would give a somewhat higher factor of safety in shear than in compression. For the two cases here considered we may then give the approximate general *working formulas:*

For the longer columns ($l/r > 50$),

$$V = 250\,A \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (53)$$

For the shorter columns ($l/r < 50$), the shear due to bending in double curvature is likely to govern. Assuming the secondary stress ratio in a lateral plane to be 40 per cent, $r/c = 0.8$ and $p = 36{,}000$, we have approximately

$$V = \frac{12{,}000}{l/r} \cdot A \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (54)$$

or, more exactly, for various values of r/c,

$$V = \frac{15{,}000}{l/r} \cdot \frac{r}{c} \cdot A \quad . \; . \; . \; . \; . \; . \; . \quad (55)$$

Thus from eq. (54), for $l/r = 30$, $V = 400\,A$, etc.

In a vertical plane, or in the plane of the main truss, the shearing stresses for the shorter column lengths are likely to be double the values given by the preceding equations, as shown by the illustration in the preceding article. These stresses are so large in such cases that they can be provided for only by the use of solid web plates, as is the usual practice for such members. This is also likely to be true in a lateral direction, and recent designers of heavy compression members have recognized this by the use of at least one web plate in this plane. (See Art. 54.)

54. Forms of Compression Members.—In Fig. 19 are shown some of the more common forms of compression members used for framed structures. The factors which usually determine the form of a compression member are cheapness of manufacture, cost of material, the efficiency of the form as a column and its suitability as regards details. Sections (*a*) to (*f*) are commonly used for top chords and end posts. The component parts are connected together laterally by a cover plate which gives lateral stiffness and which is especially desirable in these members as they are relied upon to transfer wind stress to some extent by their resistance to bending. Eccentricity of section about a horizontal axis is reduced by using larger angles at the bottom, as in (*b*), or inside angles, or flats as in (*c*), to partly balance the cover plate. Sections (*g*) to (*k*) are the usual sections for web compression members, and are symmetrical in both planes. Section (*k*) does not make an efficient strut on account of its small value of r, and is used only for small stresses. Flanges turned in as in (*h*), rather than out as in (*g*), give a more convenient strut for joint details but are somewhat more inconvenient to rivet. For very heavy struts where a symmetrical member is desired, sections such as (*n*) and (*o*) are used. A web connecting the component parts, as in (*n*), is preferable to lacing only. For still larger members, one or more intermediate webs are used, as in (*e*) or (*f*), and frequently transverse diaphragms at occasional intervals to hold the parts in a truly rectangular form. For lateral struts carrying small stresses (*k*) and (*l*) are commonly used. Form (*m*) is convenient for chords of small highway bridges. Forms (*p*) and (*q*) are often used for viaduct columns.

The economy of area of a compression member depends mainly upon its radius of gyration as compared to its area. For the same

area the greater the value of r, within certain limits, the stronger the member. The material must not, however, be arranged in too thin section, otherwise local failure will occur. The relative economy

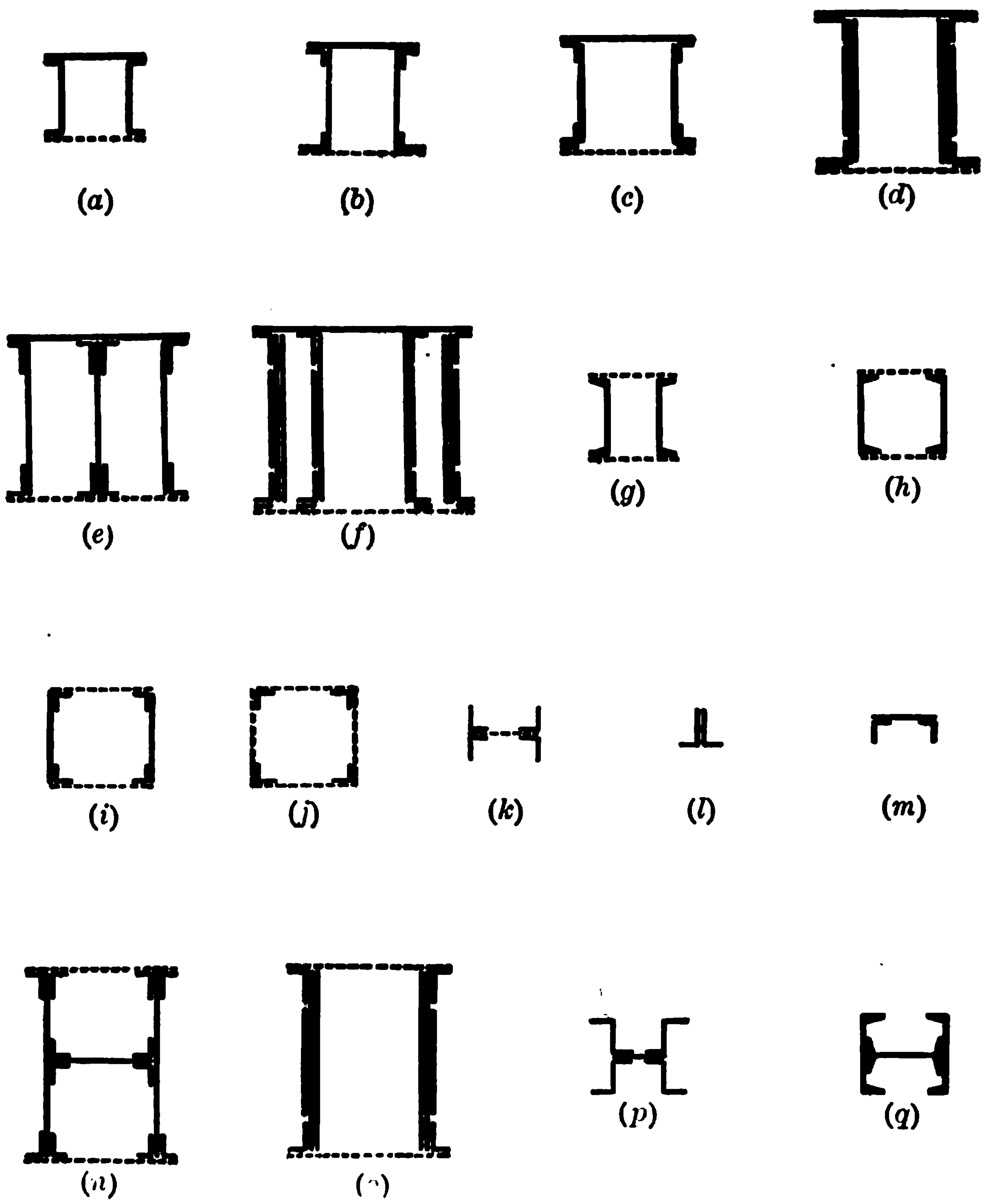
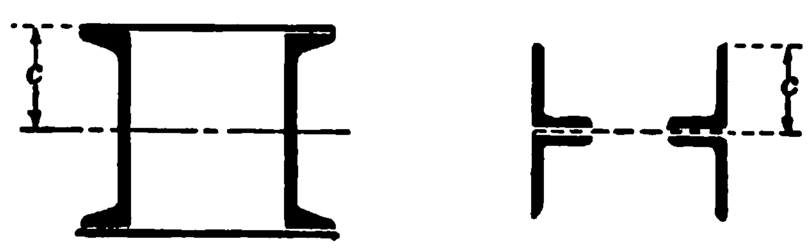


FIG. 19. Forms of Compression Members.

of the sections of Fig. 19 is approximately measured by the ratio of radius of gyration to the height (h) of the member. In forms (a) to (f) and (j) this ratio is about $\frac{4}{10}$; in forms (g), (h) and (i) about $\frac{3}{8}$; in (o) about $0.35\,h$; in (k) about $0.2\,h$; and in (l), (m), (p) and (q) about

0.3 *h*. These approximate values are convenient to use in a preliminary determination of cross-section. The cost of manufacture is dependent mainly upon the number of rivets to be driven. On this account form (*k*) is relatively economical. Forms (*a*) to (*f*), and (*p*) and (*q*) are economical in the small amount of lacing or non-effective material used.*

55. Effect of Form of Section on Column Strength.—The usual column formula involves only the ratio l/r as a variable and, therefore, gives the same results for all forms of section, provided the value of r is not changed. This relation is probably not exact, as is indicated by a study of all the elements involved. Consider the straight-line formula $p = f - k\dfrac{l}{r}$. In this formula, f is the ultimate strength of the short column, approximately the yield point of the material. The term $k\, l/r$ is the fibre stress due to bending. The total maximum



(*a*) (*b*)

FIG. 20.

fibre stress at the ultimate load is $f = p + k\, l/r$. Now the portion due to bending, $k\, l/r$, is a function of the bending moment produced and of the cross-section of the column. It is equal to $\dfrac{M\,c}{I} = \dfrac{M\,c}{A\,r^2}$, in which M = actual bending moment and c = distance to extreme fibre. Comparing two columns of the same values of l, A, and r, it may be assumed that the bending moments caused by eccentricity and other irregularities of the column are the same. Hence the fibre stresses due to bending in the columns will be proportional to c/r. A form of section in which the ratio c/r is small will, therefore, have less bending fibre stress than one in which this ratio is large. The extreme conditions are represented by the channel or box section, Fig. 20 (*a*), in which c/r is about 1.25, and the angle section, Fig. 20 (*b*), in which c/r is about 2.50, or twice as great as in the first case. As-

suming the same equivalent eccentricity in both cases the maximum fibre stress for any given load will be considerably greater in the latter form. An approximate estimate of this effect may be obtained by the application of the general formula for eccentrically loaded columns (Art. 35). An increase in the value of c, with no other change, will decrease the value of p.

For example, consider two sections of the same values of A and of I as shown in Fig. 21, one a box section composed of 6-in. channels and 3/8-in. plates and the other composed of four 5 x 3½-in. angles.

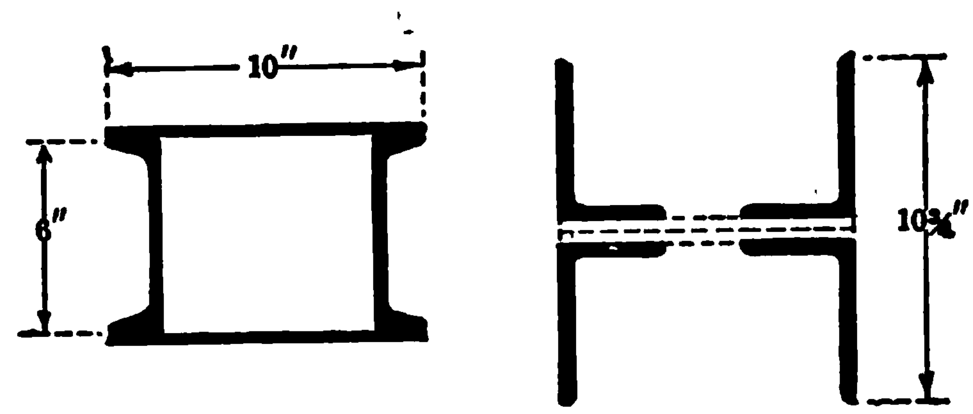


FIG. 21.

The value of r will be about the same in both cases, about 2.5. In the box section $c/r = 1.28$, and in the angle section $c/r = 2.15$. These two sections, having the same values of I, will be equally *stiff*, that is, will deflect the same amount for the same bending moment. The extreme fibre stress in the second case will, however, be $2.15/1.28 = 1.68$ times as great as in the first. Furthermore, having equal stiffness the equivalent eccentricities considered as columns will probably be about the same. Assume in both cases a length of 200 ins., and a value of eccentricity e of 0.15 in. Then $e\,c/r^2 = 0.075$ for the box section and 0.13 for the angle section. Taking $f = 36{,}000$, we get for the box section $p = 29{,}800$ lbs. per sq. in., and for the angle section $p = 27{,}200$ lbs. per sq. in., a difference of over 8 per cent. This difference is based only on the assumption that in the two columns the extent of irregularities and bends is the same, resulting in the same equivalent eccentricities. Inasmuch as both sections have the same stiffness, and the angle section is more liable to injury in handling owing to the wide outstanding angle legs, it would appear that the probable eccentricity would be fully as great in the angle as in the box section. This analysis takes no account of the tendency of the

outstanding legs to buckle or bend under compression, but assumes the stress on extreme fibre at failure to be the same in both cases. Any buckling tendency of the angle legs would cause a still greater relative difference in strength. It would appear therefore that, other things being equal (A, r and l), the strongest column is the one having the smallest width or value of c; that is, a box-shaped form will be stronger than one having outstanding webs or flanges. Extreme forms are a circular tube on the one hand, and an H section on the other.

56. Limiting Dimensions of Compression Members.—In designing the several elements that go to make up a built compression member and its connecting details, it is important that the strength of each element will be sufficient to develop the full assumed strength of the column as a whole. Now, in the analysis of a column and in the derivation of strength formulas it has been assumed that at ultimate load the maximum fibre stress in the column, due to direct stress and bending combined, is fully equal to the yield-point strength of the material. Hence, to develop this assumed strength, each element must be so designed as to have an individual ultimate strength not less than its yield-point strength. This requires the connections between the several elements to be so closely spaced as to practically eliminate long column action in the individual element, that is, to make l/r so small that the point of failure will not be less than the yield-point strength. Inspection of the diagrams in Figs. 11 and 12 indicates that if the value of l/r for any element is less than about 40 to 60, depending on end conditions, this result will be accomplished. It is not enough to require the l/r for the element to be less than the l/r for the entire column. This would provide only for a *uniformly distributed* stress on the column and not for the bending stress which occurs at ultimate load.

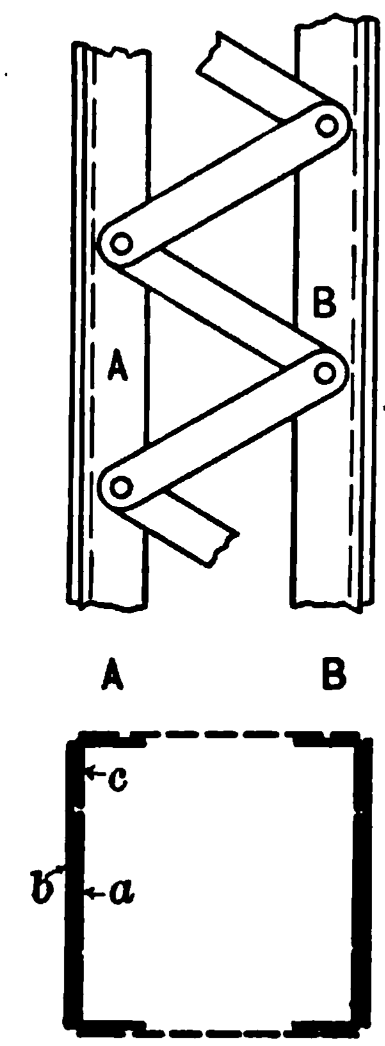


FIG. 22.

In a member like Fig. 22, each of the elements, a, b and c, acts as an independent column between rivets attaching them to another member, and the entire half A forms a column between lacing bars.

For a rectangular plate, $r = 0.29\,t$ where t = thickness. Hence if l' = rivet pitch, the value of l/r for a plate is $3.45\,l'/t$. Considered as a column such a plate is practically fixed ended, and the limit of l/r to meet the above requirements may be placed at 60, giving $3.45\,l'/t = 60$, or $l' = 17.4\,t$. A common rule is to limit the pitch in such cases to sixteen times the plate thickness, or $l' = 16\,t$, which agrees well with the above limit. The segment A may bend between lacing rivets in either direction and is, therefore, more nearly hinged ended. Its l/r should not exceed 30 or 40. The common rules for lacing will generally secure ample strength in this respect.

The unsupported width of plates and angle legs must also be limited to such proportions as will avoid local buckling. Thus in form b and g, Fig. 19, the outstanding legs of the angles may be so great as to allow them to buckle at the edges before reaching their full compressive strength. Also, if the width of plate between lines of rivets is too great local buckling may occur. The proper limits for the unsupported width of plates and angles can only be determined by experiment, but such experiments are as yet not adequate

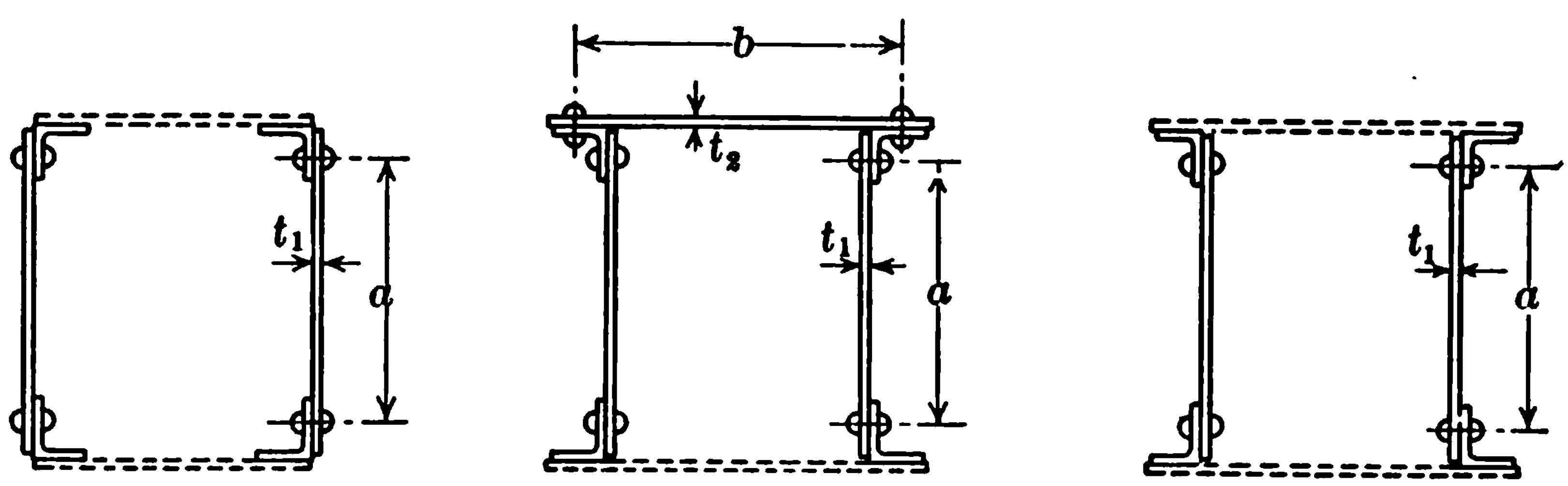


FIG. 23.

to enable definite rules to be accurately formulated. The common rules of practice are well indicated in the specifications of Appendix A.

Referring to Fig. 23 the common requirements are:

Thickness, t_1, of side plates or web should be not less than one-thirtieth the unsupported width a.

Thickness, t_2, of cover plate should be not less than one-fortieth the unsupported width b.

Width of unsupported outstanding legs of angles should not exceed twelve times their thickness, and maximum distance from edge

of plate to supporting rivets should not exceed eight times its thickness nor exceed 6 ins.

Maximum pitch of rivets between plates and shapes in the direction of the stress to be 6 ins. for 7/8-in. rivet and 5 ins. for 3/4-in. rivets. For plates in contact, maximum pitch in any direction to be 12 ins.

57. **Design of Latticing.**—The chief function of the lattice bars is to carry the shearing stresses as calculated in Arts. 22–24. For small columns the lattice bars are arranged in a single triangular system as in Fig. 24 (*a*). For columns exceeding about 18 ins. in width a double system is generally used, Fig. 24 (*b*); and for very large

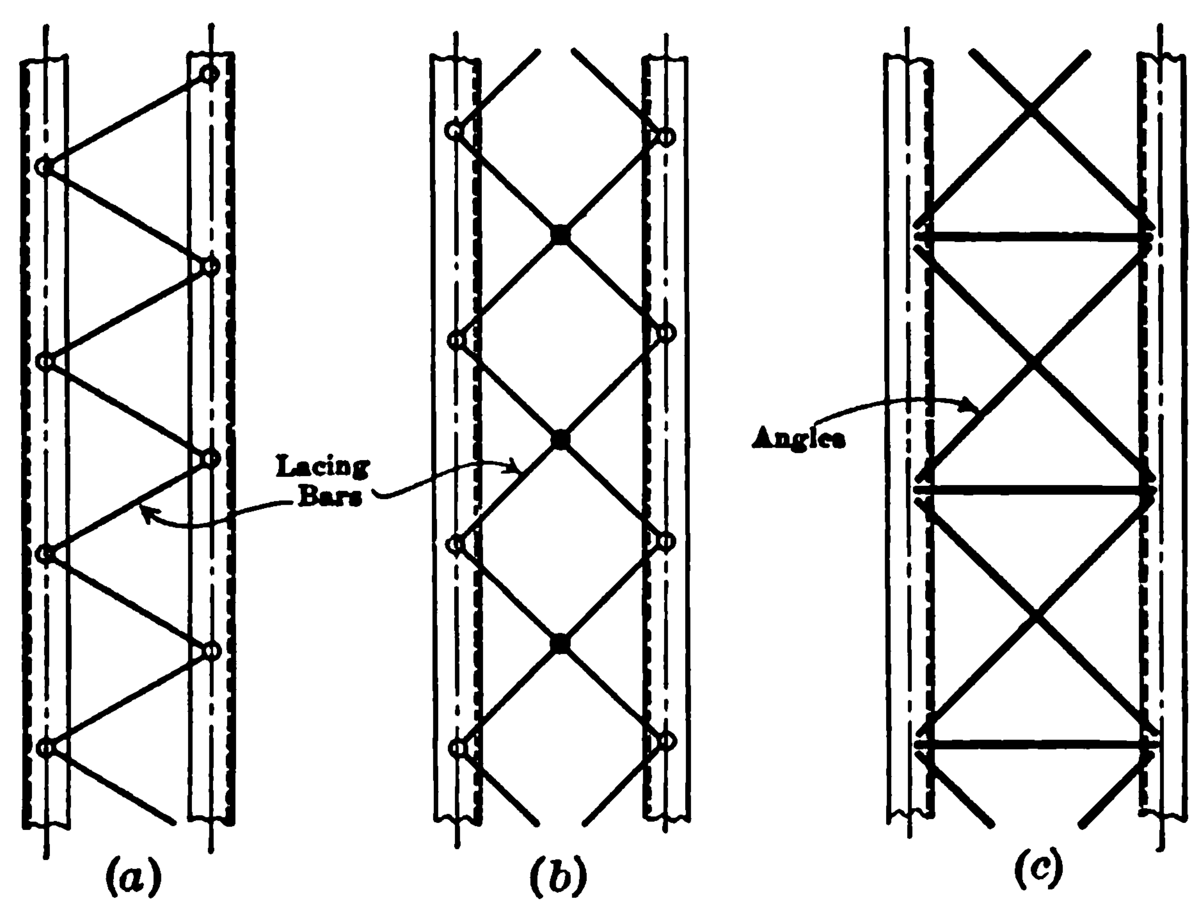


FIG. 24.

columns the lattice bars are often made of angles, or additional cross-bars of angles used as in Fig. 24 (*c*). The latter arrangement is advantageous in large members as tending to prevent certain bending or secondary effects otherwise caused by the lattice bars. (See Part II, Art. 348.) For very large members a solid web plate is desirable in lieu of or in addition to the lattice bars, as shown in Fig. 19 (*n*).

The required size and arrangement can be determined by estimating the shear as explained in Art. 24. As there stated, lattice bars should not be used on main members in the plane of the main truss, but web plates used instead.

Having the total shear V determined, the stresses in the lattice bars are found as follows:

1. *Column with two webs*, Fig. 25.—For a single row of bars on each side the stress in each bar is

$$S = \frac{V}{2} \cdot \frac{l'}{b} \quad \ldots \ldots \ldots \ldots \quad (56)$$

where S = stress;

l' = length of lattice bar;

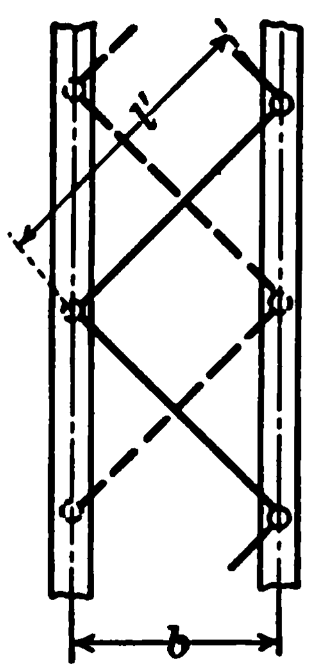


FIG. 25.

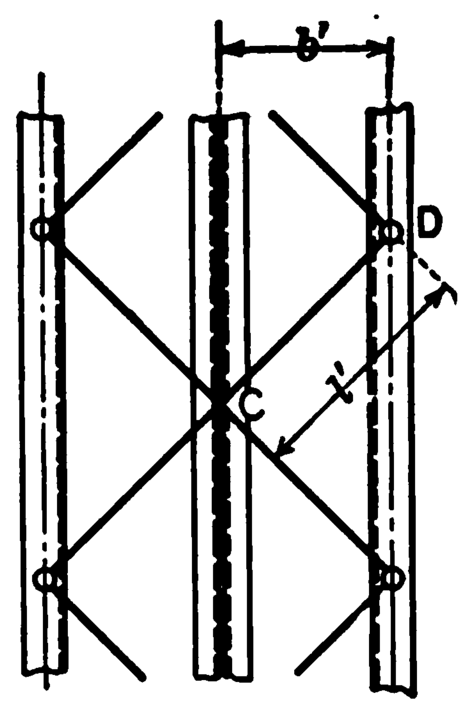


FIG. 26.

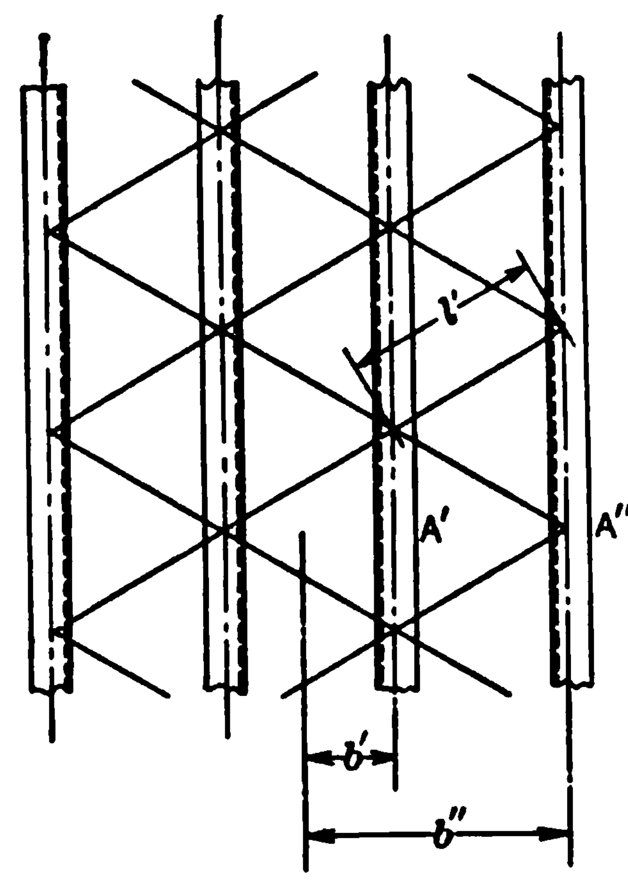


FIG. 27.

b = width between rivet lines.

For double lacing the stress in each bar is

$$S = \frac{V}{4} \cdot \frac{l'}{b} \quad \ldots \ldots \ldots \ldots \quad (57)$$

2. *Column with three webs*, Fig. 26.—The distribution of shear over the section is the same as in a beam. The intensity per lineal inch, longitudinal or lateral, at any point is given by the formula

$$v = \frac{V\,m}{I} \quad \ldots \ldots \ldots \ldots \quad (58)$$

where V = total shear;

m = statical moment of column section, outside of point in question, taken about its centre of gravity;

I = moment of inertia of column.

In the case of the three-rib column, Fig. 26, let A' = area of outer rib section, and b' = distance from its gravity centre to the column centre, then $m = A' b'$ and $v = \frac{V A' b'}{I}$. The total shear carried by a section of the lattice $C D$ is $\frac{v \times b'}{2}$, assuming similar latticing on both sides of the column. Hence the stress in a lattice bar is

$$S = \frac{v b'}{2} \times \frac{l'}{b'} = \frac{V A' l' b'}{2 I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

3. *Column with four webs*, Fig. 27.—The same general formula is to be applied as in the previous case. For the outer space $m = A'' b''$ and $v = \frac{V A'' b''}{I}$. The lacing stress is, as before,

$$S = \frac{v l'}{2} = \frac{V A'' l' b''}{2 I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

For the inner space the quantity $A'' b''$ is replaced by $A' b' + A'' b''$, giving

$$S = \frac{V l'}{2 I} (A' b' + A'' b'') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

(See specifications, Appendix A, for rules for size and spacing of latticing.)

58. Design of Tie-Plates.—At the extreme ends of latticed members the latticing is replaced by tie or batten plates of a length from one to one and one-half times the width of the member. The duty of such tie-plates is to take the shearing stresses, as in the case of the latticing, and in addition to hold each segment of the column in line and insure that the stress transmitted from the joint is uniformly distributed over each segment. The centres of gravity, a b, of each segment, Fig. 28, are not usually in line with the joint connection, whether riveted or pin-connected, thus causing a bending moment in each segment. This bending moment must be resisted by the tie-plate and the rivets connecting the tie-plate to the segments. The

tie-plates should be placed as near the end of the column as practicable. Where the segments extend considerably beyond the tie-plates, as often occurs in the case of pin-connected members, forming what are called "forked ends," the strength of the ends against bending

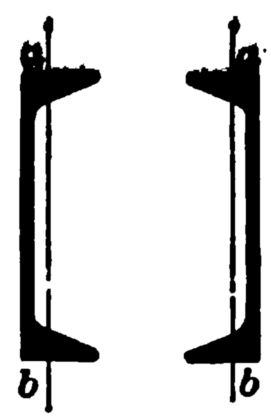


FIG. 28.

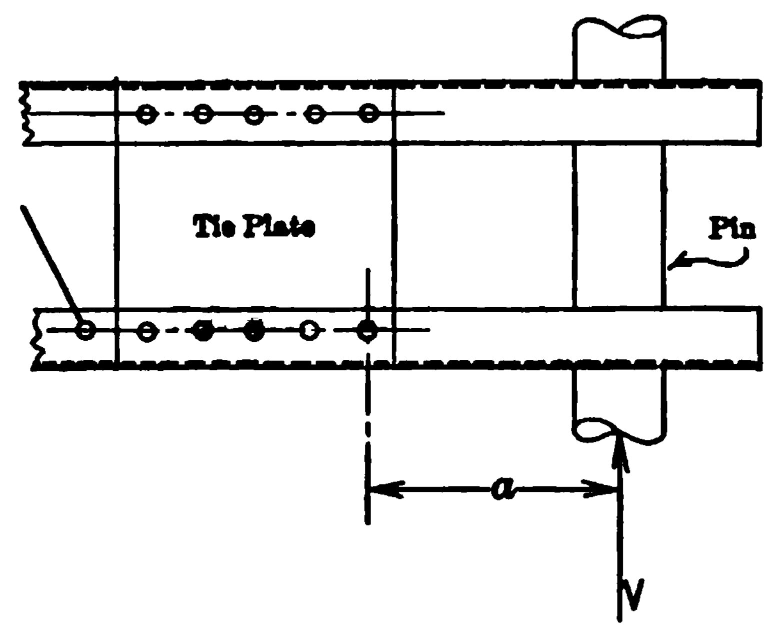


FIG. 29.

must be carefully considered. Assuming three-quarters of the load applied on one segment, this moment is equal to $\frac{3}{4}\ Va$ (Fig. 29), where V is the end shear, as calculated in Art. 53. The column strength of the single segment must also be considered in the same manner as in determining lattice pitch. Adequate strength can be furnished by reinforcing plates extending beyond the edge of the tie-plate.

CHAPTER IV

COMBINED DIRECT AND BENDING STRESSES—SECONDARY STRESSES

Combined Stresses

59. Examples of Combined Stresses.—Common examples of members subjected to both direct and bending stresses are: (1) members placed in a horizontal or inclined position, thus being subjected to cross-bending stress from their own weight; (2) chord members supporting transverse loads such as ties or joists of a floor system; (3) members in which the direct forces are applied eccentrically, due to eccentric pin-holes or joint connections, or where the load is applied on outstanding brackets, as in building construction; (4) many cases of bending due to the action of other members, which are generally included in the category of secondary stresses. Only the simpler cases of combined stresses will be considered here, and the methods used will generally be approximate. The more general cases and the more exact methods, as well as the general subject of secondary stresses, are treated at length in Part II.

60. Action of Direct and Bending Stresses.—A member subjected to transverse forces is caused to deflect a certain amount. If, at the same time, the member is subjected to a direct load, the deflection is increased if the load is compressive and decreased if the load is tensile, with corresponding effects on the bending moment. If the member is short and relatively stiff this bending effect of the direct stress is small and may be neglected; sufficiently exact results will be obtained by computing the stress due to the transverse load and that due to the direct stress separately and adding the results. If, however, the member is more slender, the effect of the direct stress on the bending moment cannot be neglected; the two effects must be calculated together. The case of the long column eccentrically loaded has already been treated in the preceding chapter. In the present chapter

approximate general formulas will be derived for both tension and compression members.

61. General Formulas for Combined Stress.—The following analysis differs from the more exact method given in Part II, in assuming the form of the deflection curve to be closely similar to that of a uniformly loaded beam, while the exact method uses the true elastic curve. It is further assumed in this article that the member is originally straight.

Let M' = bending moment at the centre, due to the transverse loads and any known eccentricity of end loads, but taking no account of deflection;

M = true bending moment;

f = fibre stress due to M;

S = total direct stress;

A = area of section;

c = distance of extreme fibre from gravity axis.

It is assumed that the centre is the point of maximum deflection.

1. *Hinged Ends.*—The deflection of a uniformly loaded beam, simply supported, in terms of maximum fibre stress is $y = \frac{5}{48}\frac{f l^2}{E c}$; and of a beam supporting a central concentrated load, $y = \frac{4}{48}\frac{f l^2}{E c}$. Adopting a coefficient of $\frac{1}{10}$ as sufficiently accurate for the purpose, we may say that, in general,

$$y = \frac{f l^2}{10\, E c} = \frac{M l^2}{10\, E I} \quad \ldots \ldots \ldots \quad (1)$$

Then the total moment M is

$$M = M' \pm S y \quad \ldots \ldots \ldots \ldots \quad (2)$$

the plus sign to be used for tension members and the minus sign for compression members. Substituting the value of y from (1) and solving, we have

$$M = \frac{M'}{1 \mp \frac{S l^2}{10\, E I}} \quad \ldots \ldots \ldots \quad (3)$$

The fibre stress due to M is

$$f = \frac{M\,c}{I} = \frac{M'c}{I \mp \dfrac{S\,l^2}{10\,E}} \quad \dots\dots\dots \quad (4)$$

In (3) the term $\dfrac{S\,l^2}{10\,E\,I}$ may be written $\dfrac{s}{10\,E}\left(\dfrac{l}{r}\right)^2$ where s = average unit stress, a form more convenient for compression members. This term takes account of the bending moment, and its relative influence can be seen by comparing its value with unity. For example, if $l/r = 40$ and $s = 12{,}000$, this term becomes equal to 0.062, showing an increase or decrease of about 6 per cent from deflection.

2. *Fixed Ends.*—If the member is continuous at the joints, or has rigid joint connections, it may be considered for this purpose as having fixed ends. The central deflection for a uniform load in that case is in terms of the centre moment, M_c, $y = \dfrac{M_c\,l^2}{16\,E\,I}$. The effect of the direct stress, S, upon the moments at both end and centre may be placed equal to $Sy/2$. Hence, if M' is the centre moment due to the transverse loads, neglecting deflection as before, we have

$$M_c = M' \pm \frac{S\,y}{2} = M' \pm \frac{S\,M_c\,l^2}{32\,E\,I} \quad \dots\dots \quad (5)$$

whence

$$M_c = \frac{M'}{1 \mp \dfrac{S\,l^2}{32\,E\,I}} \quad \dots\dots\dots \quad (6)$$

and the fibre stress at the centre is

$$f_c = \frac{M'\,c}{I \mp \dfrac{S\,l^2}{32\,E}} \quad \dots\dots\dots \quad (7)$$

If the fibre stress at the end is desired, then, in terms of the end moment, $y = \dfrac{M_e\,l^2}{64\,E\,I}$, and, as before,

$$M_e = \frac{M'}{1 \mp \frac{S l^2}{64 E I}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$f_e = \frac{M' c}{I \mp \frac{S l^2}{64 E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In (8) and (9) M' is the *end* moment in a fixed beam due to the transverse loads. For a uniform load, M' in (8) is equal to $\frac{w l^2}{24}$, and in (6) it is equal to $\frac{w l^2}{12}$, where w = load per unit length.

62. Bending Stresses in Eye-bars.—Considered as members hinged at their ends, the bending stresses in eye-bars due to their weight are considerable.

Let h = height of bar;
b = breadth;
s = working stress in tension;
S = total stress = $s\, b\, h$;
w = weight per inch = $0.28\, b\, h$.

Then applying eq. (4),

$$M' = \frac{w l^2}{8} = \frac{0.28\, b\, h\, l^2}{8}; \quad I = \frac{b h^3}{12}; \quad E = 29{,}000{,}000.$$

Substituting and reducing we get

$$f = \frac{5{,}000{,}000\, h}{s + 24{,}000{,}000 \left(\frac{h}{l}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From (10) it is seen that the bending fibre stress f depends upon the tensile working stress s and the height of the bar h. The higher the value of s the less the bending. As for h, there is a certain value for which the stress f is a maximum.

By differentiating (10) this value is found to be

$$h = \frac{l}{4{,}900}\sqrt{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and the corresponding value of maximum f is

$$f_{max} = 510\frac{l}{\sqrt{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The maximum stress therefore varies with the length l, and inversely with $\sqrt{s}$. The effect of depth of bar for different lengths, and the magnitude of the stresses are indicated in the following table. The values corresponding to a maximum stress are shown in bold-faced type.

BENDING STRESSES IN EYE-BARS DUE TO WEIGHT OF BAR

Tensile Working Stress = 16,000 lbs. per sq. in.

Length of Bar in Feet					
20		30		40	
Depth of Bar, Inches	Fibre Stress, Lbs. per Sq. In.	Depth of Bar, Inches	Fibre Stress, Lbs. per Sq. In.	Depth of Bar, Inches	Fibre Stress, Lbs. per Sq. In.
4	890	6	1320	8	1770
5	950	7	1400	9	1850
6	970	8	1440	10	1900
6.2	**970**	9	1450	12	1940
7	960	**9.3**	**1450**	**12.9**	**1940**
8	940	10	1450	14	1930
9	910	12	1410	16	1920

From the above table it is seen that the influence of depth is not great within the limits of ordinary practice; and that only for extreme lengths of 35 to 40 ft. does the stress reach 10 per cent of the assumed working stress. Such excess of stress requires no special provision in the design. For bars of less depth, such as square rods, the stress due to bending becomes quite small.

In this analysis the bars have been assumed as having hinged ends. In reality they are probably in a position of partially fixed-end members, resulting in a stress less than that given by this analysis.

63. Examples of Compression and Bending.—In the case of tension and bending the deflection is reduced by the direct forces, thus reducing the bending stress below its value for transverse forces only; in the case of compression and bending the reverse is the case, the direct

forces increase the deflections and bending. The magnitude of such stresses is indicated by the following examples. Hinged ends are assumed:

EXAMPLES. 1. Find the extreme fibre stress in bending, arising from its own weight and its compressive load, of a top chord section 30 ft. long made up as follows:

		Area
Cover plate	26 x ½ in.	13.0 sq. in.
4 angles	4 x 4 x ⅜ "	11.44 " "
2 side plates	21 x ½ "	21.00 " "
2 flats	5 x ⅜ "	7.50 " "
	Total,	52.94 " "

The direct compressive stress will be assumed to be 12,000 lbs. per sq. in. For this case the formula is

$$f = \frac{M' c}{I - \dfrac{S l^2}{10\, E}}$$

The value of I for this section is 4211, and c for the upper fibres is 10.0 in. Allowing for lacing, the weight of this section is about 200 lbs. per ft. and $M' = \frac{1}{8} \times 200 \times 30^2 \times 12 = 270{,}000$ in.-lbs. Taking $E = 29{,}000{,}000$, we have then

$$f = \frac{270{,}000 \times 10.0}{4211 - \dfrac{12{,}000 \times 52.9 \times 360^2}{10 \times 29{,}000{,}000}} = \frac{2{,}700{,}000}{4211 - 284} = 688 \text{ lbs. per sq. in.}$$

This is about 5.7 per cent of the direct stress. The effect of the deflection is shown in the term 284 in the denominator as compared to 4211. It results in increasing the stress about 7 per cent, or amounts to 0.4 per cent of the direct stress.

2. Suppose the pins are placed at the centre of the plates, and therefore 1.13 in. below the gravity axis. Find the combined effect of weight and eccentricity. The value of M' in the formula is the combined centre bending moment due to weight and eccentricity. The moment due to weight = 270,000 in.-lbs., and that due to eccentricity = $12{,}000 \times 52.9 \times 1.13 =$ 717,000 in.-lbs., of opposite sign. The resultant is 447,000 in.-lbs., negative moment. The maximum compressive fibre stress will be on the lower fibres, for which $c = 12.5$, hence

$$f = \frac{447{,}000 \times 12.5}{4211 - 284} = 1420 \text{ lbs. per sq. in.}$$

An eccentricity which would just balance the centre moment due to weight

would be equal to $\dfrac{270{,}000}{12{,}000 \times 52.9} = 0.425$ in.

The members in the foregoing example have been assumed pin-ended. The top chord of a Pratt truss is usually made pin-ended only at the hip joints, being essentially continuous at the other joints. The analysis of this case is given in the next article. Chord segments of curved chord pin-connected trusses are usually made pin-ended at all points, in which case the above analysis applies. The bending moments due to weight in such members can be well balanced by pin eccentricity.

A study of eq. (4), together with the column formula, shows that the greatest relative effect due to deflection occurs for l/r about 150, when the term $S\,l/10\,E\,I = 0.43$, thus acting to increase the moment and fibre stress about 75 per cent. For $l/r = 100$ the increase is about 50 per cent.

For the usual upper-chord section the effect of the deflection may be neglected; it is of consequence only in long slender members which may sometimes be used for web members or lateral struts.

The foregoing analysis takes no account of moment due to long-column action, but assumes members originally straight. Long-column action, combined with transverse loading, is considered in Art. 70.

64. Bending Moments in Pin-Connected Continuous Horizontal Top Chord.—The top chord of a Pratt truss is usually made with butt joints at all points except at the hip joint where the member is made pin-ended. At other joints the only stress transmitted through the pin is the chord increment coming from the diagonal at that joint. The entire chord member, besides acting as a series of columns, forms, therefore, substantially a continuous girder, and subjected to bending moments due to three causes:

1. Bending moments due to its weight and to other applied transverse loads;

2. Bending moments due to possible pin eccentricities, which causes the direct stress to be applied eccentrically; and

3. Bending moments due to the deflection of the truss as a whole.

In Part II, this problem has been treated as a problem in secondary stresses and a complete general solution given, to which the reader is referred for a study of the more exact methods of analysis. It will, however, be desirable to treat the problem here in an approximate way so that the relative effects of the various elements may

be appreciated. The three causes enumerated above will be considered separately.

The following notation will be used:

w = weight of member per unit length;
l = length of panel;
d = depth of member;
S = total direct stress in *end* segment of chord;
e = eccentricity of pins, assumed as uniform throughout and taken as positive when measured downwards from centre of gravity, the usual condition.

65. (*a*) *Bending Moments Due to Weight.*—The moments which are of greatest significance are those near the centres of the chord segments where the maximum buckling tendency occurs. For approximate results we may assume the weight as uniform and use the coefficients of $w\,l^2$ for uniformly loaded continuous girders, given on page 36 of Part II. From these values we get, approximately, for the centre bending moments:

$$\left.\begin{array}{ll}\text{For intermediate segments,} & M_c = 0.045\,w\,l^2 \\ \text{For end segments,} & M_c = 0.072\,w\,l^2\end{array}\right\} \quad \ldots\ldots \quad (11)$$

For less than five panels, the coefficient for an indeterminate segment becomes $0.036\,w\,l^2$ for four panels, and $0.025\,w\,l^2$ for three panels.

If we express w in terms of the area of cross-section and assume the distance from centre of gravity to the upper fibre to be equal to $0.4\,d$, there result the following approximate formulas for fibre stress f:

$$\left.\begin{array}{ll}\text{For intermediate segments,} & f = 0.03\,\dfrac{l^2}{d} \\ \text{For end segments,} & f = 0.05\,\dfrac{l^2}{d}\end{array}\right\} \quad \ldots\ldots\ldots \quad (12)$$

Thus if $l = 25$ ft. and $d = 20$ in., then for intermediate segments, $f = 0.03 \times \dfrac{300 \times 300}{20} = 135$ lbs. per sq. in. For end segments, $f = 225$ lbs. per sq. in. The bending moments at the intermediate joints are negative and equal to from $0.075\,w\,l^2$ to $0.105\,w\,l^2$.

Application of these formulas shows that the stresses in a continuous top chord caused by weight are small.

66. (*b*) *Bending Moments Due to Pin Eccentricity.*—The upper

chord of a seven-panel truss will be investigated for illustration. The bridge will be assumed as uniformly loaded and all pins eccentric by equal amounts. Fig. 1 shows a diagram of the top chord, with arrows indicating the moments M' and M'' introduced at joints 1, 2, 5 and 6, by reason of the pin eccentricity. At 1 and 6 the moment

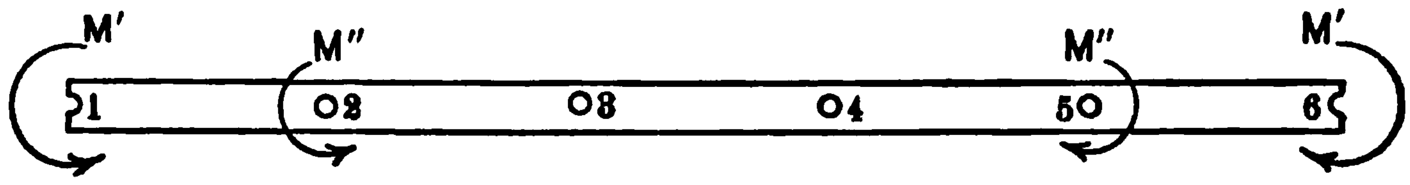


FIG. 1.

M' is equal to the total stress in the member multiplied by the eccentricity, $= S\,e$ (the end joint being fully pin-ended); at 2 and 5 the *difference* in chord stress is the only part transmitted through the pin. At 3 and 4 no additional stress is applied. For a seven-panel truss it can readily be shown that $M'' = 0.2\,M'$.

We will first consider the effect of the moments M'. Let M_1, M_2, M_3, etc., be the bending moments at 1, 2, 3, etc. From the three-moment equation, page 18, Part II,* there being no loads on any span, we have

$$M_1 + 4\,M_2 + M_3 = 0 \text{ and } M_2 + 4\,M_3 + M_4 = 0.$$

It is evident also that $M_1 = M_6 = -\,M'$; $M_2 = M_5$ and $M_3 = M_4$, whence we derive

$$M_3 = -\,{}^1\!/_{19}\,M' \text{ and } M_2 = +\,{}^5\!/_{19}\,M' \quad . \; . \; . \; . \; . \; . \quad (13)$$

To determine the effect of M'' by the use of the usual three-moment equation we may consider this moment to be caused by a couple $P \times 2a$, as shown in Fig. 3, the distance a being very small. Then $M'' = 2\;Pa$; $M_1 = M_6 = 0$; $M_2 = M_5$; $M_3 = M_4$. Applying the

* For concentrated loads the equation is $M_1\,l_1 + 2\,M_2\,(l_1 + l_2) + M_3\,l_2 =$

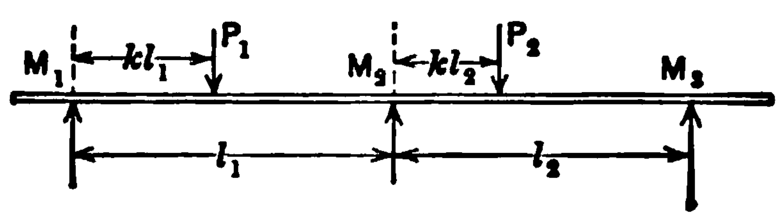


FIG. 2.

$-\,\Sigma\,P_1\,l_1^2\,(k\text{-}k^3) - \Sigma\,P_2\,l_2^2\,(2\,k - 3\,k^2 + k^3).$

equation to supports 1, 2 and 3, and noting that $k = a/l$ for span 2–3 and $\frac{l-a}{l}$ for span 1–2, we have $4 M_2 l + M_3 l = 0$. And applying it to 2, 3 and 4, we have $M_2 l + 4 M_3 l + M_4 l = P l^2 \left(\frac{a}{l} - \frac{a^3}{l^3}\right)$. As

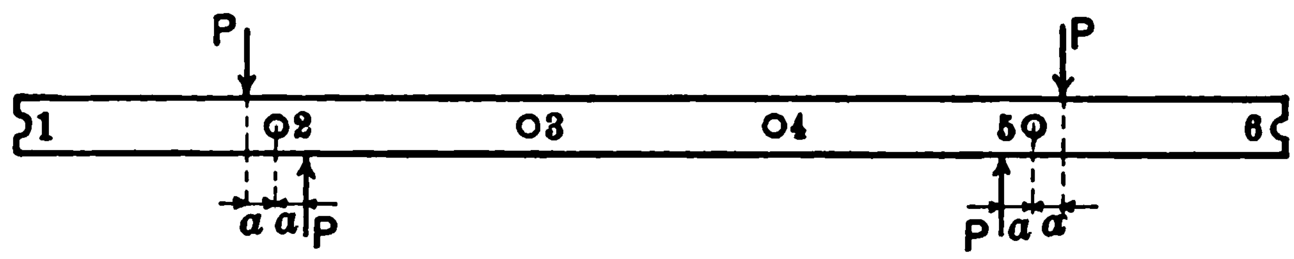


FIG. 3.

a is very small the second member becomes $P l a$ or $\frac{1}{2} M'' l$. We then derive from these two equations

$$M_2 = -\tfrac{1}{38} M'' \text{ and } M_3 = \tfrac{4}{38} M'' \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The maximum moment in span 1–2 occurs just to the left of joint 2 and is equal to $M_2 + P a = +\frac{9}{19} M''$. On the right the moment is $M_2 - P a = -\frac{10}{19} M''$. Combining these results with those of eq. (13), and substituting $S e$ for M' and $0.2\, S e$ for M'', we find the total bending moments to be as follows:

$$\left.\begin{array}{ll} \text{At joint 1} & M = -S e \\ \text{At left of joint 2} & M = +0.358\, S e \\ \text{At right of joint 2} & M = +0.158\, S e \\ \text{At joint 3} & M = +0.032\, S e \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The sign of the moment will be correctly given if e is taken as positive when the pin is below the gravity axis, the usual case. Positive bending moment corresponds to compression in the upper fibres.

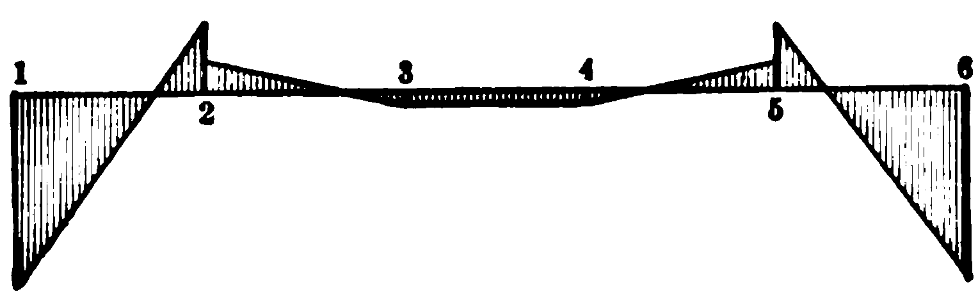


FIG. 4.

Fig. 4 shows to scale the bending moments throughout for the seven-panel truss.

In a similar manner the bending moments for a truss of any number of panels may be calculated. Calculations for trusses of 5, 6, 7 and 8 panels give results as follows:

Bending Moments Due to Pin Eccentricity.

S = Stress in end chord segment; e = eccentricity;
M_l = moment on left; M_r = moment on right of joint.

5-panel truss	End joint	$M = -Se$
	2d joint	$M = +0.20\,Se$
6-panel truss	End joint	$M = -Se$
	2d joint	$M_l = +0.34\,Se$
		$M_r = +0.22\,Se$
	3d joint	$M = -0.12\,Se$
7-panel truss	End joint	$M = -Se$
	2d joint	$M_l = +0.36\,Se$
		$M_r = +0.16\,Se$
	3d joint	$M = +0.03\,Se$
8-panel truss	End joint	$M = -Se$
	2d joint	$M_l = +0.37\,Se$
		$M_r = +0.12\,Se$
	3d joint	$M_l = +0.01\,Se$
		$M_r = -0.08\,Se$
	4th joint	$M = +0.02\,Se$

As in the case of the effect of weight, the significant bending moments are those which occur near the centres of the members. In the end segments the centre moment is equal to from $-0.32\,Se$ to $-0.40\,Se$, and at a point $3/8\,l$ from the end it is from $-0.50\,Se$ to $-0.53\,Se$. In the intermediate segments it is in all cases very small, not exceeding $0.07\,Se$ and therefore of no practical consequence.

67. (*c*) *Bending Moments Due to the Deflection of the Truss.*—If the deflection of the truss were due to bending stress alone, as is assumed in the case of beams, then the maximum bending stresses in the top chord due to this deflection would have the same ratio to the primary chord stresses as the depth of the top chord has to the depth of the truss.

The continuous top chord of a Pratt truss will necessarily deflect

practically the same amount as the truss itself, and the bending stresses in the chord will be proportional to this deflection and to the depth of the chord. If the effect of web stresses on deflection be neglected the total fibre stress at any point in the chord members would be proportional to the distance of the point from the neutral plane of the truss. Hence we may say that the bending stresses in the chord member itself (or excess of maximum fibre stress over stress at gravity axis) will bear the same ratio to the axial stress as the distance to extreme fibre in the chord bears to the distance from neutral plane of truss to chord axis. For symmetrical members and equal stresses in the top and bottom chord the bending fibre stress in the chord would be given by the equation

$$f = s.\frac{d}{h} \quad . \; . \; . \quad (16)$$

where s = unit tensile or compressive stress, d = depth of chord, and h = height of truss. For unsymmetrical conditions and unequal unit stresses the equation becomes

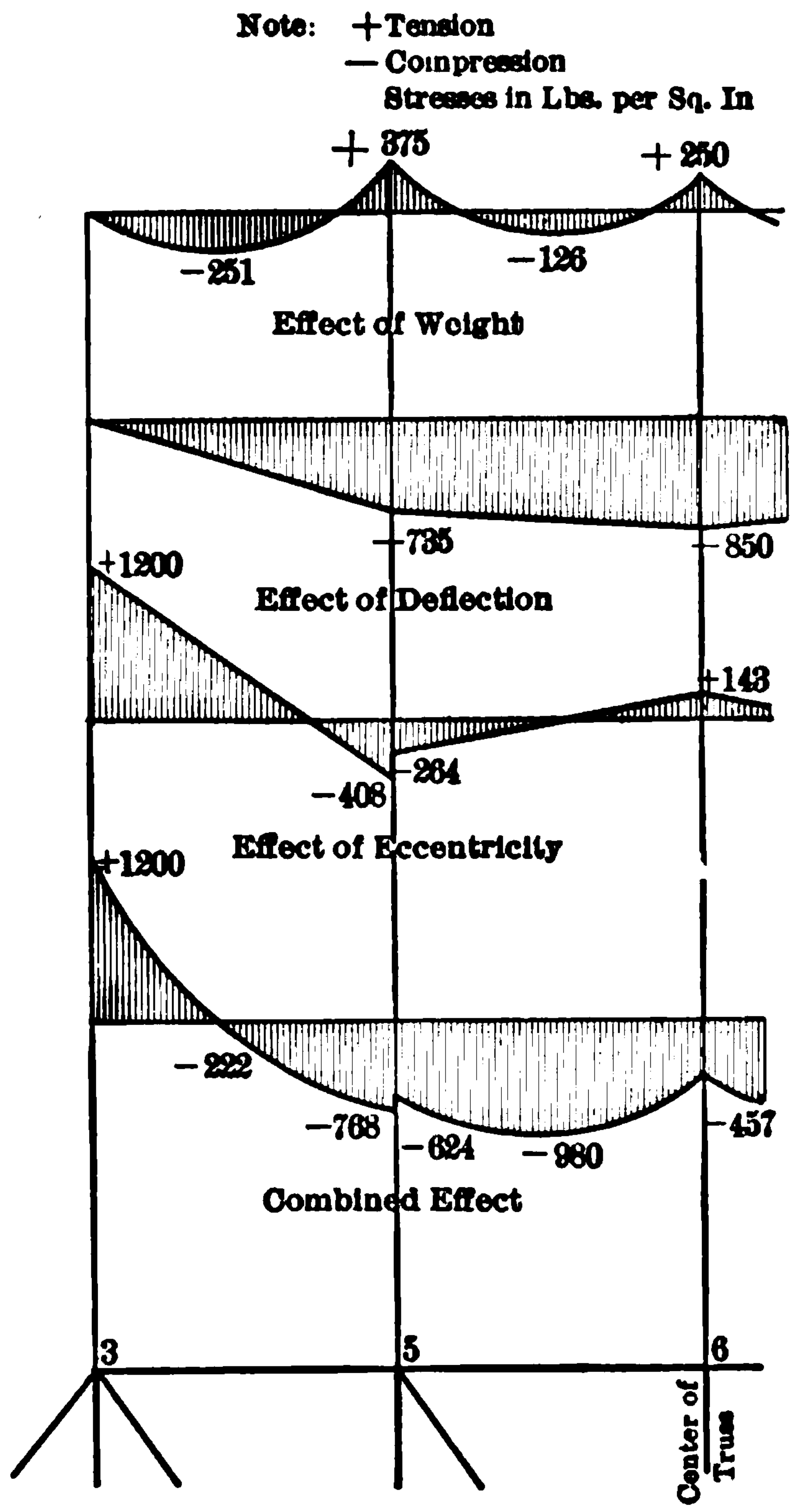


FIG. 5. Bending Stresses in Top Chord.

$$f = (s_u + s_l)\frac{c}{h} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

in which c = distance to extreme fibre whose stress is f, and s_u and s_l = unit stresses in upper and lower chord respectively, gross sections being taken.

In the above analysis the influence of the web members upon the deflection has been neglected. In Art. 217, Part I, it is shown that in the case of a Pratt truss the deflection due to web distortion is about

equal to that due to chord distortion. Hence we have, finally, as an approximate value for bending stresses in the top chord due to truss deflection:

$$f = 2\,(s_u + s_l)\,\frac{c}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

An estimate of the magnitude of these stresses can readily be obtained from eq. (18). For ordinary spans the ratio c/h will be about $1/40$ and s_u = about $0.8\,s_l$, from which we have approximately $f = 1/40 \times 2\,(2.25\,s_u) = 0.112\,s_u$, or about 11 per cent of the direct stress.

68. *Combined Effect of Weight, Eccentricity and Deflection.*—As an illustration of the relative magnitudes of the stresses here considered, there are shown in Fig. 5 the bending stresses in the top chord of a 6-panel Pratt truss due to each of the factors here considered, and the combined effect of all.* For the centre segment the effect of deflection is the only effect of consequence; in the end segment the eccentricity of pin is of some importance. The analysis in Part II, the results of which are given in Fig. 5, is made by a more exact method for the effect of weight and truss deflection than that given in the preceding articles. The student may compare results by the application of eq. (18), making $s_u = s_l = 10{,}000$ lbs. per sq. in.

69. *General Conclusions.*—From these calculations we may conclude:

1. That, except in the end segment, the moments due to pin eccentricity are relatively small and of little consequence.

2. That it is impossible in general to balance moments due to weight by making pins eccentric, but as such moment is small this is not necessary. If desired, the moment in the end segment may be balanced where it is a maximum, by using an eccentricity found by writing $0.35\,Se = 0.072\,w\,l^2$. This reduces approximately to $e = 0.06\,l^2/s$, where s = unit stress in member. For example, if $l = 300$ and $s = 9{,}000$, $e = 0.6$ in.

3. That a comparatively large eccentricity is unobjectionable except as it affects the end segment.

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* Analyzed in Part II, p. 470. Joint load assumed here 150,000 lbs. Span length, 160 ft.; height, 31 ft.; area chord, section 52.25 sq. in.; depth of chord, 23.62 in.; c to top fibre, 9.19 in.; amount of chord stress, S, in end segment = 516,000 lbs.

4. That, except in the end segment where the pin eccentricity is large, the maximum secondary stress in the top chord will not exceed about 12 to 15 per cent of the primary stress. (Compare with general discussion of secondary stresses in Art. 74.)

It should be observed that this discussion does not apply to chord members or end posts hinged at both ends. In this case the direct effect of weight and pin eccentricity can readily be calculated, and made to balance each other at the centre if desired.

70. Design of Columns Subjected also to Bending Stresses.—The foregoing articles show how to calculate the bending stresses in tension and compression members subjected also to transverse forces, on the assumption that the members were originally straight and the deflection due to long-column action was neglected. The results thus obtained are sufficiently close as regards the *additional* stress due to bending, but in a problem of design the question arises as to the proper method of applying the column formula, and, at the same time, taking into account the direct bending stresses. The rational method may be arrived at as follows:

The maximum fibre stress in a column whose average stress is $s = S/A$, is

$$f = \frac{S}{A} + \frac{S \Delta c}{A r^2} \quad \ldots \ldots \ldots \ldots \quad (19)$$

in which the term $\frac{S \Delta c}{A r^2}$ represents the stress due to flexure, and Δ represents the deflection of the column, or the eccentricity of line of pressure at the section in question (equivalent eccentricity of Art. 36). If this column is now subjected to the action of transverse forces it will then deflect an additional amount Δ'. Let M' = bending moment at centre produced by such transverse forces, then the total fibre stress is

$$f = \frac{S}{A} + \frac{S \Delta c}{A r^2} + \frac{(S \Delta' + M')c}{A r^2} \quad \ldots \ldots \quad (20)$$

The first two terms represent the effect of column action as in eq. (19) and the other term the direct and indirect effect of beam action as in eq. (2), Art. 61. Solving for the required area we have

$$A = \frac{S\left(1 + \frac{\Delta c}{r^2}\right)}{f} + (S\Delta' + M')\frac{c}{f r^2} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The first term is the area required by the column formula which may be used, and the second part is the area required for the bending moment $S\Delta' + M'$, or the total moment M, as given by eq. (2). We find, therefore, that *the rational process of design is to add to the area required by the column formula, the area required as a beam*, calculated from eq. (21), and using the usual value for the allowable stress f. The term $S\Delta'$ need rarely be taken into account, as shown in Art. 63. The value of r is evidently to be taken for the entire section.

In the foregoing it is assumed that the bending moment due to transverse load or known eccentricity is a maximum at or near the centre of the column. For the usual transverse loads this is sufficiently accurate. Eccentric application of direct stress may be, and often is, of opposite sign at the two ends, thus producing maximum moments at ends and small or no effect at the centre.

EXAMPLE.—Let it be required to design a top chord section 15 ft. long in which the direct compressive stress is 400,000 lbs. and which sustains a transverse load of 3,000 lbs. per ft. The stress to be used in direct compression is to be determined by the column formula $p = 16{,}000 - 70\, l/r$, but not to exceed 14,000 lbs. per sq. in.; the allowable fibre stress in bending is 16,000 lbs. per sq. in.

In eq. (21) c and r are unknown. It will be convenient to make a preliminary determination by assuming a depth of the member. Try 24 in. Then take approximately $c = r = 0.4$ (depth) $= 9.6$ in. $l/r = 180/9.6 = 18.8$. The bending moment $= 1/8 \times 3{,}000 \times 15^2 \times 12 = 1{,}010{,}000$ in.-lbs. The allowable column stress $= 16{,}000 - 70\, l/r = 16{,}000 - 70 \times 18.8 = 14{,}700$ lbs. The value of 14,000 must then be used. Then for the direct stress, $A = 400{,}000/14{,}000 = 28.5$ sq. in. For bending, $A = \frac{Mc}{fr^2} = \frac{1{,}010{,}000 \times 9.6}{16{,}000 \times 9.6^2} = 6.5$ sq. in. Total area required $= 35.0$ sq. in. This being a comparatively small area for a 24-in. depth, a 20-in. depth will be tried. Take $r = c = 8$ in. Then $l/r = 22.5$ and the column formula gives $p = 16{,}000 - 1{,}580 = 14{,}420$. Use 14,000. The area for compression will be equal to 28.5 sq. in. For bending, $A = \frac{1{,}010{,}000 \times 8}{16{,}000 \times 8^2} = 7.9$ sq. in. Total $= 36.4$ sq. in. The next step is to make up a section

in detail 20 in. deep and about 36.4 sq. in. in area, and determine exact values of c and r and recalculate the required areas. If the result does not check, the section will need to be revised and possibly a new depth chosen to get the most satisfactory result.

No account has been taken here of the effect of deflection. If this is calculated by the use of eq. (4) it will be found to be very small.

Secondary Stresses

71. Principal Classes of Secondary Stresses.—The general subject of secondary stresses, including detailed methods of calculation, is fully presented in Part II of this work, to which the reader is referred for details. It will be desirable to recapitulate here briefly some of the more important relations brought out in the detailed study, together with some results of calculations, in order to discuss intelligently the relative merits of designs with respect to this matter. It will be convenient to consider the secondary stresses under the following heads:

(1) Bending stresses in the plane of the main truss due to rigidity of joints.

(2) Bending stresses in members of a transverse frame due to the deflection of floor-beams.

(3) Stresses in a horizontal plane due to longitudinal deformation of chords, especially the stresses in floor-beams and connections.

(4) Variation of axial stress in different elements of a member.

72. Secondary Stresses in the Plane of the Truss due to Rigidity of Joints.—Where the members of a truss are rigidly connected by means of riveted joints, or continuity of construction, the longitudinal deformation of the members due to their primary stresses causes a certain amount of bending, thus giving rise to bending stresses which have a maximum value at or near the joints. These bending or secondary stresses are sometimes of large amount and require careful consideration. It is generally possible and sufficient so to design a structure as to keep these stresses within reasonable limits and then to neglect them in the calculations, but in many special cases, and in large and important structures, they should be calculated.

The detailed analysis in Part II leads to certain general results which are of assistance in determining upon a design without going

into the detailed calculations. The most important of these principles are as follows:

(1) The secondary stresses are in general proportional to the primary stresses, and, therefore, are conveniently expressed in percentage of primary stress.

(2) Other things being equal or similar, the percentages of secondary stress are proportional to the distances from gravity axis to outer fibre in the plane of bending, and inversely proportional to the lengths of the members. When the members are symmetrical the secondary stresses are proportional to the ratios of *widths* to lengths. Thus if two trusses are compared whose general dimensions and moments of inertia of members are proportional, but the ratio of width to length of the various members of one truss is in all cases twice this ratio in the other truss, then the percentages of the secondary stress in the first truss will be twice the percentages in the second truss.

(3) The more uniform the proportions of a truss the less, in general, will be the secondary stresses. Sudden changes in length, width, or in moment of inertia, are likely to result in relatively large secondary stresses.

(4) Trusses consisting of approximately equilateral triangles, and without hangers or vertical struts, present the most uniform conditions and will have, in general, the lowest secondary stresses. A truss composed of right-angle triangles will show somewhat higher secondary stresses, and such stresses will be large if the ratio of height to panel length is large.

(5) Wherever hangers or vertical struts are used to support single joint loads, as in a Warren girder with verticals, or in a Pratt truss, at the hip vertical (or at the centre vertical in the case of a deck bridge), the secondary stresses in the adjacent chord members are likely to be considerably larger than elsewhere. The best arrangement, so far as secondary stresses are concerned, is where each web member forms an integral part of the entire truss so that its stress will gradually change as the load progresses.

(6) From the fact that secondary stresses are, in general, proportional to the ratio of width to length of member, it follows that these stresses in the loaded chord of a truss with sub-panels are likely to

be relatively high, as such a chord will be relatively deep and the distortion of hanger or sub-strut relatively great.

(7) The secondary stresses due to eccentric connections are likely to be large at the joint in question, but this effect does not extend to a large degree to surrounding joints.

73. **Secondary Stresses in Typical Trusses.**—From results of calcutions of typical trusses of different forms, certain general conclusions may be drawn as to the amount of secondary stresses in trusses of usual forms and proportions. A considerable number of results of such calculations are given in the report of the Iron and Steel Committee of the Am. Ry. Eng. Assn. in Bul. No. 163, Jan., 1914, from which a number of the following illustrations are taken.

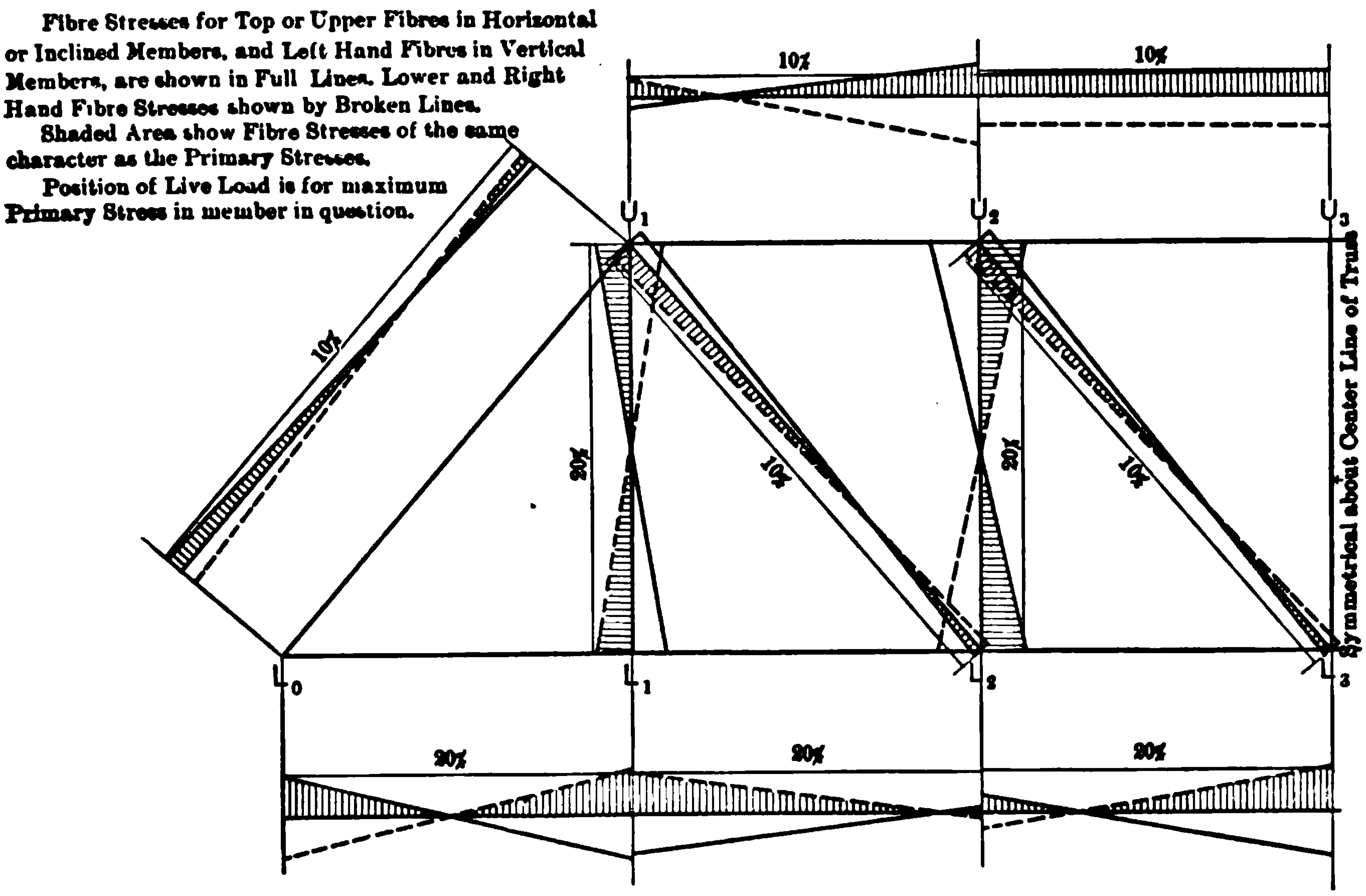


FIG. 6. Secondary Stresses in a Riveted Pratt Truss.
6 Panels at 26′ 8″ = 160′ 0″. Height = 31′ 0″.

74. *Single Intersection Pratt and Warren Trusses.*—Fig. 6 shows diagrammatically the amount of the secondary stresses in an ordinary Pratt truss of 150 ft. span length and 31 ft. depth. The results are shown as percentages of the primary stresses. Fig. 7 shows the direction of bending of the various members. Fig. 8 shows the results for

a pony riveted truss in which the relative width of the various members is considerably greater than in Fig. 6. The maximum values in the first case amount to about 20 per cent for tension members and about 15 per cent for top chord; and in the latter case about 40 per cent

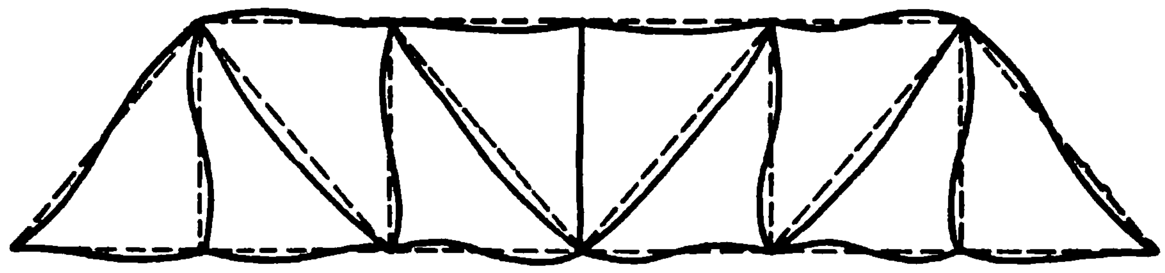


FIG. 7.

for tension members and 30 per cent for top chord. The difference is due to the greater ratio of width to length of members in the second

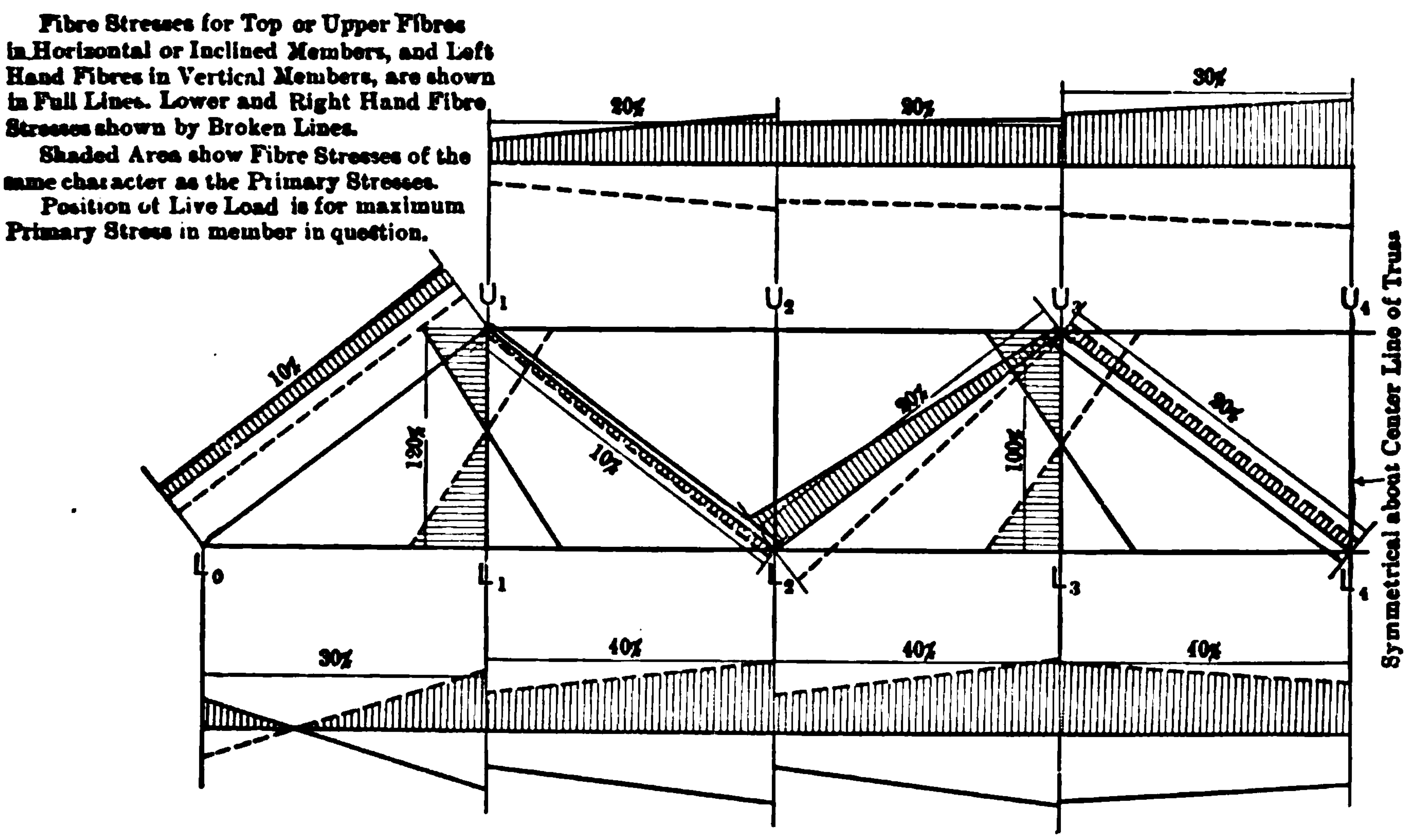


FIG. 8. Secondary Stresses in a Riveted Warren Truss.
8 Panels at 13′ 1½″ = 105′ 0″. Height = 10′ 0″.

design. In both cases the maximum percentage is, roughly, equal to $300 \times \left(\frac{\text{width}}{\text{length}}\right)$ for tension members, and $200 \times \left(\frac{\text{width}}{\text{length}}\right)$ for main compression members. Calculations of other Pratt and Warren truss designs indicate a somewhat greater value reaching 30 to 40

per cent and about 400 to 450 $\times \left(\frac{\text{width}}{\text{length}}\right)$.

The principal cause of secondary stress in the chord members of single intersection trusses is the deflection of the truss as a whole, thus causing a greater stress in the outer fibre than at the gravity axis, as in the case of any beam. The deflection of a truss will be about twice that of a beam for the same chord or flange stress (on account of web effect), hence the excess of stress on extreme fibre over the axial stress (the secondary stress) will be approximately twice the difference which would occur in a plate girder with flanges of the same form as the chord sections. This gives a secondary stress ratio due to this cause of

$$\alpha = 4\,c/h \quad \text{. . . . .} \quad (22)$$

where α = ratio of secondary to primary stress, c = distance from axis of chord to remote fibre, and h = depth of truss.

Applying this to the truss of Figs. 6 and 8 the value of c = about 9 ins. for both top and bottom chord. Then $\alpha = \frac{36}{31 \times 12} = 10$ per cent. In Fig. 8, c = 10.25 ins. for top chord and 9.25 ins. for bottom chord. Then $\alpha = \frac{4 \times 10.25}{10 \times 12} = 34$ per cent, and $\frac{4 \times 9.25}{10 \times 12} = 31$ per cent, respectively. The actual values vary considerably from these figures, due to other factors, especially the effect of vertical members. These results, however, are a good measure of the general average of stress and show well the fundamental cause of secondary stress in chords of well-designed trusses.

75. *Trusses with Subdivided Panels.*—Fig. 9 gives the results of calculations of a subdivided Pratt truss. The stresses in the lower chord and end post run to 50 and 60 per cent, equal to about 500 $\left(\frac{\text{width}}{\text{length}}\right)$. The high values are due partly to the very short panel length of the loaded chord, making the ratio of width to length large, and partly to the direct effect of the distortion of the suspenders. The top chord shows only the usual amount of stress. A subdivided double triangulation truss calculated in Bul. No. 163 showed results

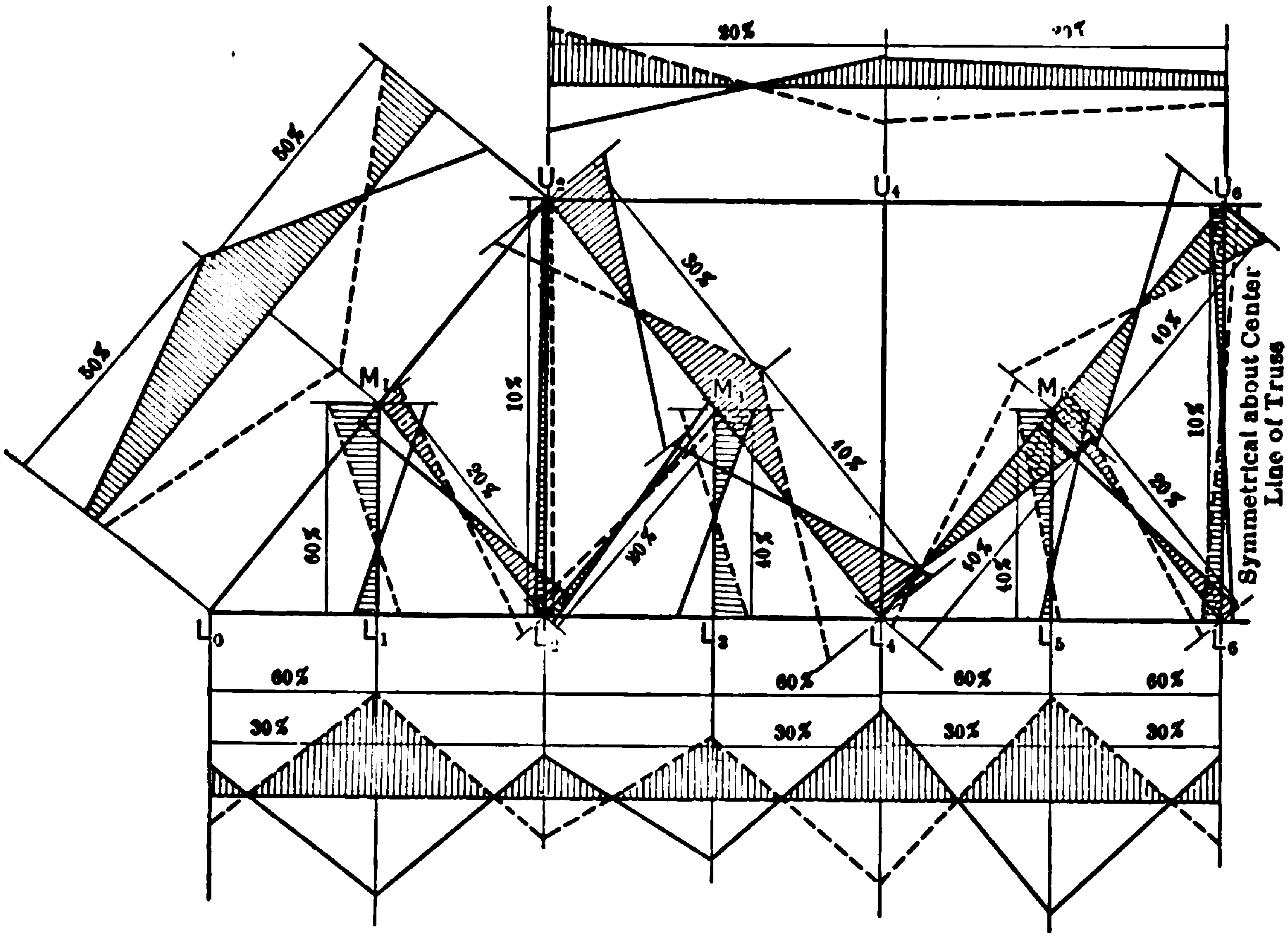


FIG. 9. Secondary Stresses in a Subdivided Pratt Truss.

Light Lines show Undeformed Truss.
Heavy Lines show Deformed Truss for a uniform live load over entire span.

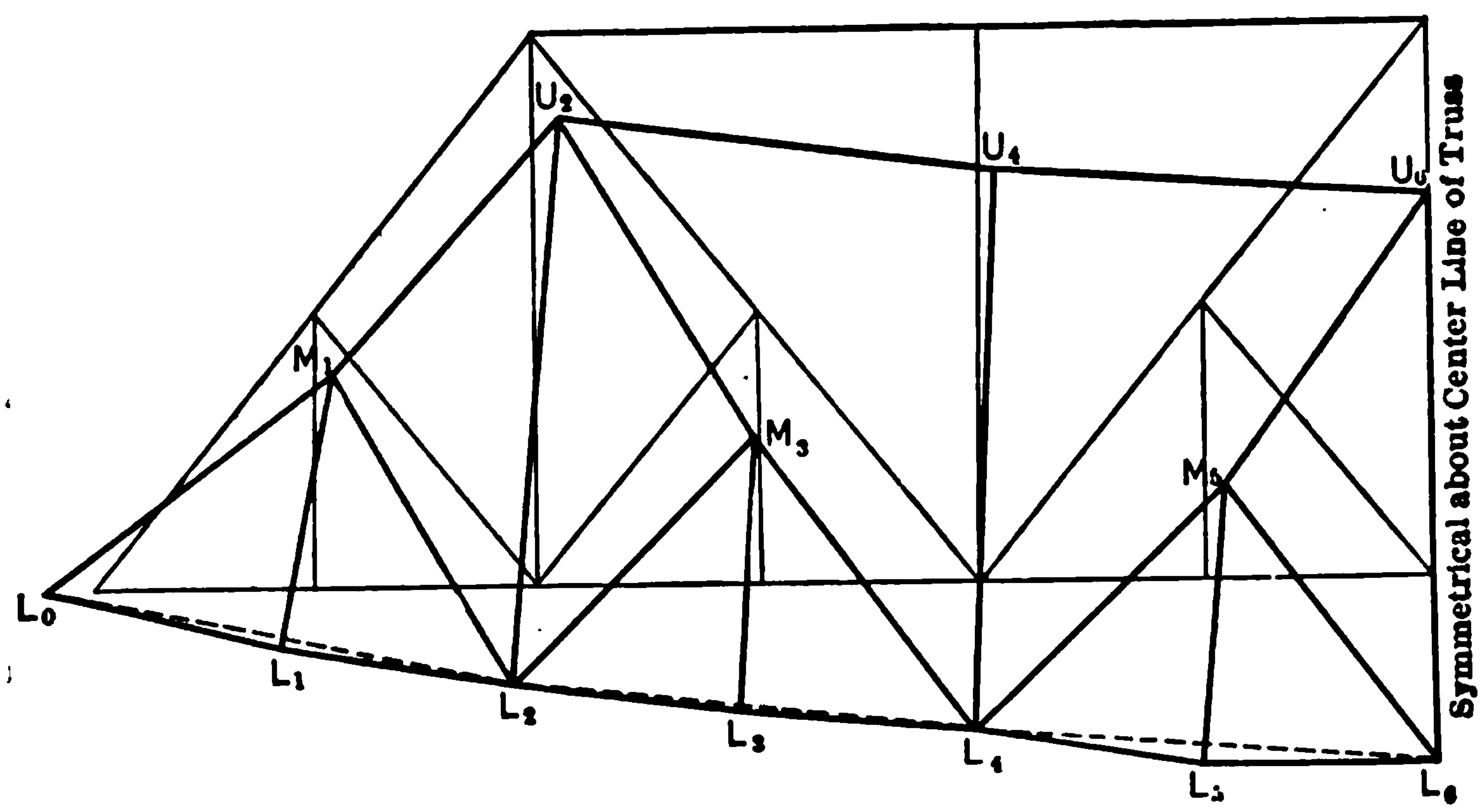


FIG. 10. Deflection Diagram of a Subdivided Pratt Truss.

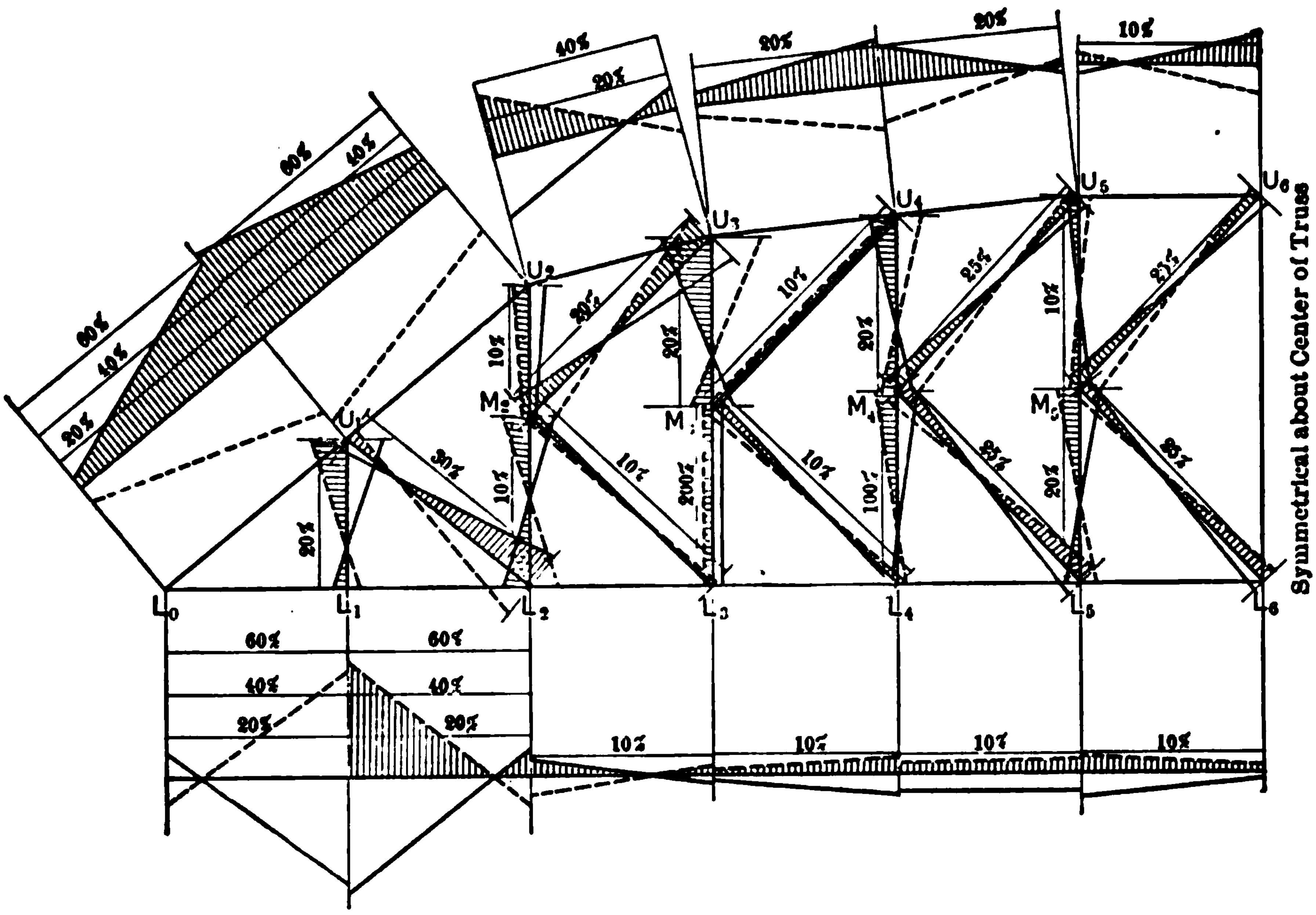


FIG. 11. Secondary Stresses in a K-Truss.

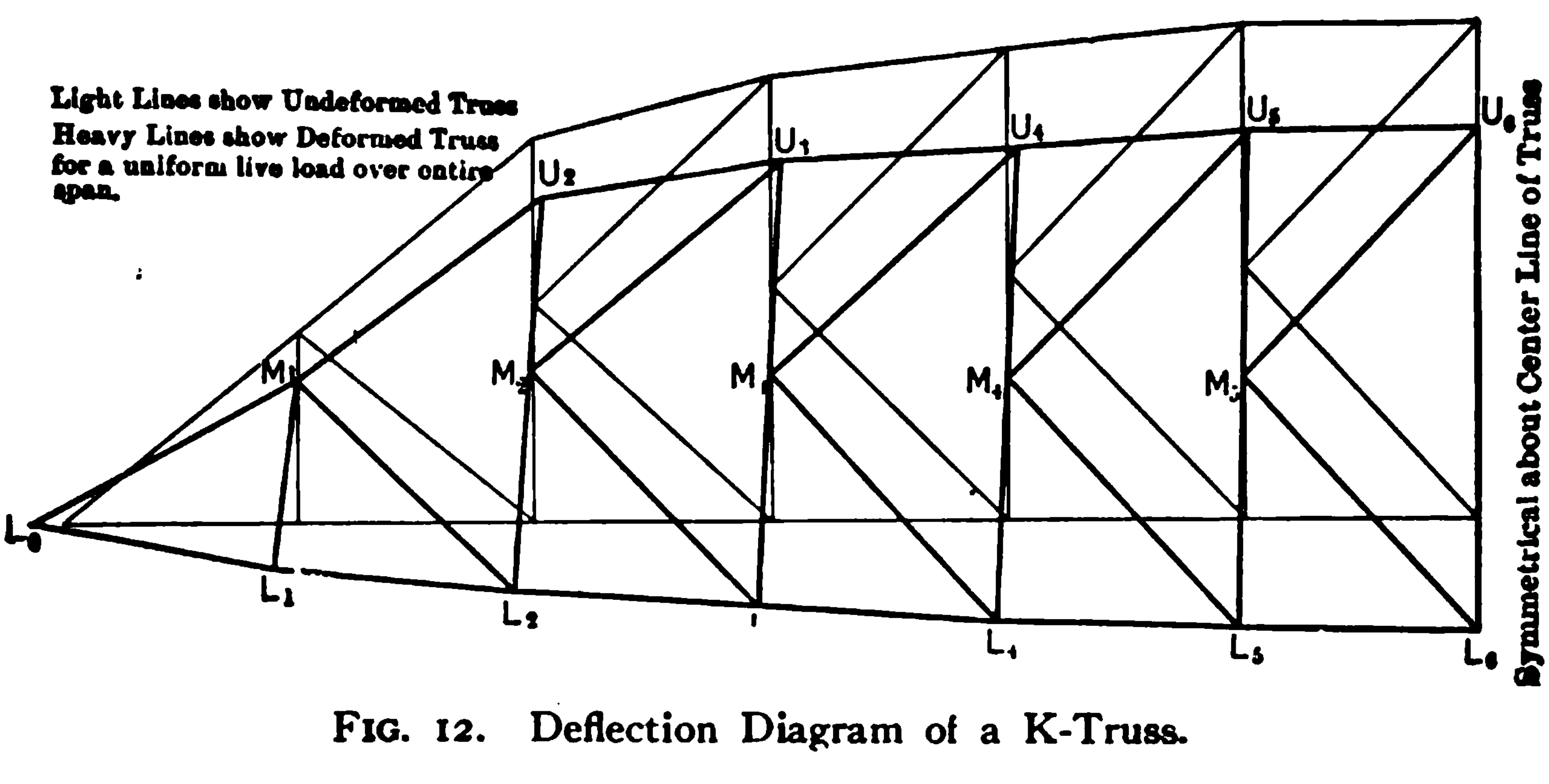


FIG. 12. Deflection Diagram of a K-Truss.

as high as 100 per cent where the panel length was only 8 ft. 4 ins. and depth of chord 24 ins. The stress due to deflection alone would be only from 25 to 30 per cent. The reason for such high values in a truss with sub-panels is clearly brought out in the sketch of Fig. 10, which shows to an exaggerated scale the deflections or movements of the various joints of the truss of Fig. 9. This brings out the great effect of the distortion of the hangers and sub-struts on the chord members.

76. *Truss with "K" Type of Bracing.*—Figs. 11 and 12 show the secondary stresses and deflection diagram of a "K" type of truss, for uniform load. This example is noteworthy in the relatively small stresses. This truss is adapted to long spans as it gives a short-panel length without undue inclination of the web members. In this respect it has the advantages of the double intersection truss without its disadvantages. As compared to the subdivided truss, Figs. 9 and 10, it is much freer from secondary stresses.

77. *Double Intersection Trusses* are liable to show high secondary stresses in the chord members from the effects of concentrated loads,

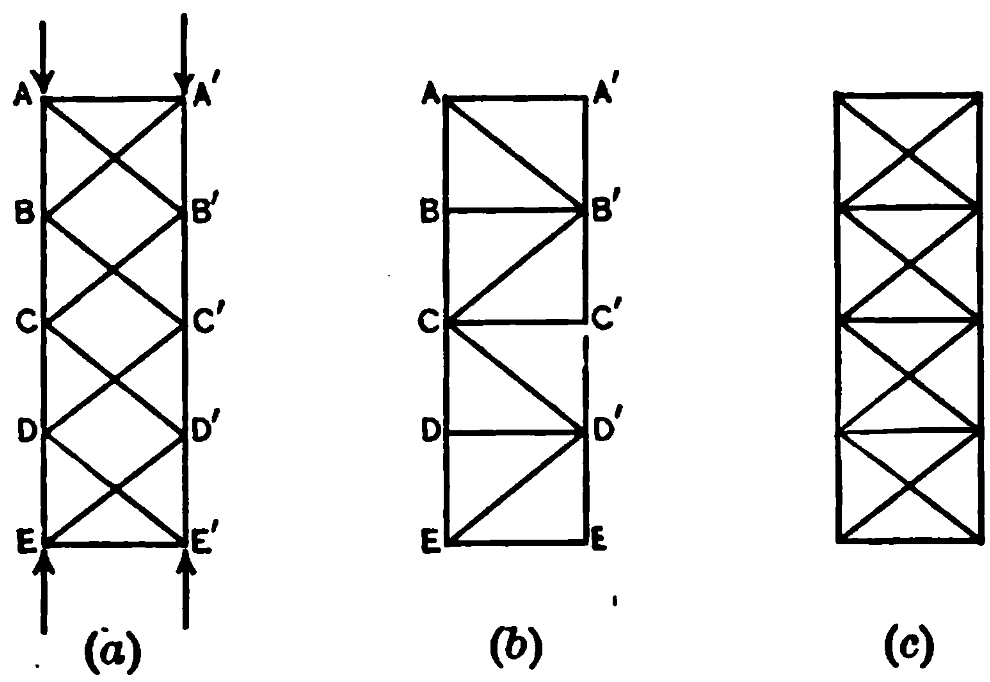


FIG. 13.

due to the independent action of each web system. They should be used, if at all, only for long, heavy spans where the dead load is relatively great.

78. *Trestle Towers and Lateral Bracing Between Chords of Trusses.*—A considerable amount of secondary stress may be caused in tower legs and chord members, in a lateral direction, by the use of single bracing, or double bracing without transverse members. Figs. 13

(*a*), (*b*) and (*c*) illustrate three systems of bracing in common use. In (*a*) and (*b*) considerable lateral displacement of points B and B' and D and D' is caused by the compressive deformation in the main (upright) members. The points A and E and C and C' tend to stand fast, which results in a considerable lateral bending in the main members. In type (*c*) this effect is nearly eliminated by the lateral ties placed at each joint. The conditions here described arise in the

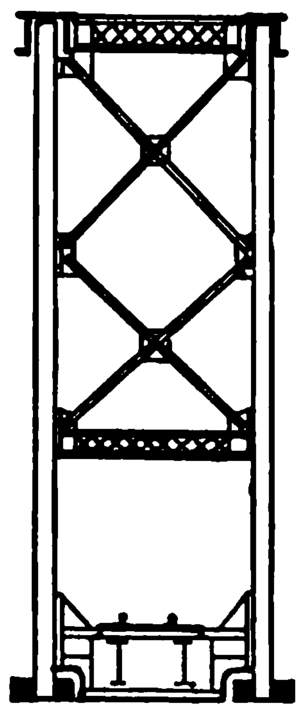


FIG. 14.

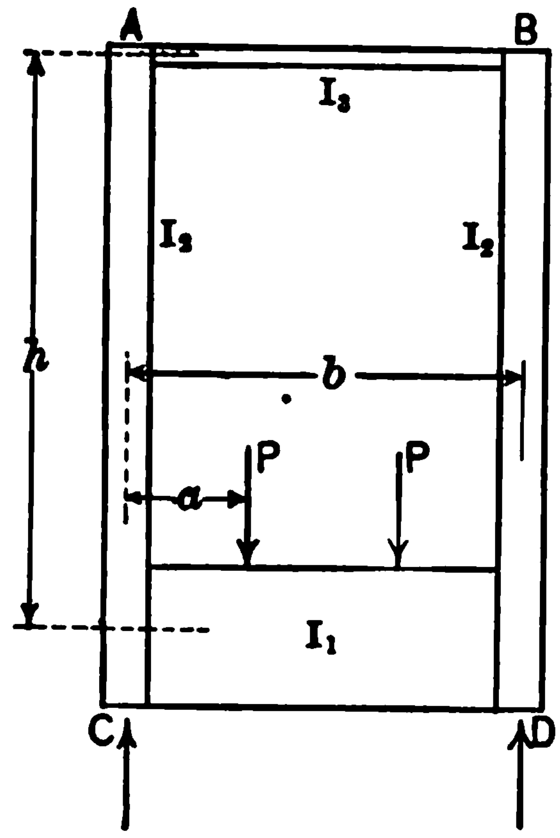


FIG. 15.

case of any set of parallel members, both of which are stressed in the same direction, such as the upper or lower chords of a bridge, or the long vertical posts forming the two verticals of a transverse frame of a high truss bridge (Fig. 14). In all of these cases transverse ties or struts at all joints are desirable. The same principles apply to very large columns whose segments are connected only by diagonal lacing.

79. Bending Stresses in Vertical Posts Due to Deflection of Floor-beams.—Where floor-beams are rigidly connected to vertical posts, as in the usual modern design, the deflection of the floor-beams produces certain deflections of the posts in a transverse plane, with corresponding bending stresses. This problem can readily be approximately analyzed as follows:

Fig. 15 shows a transverse frame consisting of beam, posts, and overhead transverse bracing. We may consider two extreme cases:

first, if the transverse bracing is so slender, or the connection such that the post may be considered as hinged at A and B; and second, if the bracing and connection are sufficiently rigid that the post may be considered as fixed at A and B. The first corresponds to a low truss where $A\ B$ is only a light lateral strut; the second corresponds to a deep truss where the posts above the line $A\ B$ are securely held in line by transverse bracing. The actual conditions generally fall somewhere between these two extreme assumptions.

Taking into account the deflections of the beam and posts, and placing equal to zero the deflection of A relative to B, the following formulas for the bending moments at C and D are derived.*

$$\text{First case (hinged at } A \text{ and } B\text{)} \quad M = P\,\frac{3a\,(b-a)\,I_2}{2\,h\,I_1 + 3\,b\,I_2} \quad . \quad . \quad (23)$$

$$\text{Second case (fixed at } A \text{ and } B\text{)} \quad M = P\,\frac{2a\,(b-a)\,I_2}{h\,I_1 + 2\,b\,I_2} \quad . \quad . \quad (24)$$

It can be shown that under the above assumptions, and for the usual spacing of track stringers in single-track bridges, the ratio of fibre stress in the post to the fibre stress at the centre of the beam is approximately as follows:

$$\frac{f_p}{f_b} = 0.7\,\frac{c_2}{c_1} \text{ to } 1.0\,\frac{c_2}{c_1},$$

in which f_p = fibre stress in post, f_b = fibre stress at beam centre, c_1 = depth of beam, and c_2 = width of post. Thus, the ratio of post stress to maximum beam stress is nearly equal to the ratio of widths of members. For example, if the depth of beam is 48 ins. and width of post 12 ins., the ratio of the respective widths is 0.25, and, therefore, the bending stress in the post will be approximately from 20 to 25 per cent of the bending stress in the centre of the beam, assuming the usual spacing of stringers.

Results of observations bear out these theoretical conclusions. Bending stresses in posts have been observed as high as 40 per cent of the floor-beam stress, and invariably the observations have shown quite large values. In the case of compression verticals the maximum

* See Part II, Art. 350.

bending stresses do not occur simultaneously with the maximum primary stress; however, an increase of 20 to 25 per cent in the maximum fibre stress may be expected from this cause. The narrower the posts and the deeper the beams the less this secondary stress. In the case of tension verticals, such as hip verticals, the maximum bending stress occurs simultaneously with the maximum primary stress, thus increasing very greatly the maximum fibre stress.

80. Stresses in Floor-beams due to Longitudinal Deformation of Chords.—Let Fig. 16 represent in plan one panel of a through bridge,

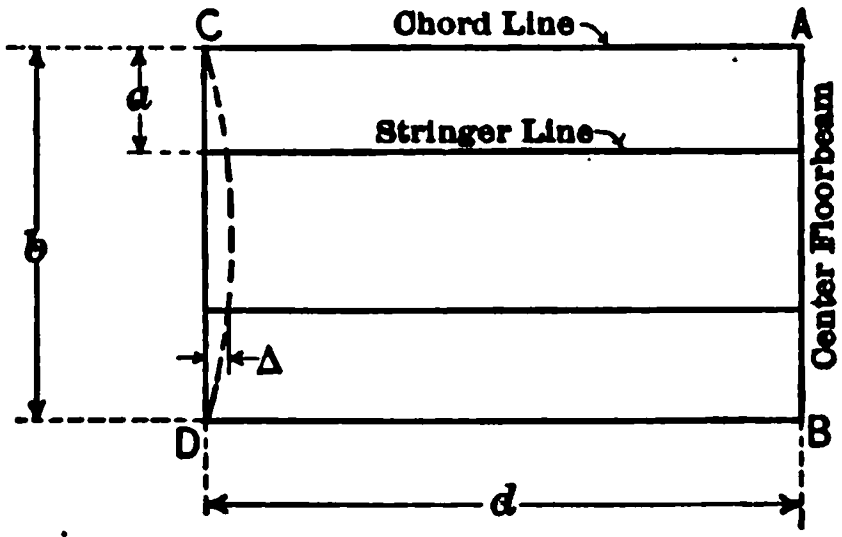


FIG. 16.

A B being a centre floor-beam. The stringers are assumed to be rigidly connected to the beams as usual. When the chords A C and B D elongate under stress, the stringers will remain substantially unchanged in axial length and the result will be that the floor-beams will be bent horizontally an amount corresponding to the elongation of the chords. If there are no expansion joints in the stringers the centre beam will tend to stand fast and the other beams will bend toward the centre, the maximum deflection taking place in the end beams. If it is assumed that the axis of the stringers do not elongate, that the stringer connections are unyielding, and that the ends of the beams remain vertically over the joint centres, then the deflection of the first beam from the centre will be equal to the elongation of one panel of the chord, the deflection of the next beam will be equal to the elongation of two panels, etc.

In Fig. 16, C D represents the first beam from the centre. The deflection Δ is taken equal to the elongation of chord $C\,A = d\,s/E$, where s = unit stress in the chords and E = modulus of elasticity. Although the joints at C and D are more or less rigid, it may be as-

sumed that, as regards horizontal bending, the beam is free to turn at the ends. In this case the deflection of the beam in terms of maximum fibre stress is

$$\Delta = \frac{f\,a}{6\,E\,c}(3b - 4a),$$

in which f = fibre stress, c = half of flange width. Placing this deflection equal to $d\,s/E$, and solving for f, we derive

$$f = \frac{6\,c\,d}{a(3b - 4a)} \times s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

Assuming, for example, d = 300 ins., c = 6 ins., b = 192 ins., $a = b/4$, we find that $f = 0.58\,s$. For the second beam $f = 1.16\,s$, etc.

In these calculations it has been assumed that the stringers are not elongated at all and that the connection to the beams permits of no deformation whatever. Practically, the riveted joint is not entirely rigid, as the connection angles, the rivets, and the web of the beam all contribute to the deformation. The stringers, also, receive some longitudinal stress, although the amount per square inch of section is small. On the whole, the deflections and stresses in the beam are not as great as deduced from the above calculations, but they are often large and of much importance. As the calculations show, they increase with the width of the beam and with the number of panels, and are greater as the distance a becomes smaller.

Actual strain measurements which have been made on floor-beams show the presence of large bending stresses. On the following page are given some results obtained on an 8-panel bridge of 105-ft. span length. The stresses are those due to longitudinal bending alone.*

In some designs an attempt is made to resist the bending by a horizontal truss at the ends of the span. Such a plan would appear to be unwise. The elongations of the chords cannot be prevented, and the beams can only be held in line by forcing the stringers or their connections to distort an equal amount. An attempt to do this results in heavy stresses on rivets and connections, with little benefit. Excessive stresses can be prevented by the use of expansion joints in stringer connections, and these are frequently used in long-span bridges. The effect of dead load can also be eliminated by fitting

* Bul. Am. Ry. Eng. Assn., No. 163, January, 1914, p. 489.

the stringer length to the chord length under dead load, and making stringer connections after the truss is free of the falsework. Conditions are especially unfavorable in double-track structures where the distance between outside stringers and end beam is relatively small. Where it is desirable to introduce horizontal trusses in the floor system to resist the traction or braking stresses, the most suitable place for such a truss is at one or two beams near the centre of the span, or at the centre of the space between expansion joints in stringers where such are used.

		Observed Flange Stresses, Lbs. per Sq. In.	Calculated Flange Stresses, Lbs. per Sq. In.
Center beam		470	0
First beams from center	Left	1140	940
	Right	1100	
Second beams from center	Left	1870	1880
	Right	1770	
Third beams from centre	Left	2950	2820
	Right	2400	
Fourth or end beams	Left	2900	3760
	Right	2700	

Besides the stresses in beams and connections, the extension of the chords gives rise to considerable stress in the lateral members and their connections. With fairly rigid joints the intensity of stress in the laterals may easily reach one-third to one-half the stress in the chords themselves. In the end panels, where the chord section is small and the laterals relatively large, a considerable proportion of the chord stress will be carried by the laterals. This consideration shows the importance of good lateral connections, especially near the ends of the span. Observations on end posts and end sections of lower chords have shown very high secondary stresses in these members, due to eccentric connections of lower laterals.

The connection of lower laterals to stringers is of doubtful value. Owing to the relative movement between chords and floor system here discussed, such a connection will cause considerable lateral bending in the stringers, especially near the ends of the span.

81. Variation in Axial Stress in the Different Elements of a Member.—In addition to the secondary stresses in a vertical plane

due to rigid joints, there will exist more or less bending in a direction at right angles to the vertical plane. In the case of eye-bar members this lateral bending will be shown in the inequality of stress in the various bars; in the case of riveted members it will be shown as a lateral bending similar to the bending in a vertical plane. It is impossible to apply theoretical analysis to any great extent in this case, but results of observations have shown such variation to be considerable, especially at lower ends of end posts, due probably to the effect of the laterals. In other riveted members variations generally of 5 to 10 per cent have been noted. In eye-bars a variation of 10 to 20 per cent from the average is not uncommon.

82. Conclusions Regarding Secondary Stresses.—The following is a résumé of the more important results of the foregoing discussion.

1. The amount of secondary stress to be expected in main truss members of ordinary Pratt and Warren trusses ranges from about 15 to about 35 per cent. If members are exceptionally deep in the plane of the truss this percentage may easily reach 50 or 60 per cent.

2. Secondary stresses in trusses with subdivided panels are usually higher than in a simple Pratt truss. Excessive stresses can be avoided by using sub verticals and sub struts of liberal cross-section or by modifying the lengths of these members so as to equalize the distortion of chord members under live load.

3. To avoid excessive bending stresses in posts to which floor-beams are riveted the posts should be made only of moderate width in a transverse direction and the beams should be made relatively deep. In the best designs these bending stresses are likely to be as much as 25 per cent of the primary stresses.

4. To avoid excessive stresses in floor-beams due to chord action expansion joints for stringers should be used in long spans. If horizontal trusses are used in the floor system to resist traction stresses these should be placed near the centre of a span or midway between expansion joints.

5. The end lateral connections should be as concentric as practicable. This is very important in the case of the end panel of the lower chord as eccentric connections at this point result in heavy secondary stresses in end post and lower chord.

CHAPTER V

RIVETED JOINTS

83. General Requirements.—The design of the riveting and other connecting details of a steel structure is a matter of equal importance to the determination of the proper forms and sectional areas of the members. The design of the details is, in fact, the more difficult part of the process, and good results are much more dependent upon the experience and skill of the designer. The stresses in main members and the sections required are fairly well determined by well-known methods of analysis, but in joint details the distribution of stress is complex and cannot be very closely determined by theoretical analysis. Good results are largely dependent upon the exercise of correct judgment as to the general behavior of the various parts of a detail in the transmission of stress,—a sense of proportion and fitness, rather than upon mathematical calculations. In this problem the element of distortion, or strain, enters as a very important factor in controlling the distribution of stress, and it is often necessary to look upon stress as dependent upon the possible or relative distortion rather than upon distortion as a function of stress.

True economy of design requires that the details of a structure should be at least as strong as the main members; hence, to secure good results it is necessary to use a little greater factor of safety in designing the details than is used in the main members. The purpose of a joint in general is to transmit stress from one plate or element into one or more other elements or members, or to a support. To be most effective the transmission of stress must be so made that no part of the member or joint shall be overstressed and so as to secure at the same time the greatest economy of material and labor. It is important also that the joint be so made as to be free from permanent distortion, that is, it should be elastic, and it should not be subject to wear.

Joint connections are made by three general methods: by rivets,

by bolts and by pins. Rivets are employed for connecting the various elements of "built-up" members, for connecting individual members in riveted structures and for many of the joint details of pin-connected trusses. Bolts are used in temporary fitting-up work in the shop and in erection, and sometimes in place of rivets for permanent work. Pins are used for joint connections in pin-connected trusses (see Chapter VII). The most generally used connecting element is the rivet, and a study of the subject of riveted connections is therefore of special importance.

84. Kinds of Riveted Joints and Stresses Involved.—Riveted joints and connections may be divided into two general types: (1) the *lap joint* and (2) the *butt joint*. In the lap joint, Fig. 1, the princi-

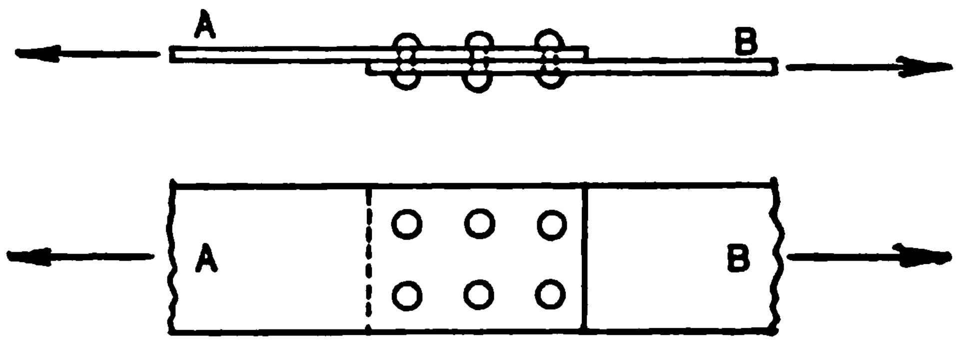


FIG. 1.

pal stresses on the rivets are the shearing stress on the plane between the plates (single shear), and the bearing pressure on both plates A and B. In the butt joint, Fig. 2, the rivets are stressed in shear

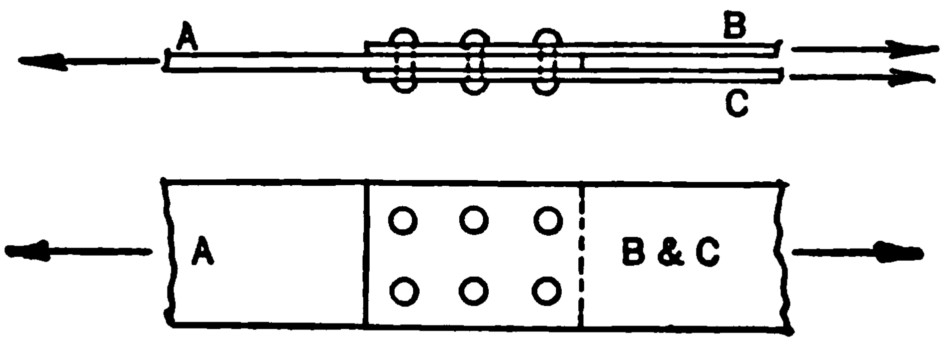


FIG. 2.

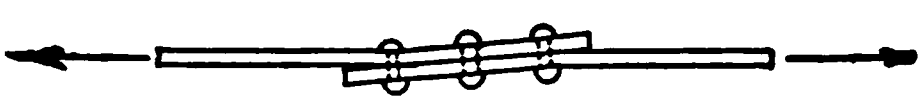


FIG. 3.

on two planes (double shear), and also in bearing on all three plates. If the thickness of plate A is less than the sum of B and C, as is commonly the case, the intensity of bearing pressure on A is the greatest. In both the joints here illustrated the rivets are relatively short and have little bending stress, but where the number of plates is large

or their thickness great the bending stresses in the rivets become important. In Fig. 1, the direct stress tends to bend the plates as indicated in Fig. 3, thus causing bending stresses in the plates and some direct tension in the rivets of the end rows. This joint is, to a certain extent, eccentric, and in this respect is not as good as the butt joint. In (1) the shearing strength of the rivets generally governs the number required; in (2) the bearing pressure on plate A controls the design unless this plate is relatively thick, in which case the double shearing strength may control. In bridge work, splices in plates are generally made by means of butt joints; connections at joints are often made in the form of lap joints.

Besides the stresses on the rivets, the effect of the rivet holes on the strength of the member, or the effective section of the member, must be determined.

85. Size and Proportions of Rivets.—The diameter of rivets is determined in a general way by the thickness of the plates to be connected. Where the rivet value is determined by the shearing strength, the strength of a rivet is proportional to the square of the diameter; and where the value is determined by bearing, the strength is proportional to the first power of the diameter. The required number of rivets in a joint is thus rapidly reduced as the size is increased until the bearing strength is involved, after which any further increase in size has much less effect on the number. In the lap joint the shearing strength is equal to the bearing strength when $\frac{\pi d^2}{4}. f_s = d\ t\ f_b$, where d = diameter, t = plate thickness, and f_s and f_b are, respectively, the allowable shearing and bearing stresses. The value of f_s is usually taken at one-half f_b, hence for equal strength in shear and bearing, $d = \frac{8}{\pi}. t = 2.55\ t$. In the butt joint, where the rivet is in double shear, the strengths are equal when $d = \frac{4}{\pi}. t = 1.27\ t$, where t = thickness of main plate. In practice, a diameter of rivet is selected which will range from once to twice the thickness of the general run of plates to be connected. In light structural work, with plates ¼ to ½ in. thick, rivets are ½ to ¾ in. in diameter; in ordinary bridge

work with plates 3/8 to 3/4 in. thick, rivets are 3/4 and 7/8 in.; in heavy work where 1-in. plates are used, 1-in. rivets are commonly employed; and in exceptionally heavy work, with numerous plates 1 in. or more in thickness, the rivets are still larger. In the Hell Gate bridge the rivets are 1 1/4 ins.

Outside of the question of shear and bearing, large rivets are advantageous in their bending resistance, so that where long rivets are required the diameter should be relatively large. On the other hand, large rivets are more difficult to drive than small ones and tend to reduce the effective section of the member to a greater extent. The size of rivet is often determined by the size of shapes used, on account of necessary clearance or suitable proportions for stress transmission. Then also, in the same structure or class of members, it is desirable to use only one or two different sizes. The specifications given in Appendix A provide as follows:

(41) The diameter of the rivets in any angle carrying calculated stress shall not exceed one-quarter the width of the leg in which they are driven. In minor parts 7/8-in. rivets may be used in 3-in. angles and 3/4-in. rivets in 2 1/2-in. angles.

Rivets are made with full-button heads, countersunk heads, and flattened heads. For proportions see Appendix B, Table II. Countersunk rivets are used where riveted members must fit closely against others; flattened heads are used where the clearance is less than that required for full heads, but greater than 5/16 in.

86. Transmission of Stress in Riveted Joints.—To secure the best and most economical results the details of a joint should be so arranged that the stress in the individual members and in the several parts of the joint will be as uniform as possible. Uniform stress in the main members requires that the gravity axes of the members shall pass through the centre of resistance of the rivet group. Neglect of this requirement results in eccentricity which produces bending in the members and uneven stress in the rivets. Economy of material requires the rivets to be so arranged as to reduce the sectional areas of connected members as little as practicable. Uniformity of stress in rivets and connecting plates is difficult and often impossible to secure, but the nearer it is approached the more permanently elastic and durable will be the structure.

Consider the joint of two rivets, Fig. 4. Suppose the cover plates, B and C, are each one-half as thick as plate A. Stress in $A = P$, and stress per square inch in all plates $= p$. Assume that each rivet

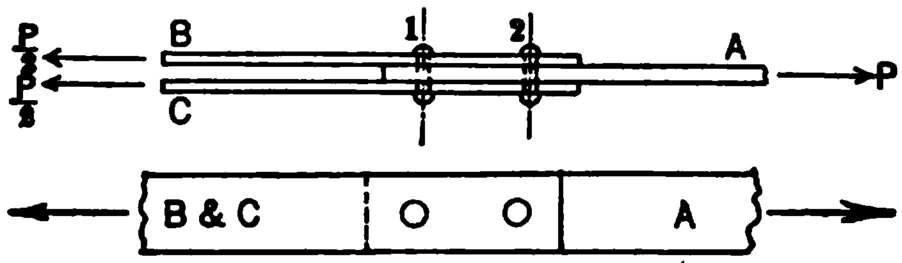


FIG. 4.

takes one-half the stress. Then it follows that the stress in plate A between sections 1 and 2 is $P/2$, and the unit stress on all plates in this space is $p/2$. Let us now investigate the question of distortion.

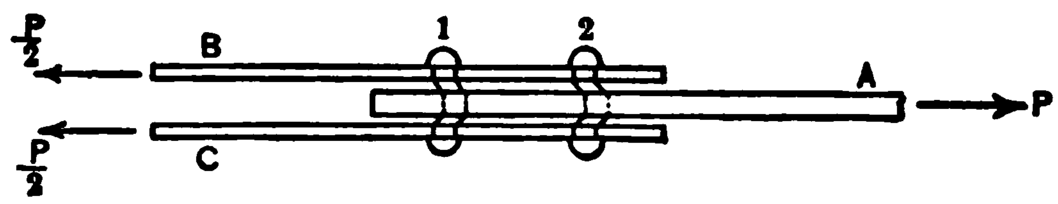


FIG. 5.

Under the stresses assumed, both plates and rivets will undergo some distortion (Fig. 5). To develop resistance in the rivets requires some slight slip or movement of one plate on the other. Under the assumed

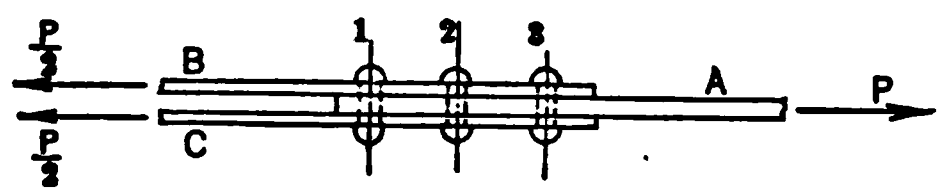


FIG. 6.

conditions, there is the same amount of stress and stretch in all three plates between rivets 1 and 2, hence whatever slip occurs at 1, the same amount occurs at 2. This gives equal rivet stress as assumed, hence the assumed conditions are consistent and the rivet stresses are undoubtedly equal.

Consider now the three-rivet splice, Fig. 6. As before, plates B and C are each one-half as thick as A, and hence are equally stressed, the unit stress being p. Assume first that the rivets are equally stressed, the load carried by each being $P/3$. Then the unit stresses in the several plates in the joint will be as follows:

	Plate A	Plates B and C
Space 1–2	$\frac{1}{3} p$	$\frac{2}{3} p$
Space 2–3	$\frac{2}{3} p$	$\frac{1}{3} p$

The stretch of the metal in each space will be proportional to the stress and will be equal to stress multiplied by l/E, where l = length and E = modulus of elasticity.

Assume that the slip or rivet distortion at No. 1 is k, then the slip at 2 will be k, plus the stretch of A, minus the stretch of B and C, in space 1–2. This will be equal to $k + \frac{1}{3}\ pl/E - \frac{2}{3}\ pl/E = k - \frac{1}{3}\ pl/E$. Then at No. 3 the slip will be equal to that at 2, plus the stretch in A,

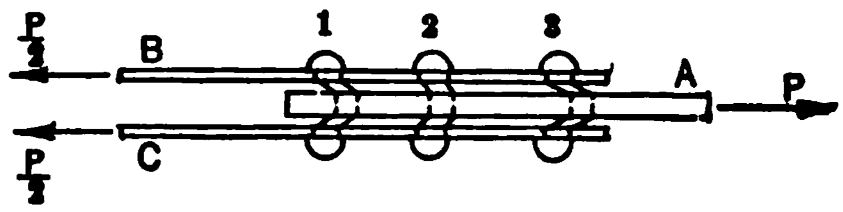


FIG. 7.

minus the stretch in B and C, $= (k - \frac{1}{3} pl/E) + \frac{2}{3} pl/E - \frac{1}{3}\ pl/E = k$. Thus we find that the slip at 1 is the same as that at 3, but the slip at 2 is less, as indicated in Fig. 7. It follows, therefore, that the stress in rivet 2 is less than that in 1 and 3, and the stresses in the plates are not distributed as assumed. If the ratio of slip to rivet stress were known it would theoretically be possible to calculate the exact distribution of stress in rivets and plates, but practically this is not possible. If the plates are very rigid and the rivets flexible, like long bolts, then the rivet stress will be practically uniform; if the rivets are rigid and the plates relatively flexible, then rivet No. 2 will receive very little stress. This analysis shows, therefore, that in joints with three or more rows of rivets in the line of stress, the end rivets are stressed the most, the second row will be stressed less, and the rivets near the centre will be stressed least. The results of tests bear out these conclusions.

Further analysis of Fig. 4 shows that if plates B and C are thicker than one-half of A, the stretch in these plates in the space 1–2 will be less than the stretch in plate A, hence the stress in rivet 1 will be less than in rivet 2. It is seen, therefore, that in such a case the rivet in the joint toward the thinner plate will receive the greater stress.

The above analysis is based on the assumption of elastic conditions and no friction between plates. In reality there is a large frictional resistance between the plates due to the clamping action of the rivets, and, until this resistance is overcome, the rivets are not greatly stressed in shear. Furthermore, the rivets frequently do not fill the holes completely, so that some little inelastic movement is required before all come to a firm bearing. After this occurs the rivet stress will be distributed more nearly in accordance with the elastic theory outlined above.

When we consider the question of *ultimate* strength the same principles do not apply. After the rivets yield a considerable amount their stress becomes nearly equal, and a joint which fails by the shearing of the rivets will show a strength very nearly proportional to the number of rivets. While the ultimate strength of a joint, therefore, may not be greatly affected by the manner of distributing the rivets, nor in fact by the exact fit of the rivets in the holes, the elastic strength may be considerably influenced thereby, and to secure the best results requires some attention to this matter.

87. Methods of Promoting Uniformity of Stress.—Uniformity of stress in the several plates at any given section tends to uniformity

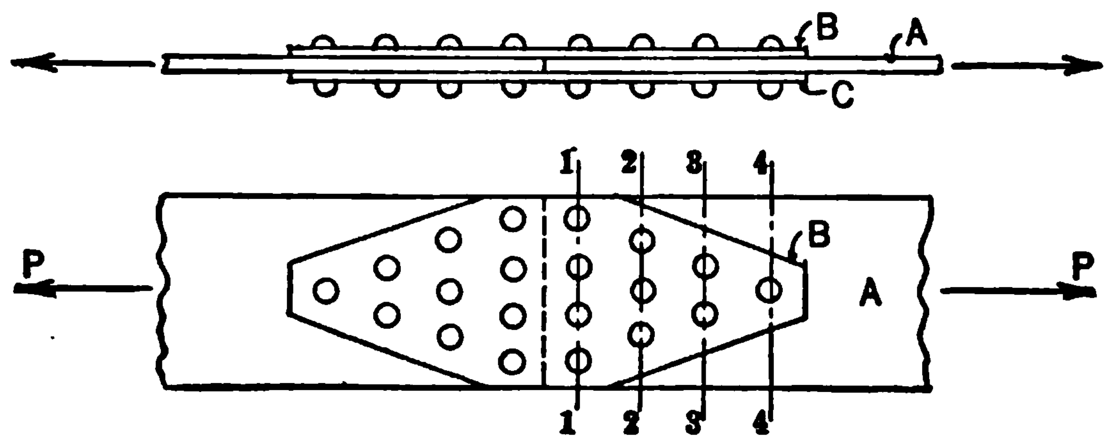


FIG. 8.

of rivet stress. It also tends to secure a maximum of frictional resistance, as the slipping force is evenly distributed instead of being concentrated over a part of the area. Hence uniformity of stress in the plates is a desired end to keep in mind. As shown in the preceding article, uniformity of stress can easily be secured in a joint having only two rows of rivets. For a greater number of rows something may be gained by varying the width of one of the plates, so that the cross-section will increase somewhat with the amount of

stress taken up by the rivets. Thus in Fig. 8, the splice plates *B* and *C* may be cut off as shown, and the number of rivets on the cross-section reduced from the maximum at section 1–1 to a single rivet at the end section 4–4. Assuming the rivet stresses to be equal, the

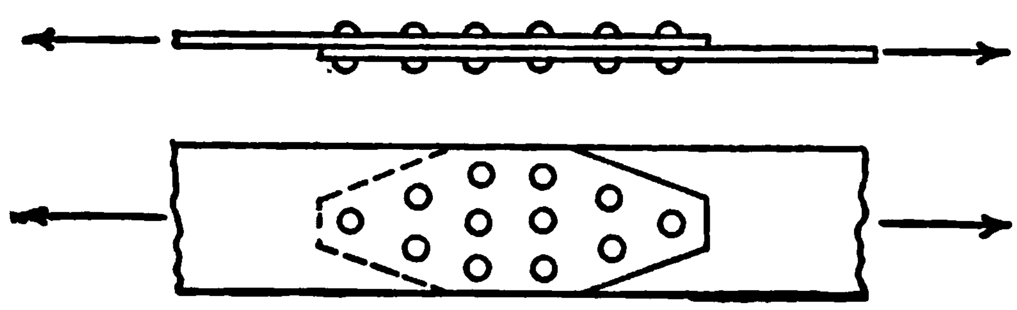


FIG. 9.

stress in plates *B* and *C* between sections 3–3 and 4–4 will be $\frac{1}{10}P$, and the stress in plate *A* will be $\frac{9}{10}P$. Then between 2–2 and 3–3 the stress in *B* and *C* is $\frac{3}{10}P$ and in plate *A*, $\frac{7}{10}P$; and from 1–1 to 2–2 the stresses are respectively $\frac{6}{10}$ and $\frac{4}{10}P$. Theoretically the cross-section of *B* and *C* from 3–3 to 4–4 should be $\frac{1}{9}$ of plate *A*; from 2–2 to 3–3 it should be $\frac{3}{7}$ of plate *A*; and from 1–1 to 2–2, $\frac{6}{4}$ of plate *A*. An approach to these conditions can be secured by tapering off the splice plates as shown, although exact equality can hardly be obtained. Such an arrangement will, however, give a considerably better distribution of stress than a square joint. The same general arrangement can be used for a lap joint (Fig. 9). The placing of a diminishing number of rivets toward the end of a

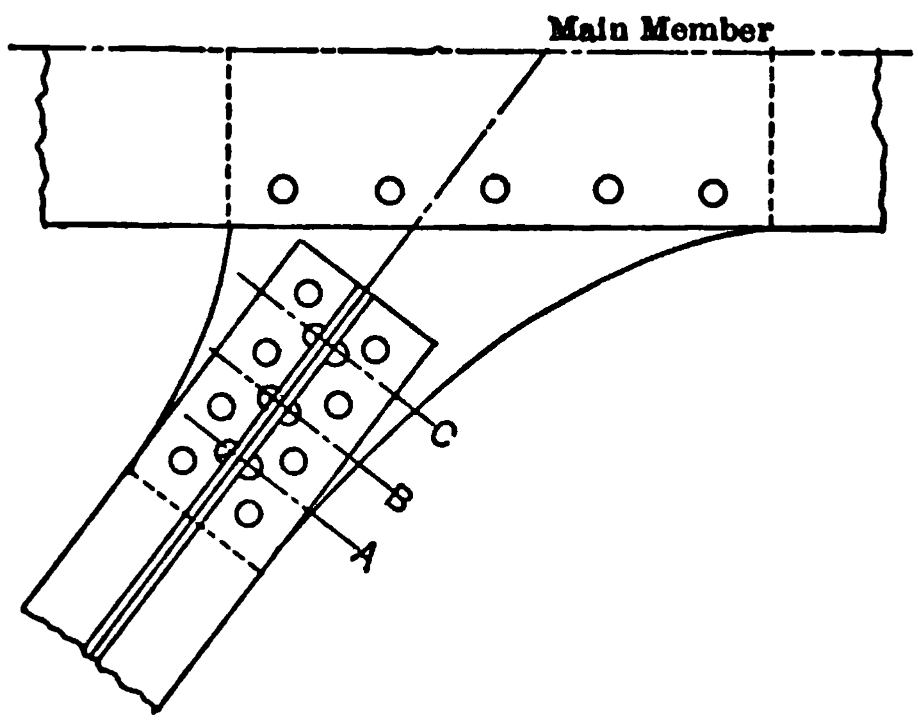


FIG. 10.

splice has also the merit of cutting out as little area as possible from the cross-section of the plates. (See Art. 93.)

In the design of joints such as shown in Figs. 10, 11, and 12, the beveled or curved outlines of the gusset plates tend to secure uniformity of rivet stress. Thus in all cases the gusset plate area through section B is greater than A, and through C greater than B, etc., cor-

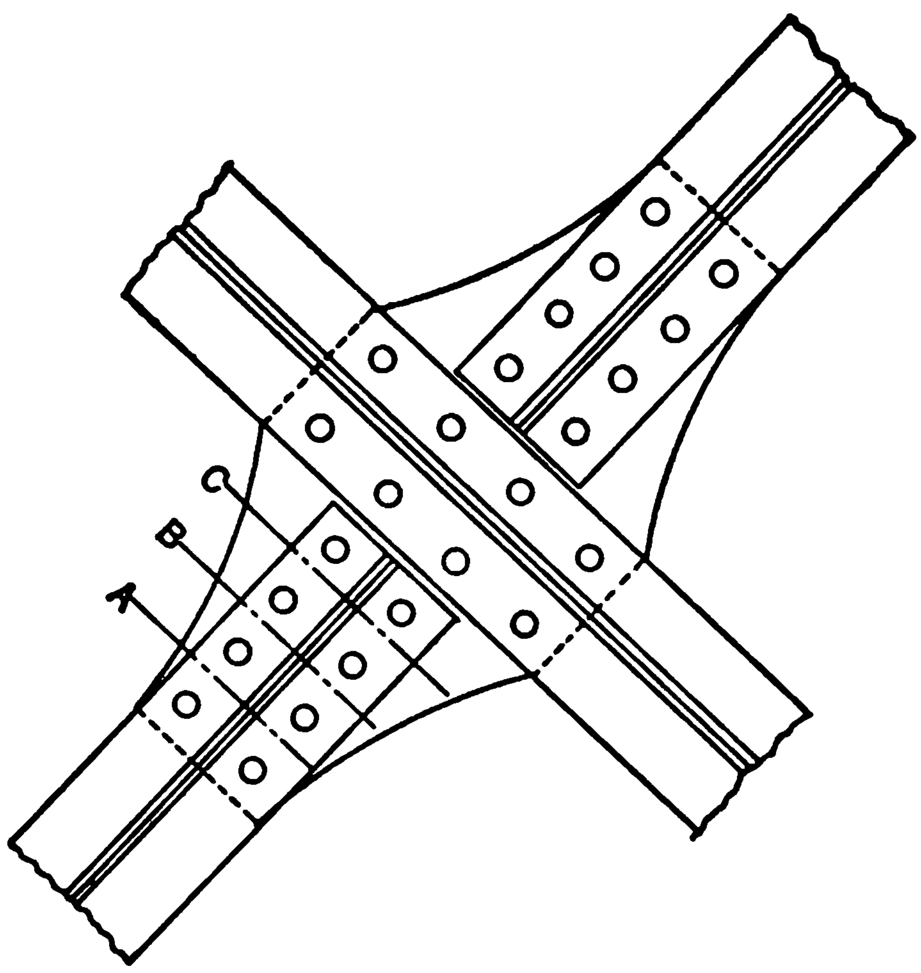


FIG. 11.

responding in a measure to the increased load carried by the plate. The edges of the plates are usually cut in straight lines but sometimes for sake of appearance they are curved as shown in Figs. 10 and 11. In Fig. 12 the curved edges shown by dotted lines give a better stress distribution than the straight edges. Where modification of sectional area cannot be made as illustrated in the preceding examples, then it should be kept in mind that, in general, the end rivets will be stressed more than the intermediate rivets. Where for any reason it is desirable to use long splices or reinforcing plates the principle should therefore be followed of spacing the rivets relatively close together near the two ends of the joint and further apart in the intermediate area. The latter rivets will not receive much stress under elastic conditions. For examples of such joints see Chap. VIII.

88. Effect of Fillers and Indirect Transmission.—Sometimes it is necessary to separate the main plate from the splice plates by one or more fillers. In this case the fillers should be arranged as in Fig. 14 rather than as in Fig. 13. In Fig. 13 the rivets are subjected to large

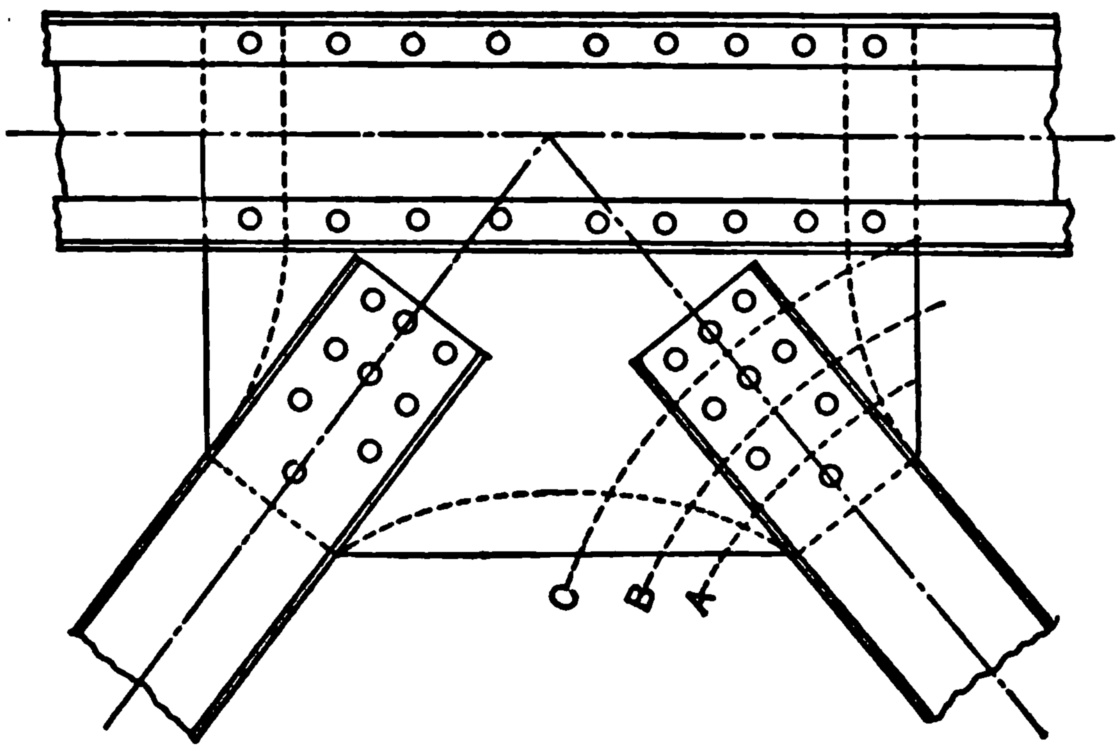


FIG. 12.

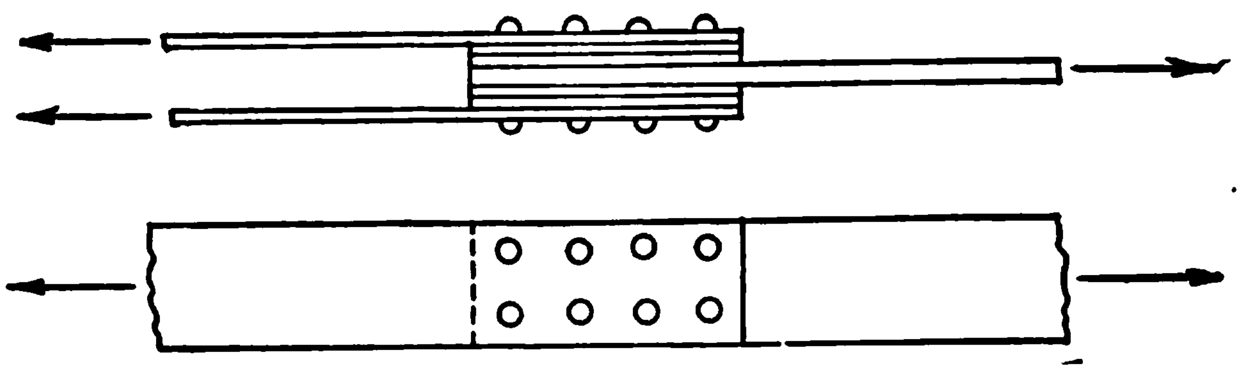


FIG. 13.

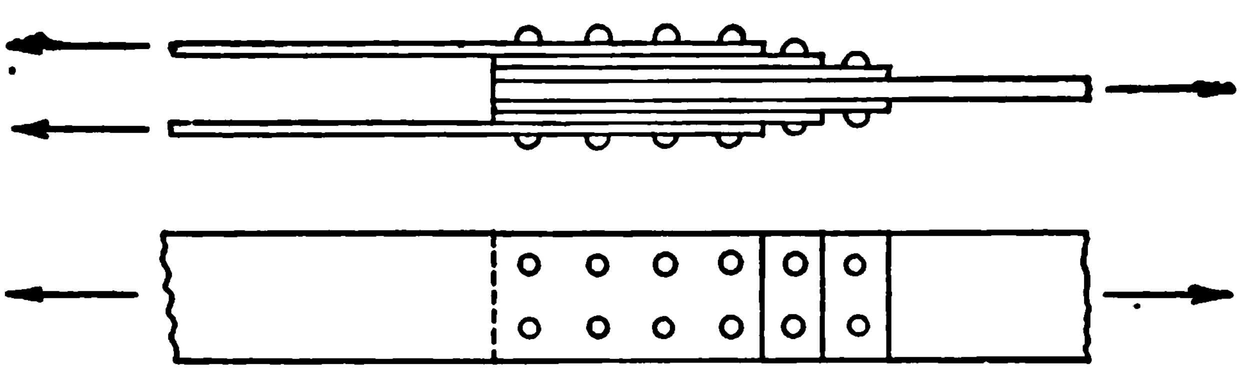


FIG. 14.

bending moments and, besides, the stresses on the rivets are very unevenly distributed. The result is a low frictional resistance and a very considerable yielding of the joint before the full shearing re-

sistance of the rivets is developed. In Fig. 14 the fillers are separately fastened by short rivets and the stress is much more evenly distributed. This arrangement also increases very greatly the frictional resistance and reduces the bending moment on the long rivets. See results of tests in Art. 89.

The use of excessively long rivets and indirect transmission is guarded in the specifications by the following:

(42) Rivets carrying calculated stress and whose grip exceeds four diameters shall be increased in number at least 1 per cent for each additional $^1/_{16}$-in. of grip.

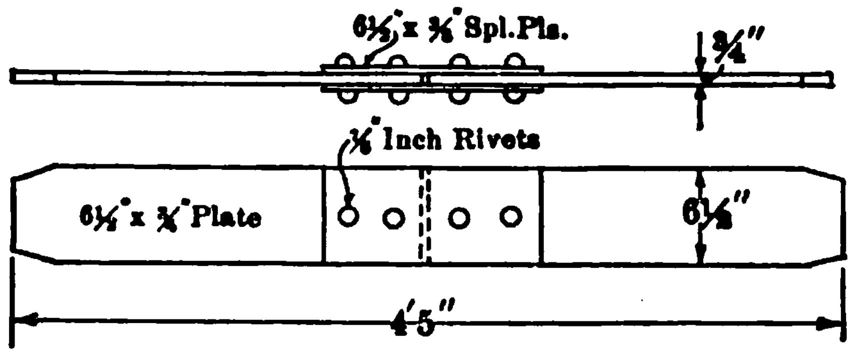
(57) Where splice plates are not in direct contact with the parts which they connect, rivets shall be used on each side of the joint in excess of the number theoretically required to the extent of one-third of the number for each intervening plate.

(58) Rivets carrying stress and passing through fillers shall be increased 50 per cent in number; and the excess rivets, where possible, shall be outside of the connecting member (that is, through the filler as shown in Fig. 14).

89. Friction in Riveted Joints.—Tests on riveted joints show that the distortion of the joint does not progress uniformly with increase of load, as in the case of a tension test of the material. Until the load reaches a considerable amount, depending upon conditions, there is no slipping of plates and very slight distortion of rivets. The plates are held by the friction developed from the clamping action of the rivets. When this frictional resistance is overcome, the plates begin to slip and the rivets begin to offer resistance in shear. Owing to imperfect fit of rivets, due to shrinkage in cooling and imperfect upsetting, an appreciable movement will take place before all rivets come into full action. After this, the deformation is about proportional to load until the rivets reach their yield point or the material is overstressed in tension. Failure takes place by shearing of the rivets, by excessive bearing pressure crushing the sides of the holes, or by a tension failure of the plates. Fig. 15 shows a typical stress-deformation curve of a test on a riveted joint arranged as shown.* Up to a shearing stress of about 7,000 lbs. per sq. in. on the rivets the distortion was exceedingly small; the plates were held principally by friction. During the increase of load from 7,000 to

* Proc. Am. Ry. Eng. & M. of W. Assn., Vol. 6, 1905, p. 272.

10,000 lbs. per sq. in. a very considerable slip and permanent set occurred, amounting to about 0.01 in. From this point the distortion increased at a less rate until the stress reached about 20,000 lbs. per sq. in. From this point the permanent distortion began again to increase more rapidly and continued to do so up to rupture at a shearing stress, in this case, of 47,270 lbs. per sq. in.



TEST JOINT

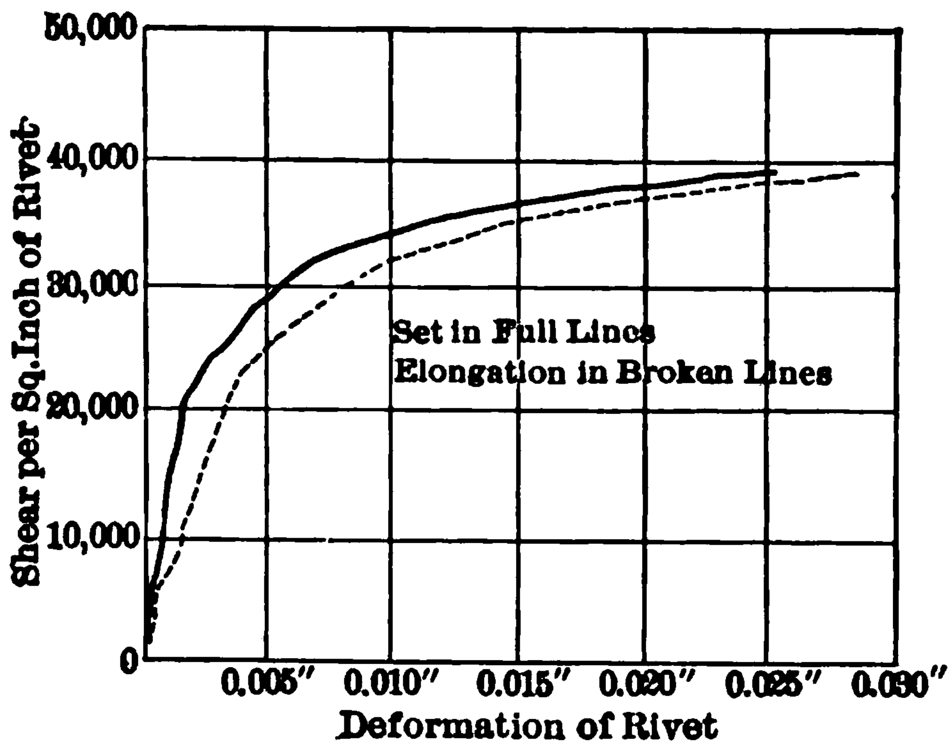


FIG. 15.

Other experiments on similar joints have shown a first slip, or frictional resistance, corresponding to a shear stress of from 7,000 to 20,000 lbs. per sq. in., depending largely upon temperature of rivet when driven, manner of driving, and the nature of the surfaces in contact. From 7,000 to 12,000 lbs. per sq. in. are common values.* Tests of both ordinary and nickel steel riveted joints, made at the University of Illinois, gave about equal results for frictional strength, namely, about 10,000 lbs. per sq. in. of rivet shear area.† These

* See paper by J. S. Von der Kolk, Zeit. des Ver. deut., Ing., 1897, p. 739; also Proc. Am. Ry. Eng. & M. of W. Assn., 1905, p. 421.

E †*ng. News*, May 4, 1911, p. 526.

tests were made with especial reference to the effect of repeated stresses and of alternating stresses. Repeated loads in one direction had no noticeable effect on the ultimate strength. After the application of any particular load the joint was relatively rigid for any load below this limit. Under alternating stresses exceeding the stress necessary to produce a slipping of the joint, the slipping became

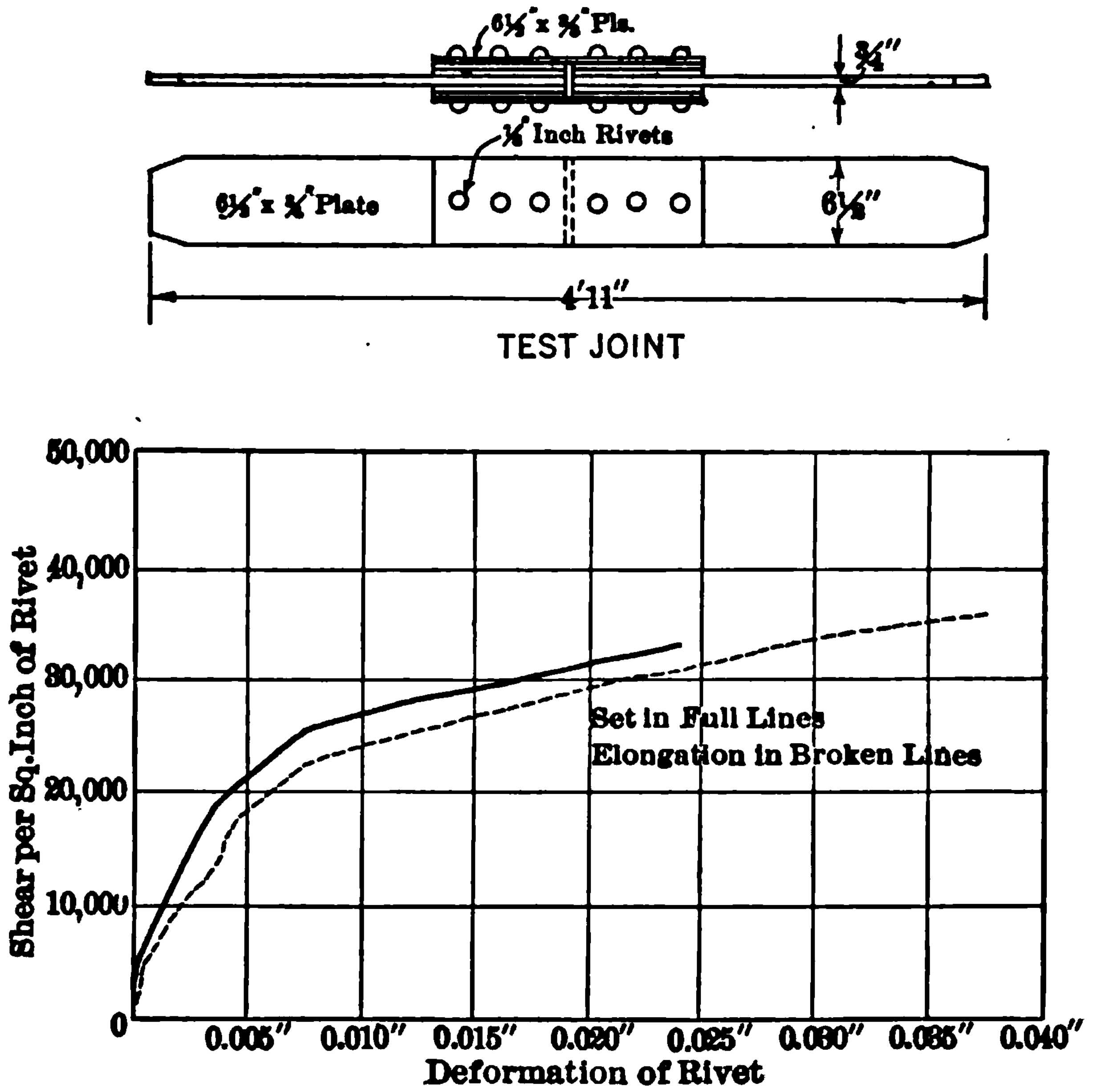


FIG. 16.

progressively easier, or the joint looser, the extent of this action depending upon the maximum stresses.

The effect of fillers on the deformation of a joint is shown in Fig. 16, giving the stress-deformation curve of the joint there shown.* Fig. 17 shows the beneficial effect of additional rivets placed directly through fillers.

The results from these and other tests show that, at a usual working stress of about 10,000 lbs. per sq. in. of shearing area, there is likely to be a slight slipping of the joint, but with good workmanship

* Proc. Am. R. E. & M. W. Assn. 1905, p. 383.

and design this will be very slight and in fact the friction may not be overcome. In the case of alternating stresses, a loosening of the joint will occur if slip takes place, and therefore the maximum working stresses should be kept as low as 7,000 to 8,000 lbs. per sq. in. (see Specification, Art. 22).

90. Working Stresses.—The stresses usually considered in deter-

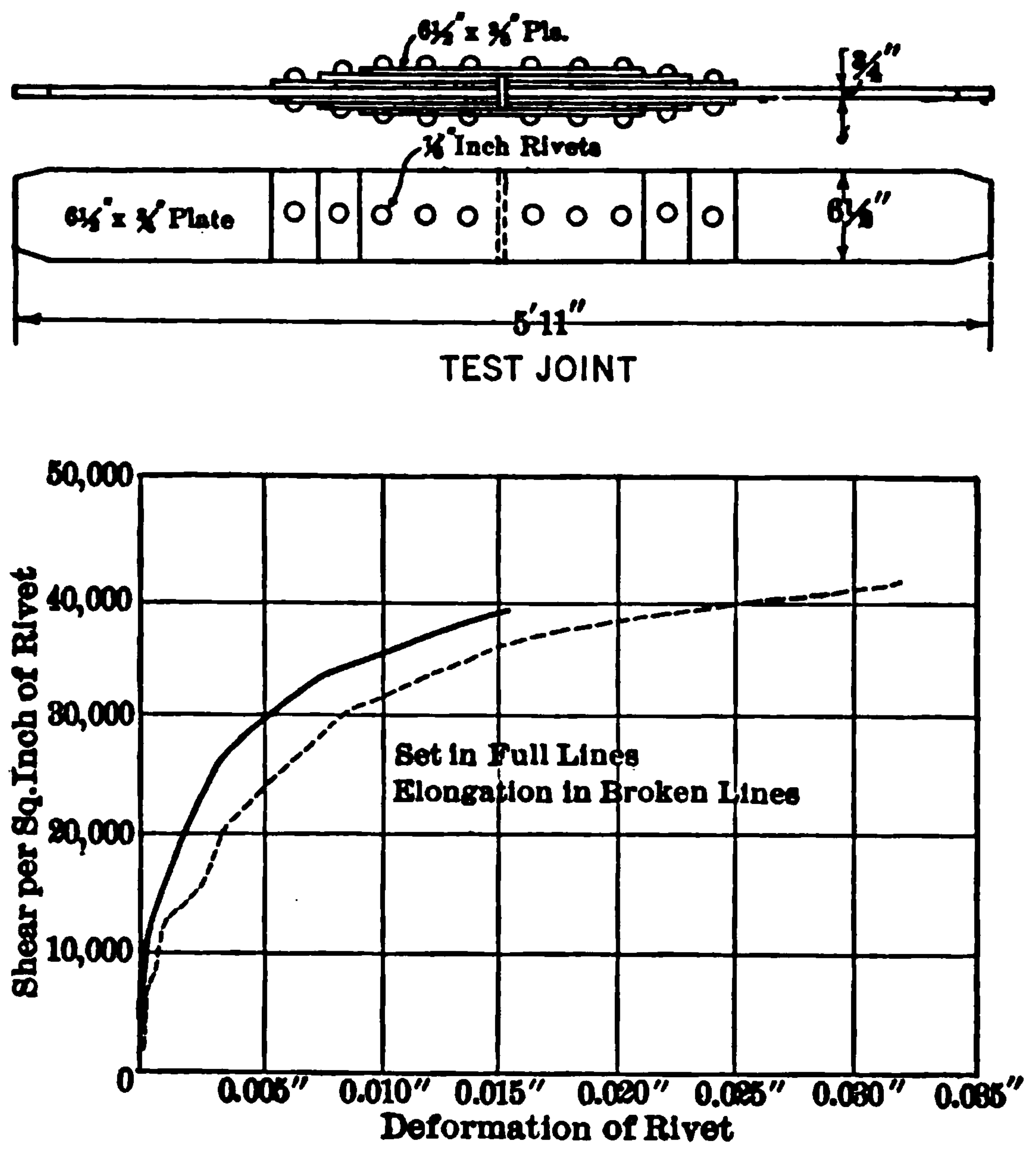


FIG. 17.

mining the strength of a rivet are the shearing stresses and the bearing pressure of the rivet against the plates. Rivets are also subjected to bending stresses, which are of much importance in the case of long rivets, and also to tensile stresses due to shrinkage of the rivet in cooling and often to the forces acting on the joint. The shearing and bearing strengths are definitely specified and used in the calculations; the bending and tension are usually more or less definitely provided for by certain limitations of length of rivet and arrangement of joint details.

The material used for rivets is generally somewhat softer and more ductile than the material used in the members themselves.

For the standard structural steel of the Am. Ry. Eng. Assn. specifications the rivet steel has an ultimate strength of about 50,000 to 52,000 lbs. per sq. in., and a shearing strength of 40,000 to 45,000 lbs. per sq. in. The allowable stresses are:

Shearing:	Shop	driven	rivets	12,000	lbs.	per	sq.	in.
	Field	"	"	10,000	"	"	"	"
Bearing:	Shop	"	"	24,000	"	"	"	"
	Field	"	"	20,000	"	"	"	"

These stresses correspond to 16,000 lbs. per sq. in for the unit tensile stress for dead load and live load (including impact). The shearing

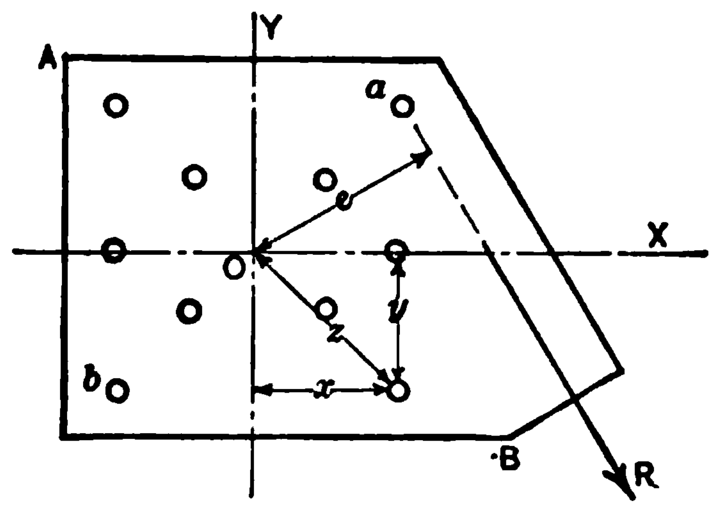


FIG. 18.

strength for shop rivets is thus three-fourths the working tensile strength, a usual ratio, and the bearing strength twice the shearing. To guard against undue bending stresses and the effect of fillers, special provisions are made (see Art. 58).

For alternating stresses a low working stress is provided for by requiring that "the connections shall be in all cases proportioned for the sum of the stresses." For equal stresses of opposite sign this results in a shearing working stress of 6,000 lbs. per sq. in., which is within the friction limit of a well-designed joint.

91. Eccentric Rivet Connections.—To secure the most uniform stress in a joint the lines of action of the several members to be connected should pass through the centre of resistance (centre of gravity) of the group of rivets. It is not always practicable, however, to accomplish this, and the joint will be more or less eccentric. In such a case the stresses on the rivets will not be uniform and it is important to determine what the actual distribution will be.

Fig. 18 represents an eccentric rivet connection. The joint plate A B is riveted to some large member which acts as a relatively rigid

support. The resultant of the other forces acting on the plate is represented by the force R, applied eccentrically with respect to the centre of gravity of the group of rivets. The problem is to determine the stresses on the various rivets and especially the maximum rivet stress. Suppose the point O is the centre of gravity of the group. If the line of application of the force R passed through O then, according to the usual practice, it would be assumed that the stress would be uniformly distributed, and the stress on each rivet would be equal to R divided by the number of rivets. In this case the force R is applied with an eccentricity e, and hence produces a turning moment on the plate equal to Re, which causes an additional stress in each rivet. The total stress on any rivet will be the resultant of the direct stress and the stress due to the turning moment.

Let n = number of rivets;
r_d = direct stress on each rivet = R/n;
r_m = moment stress on any rivet;
r_0 = moment stress on rivet at unit distance from O·
r = total stress on any rivet;
x, y = co-ordinates to any rivet referred to any convenient rectangular axes $O\,Y$ and $O\,X$ passing through O;
z = distance of any rivet from O.

Assuming the plate and its support relatively rigid, the moment stress on each rivet will be proportional to its distance from the centre of gravity, and its resisting moment proportional to the square of this distance. Therefore, the moment stress is $r_m = r_0\, z$ and the moment of resistance of this stress is $r_m z = r_0\, z^2 = r_0(x^2 + y^2)$. The total resisting moment is equal to the turning moment Re, hence we have

$$r_0(\Sigma\, x^2 + \Sigma\, y^2) = Re \quad \text{(1)}$$

whence

$$r_0 = \frac{Re}{\Sigma\, x^2 + \Sigma\, y^2} \quad \text{(2)}$$

Also, for any rivet, the moment stress, as above stated, is

$$r_m = r_0\, z \quad \text{(3)}$$

The direct stress on any rivet is

$$r_d = R/n \qquad (4)$$

The total stress is the resultant of r_m and r_d. The direction of r_m is at right angles to the radius from rivet to centre of gravity, and the direction of r_d is parallel to R. The resultant is readily found graphically, or may be found algebraically by resolving both r_m and r_d into

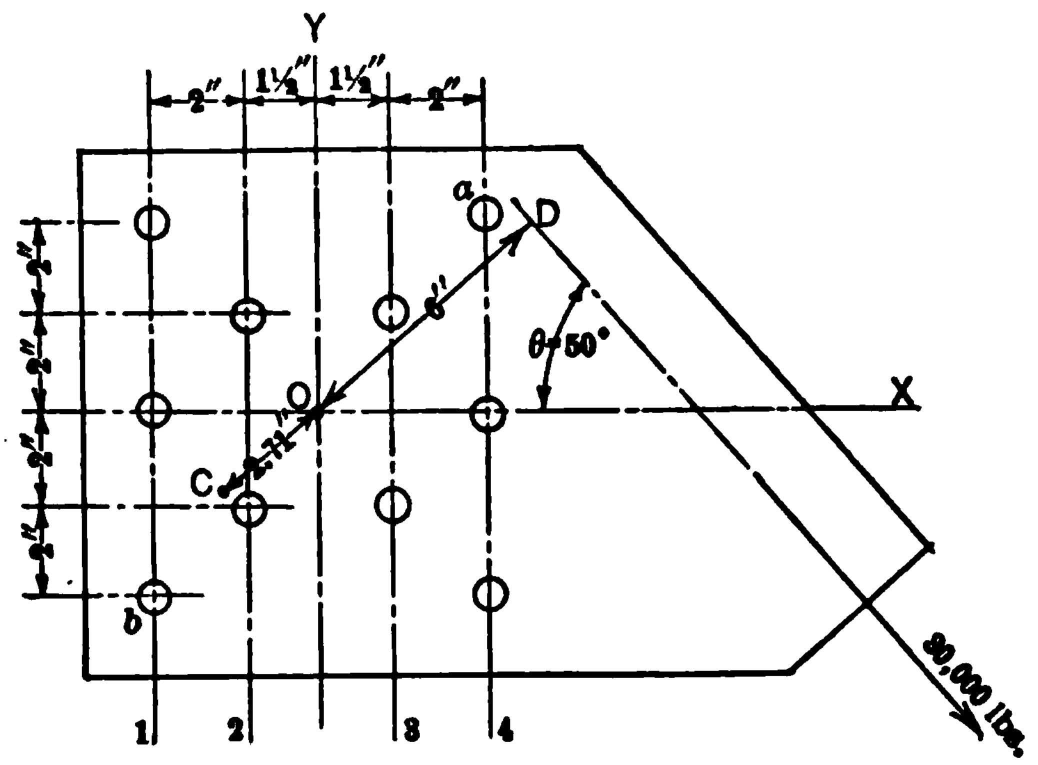


FIG. 19.

components parallel to $O\,X$ and $O\,y$ and then getting the resultant of the sums.

In Fig. 18 the most stressed rivet is evidently rivet a. Rivet b will either be the least stressed, or will have a stress opposite in direction from the stress on a. In many cases the centre of gravity of the rivet group is determined by symmetry of arrangement. If not, it must be calculated in the same manner as the centre of gravity of areas, assuming each rivet a unit. Any convenient set of rectangular axes may be taken.

EXAMPLE.—1. Assume dimensions and quantities as shown in Fig. 19. The centre of gravity is at O and $O\,X$ and $O\,Y$ are convenient axes. The value of $\Sigma\, x^2$ for row 1 is $3 \times 3.5^2 = 36.8$, and for row 2 is $2 \times 1.5^2 = 4.5$. Total value of $\Sigma\, x^2 = 2(36.8 + 4.5) = 82.6$. The value of $\Sigma\, y^2$ for row 1 is $2 \times 4^2 = 32$, and for row 2 is $2 \times 2^2 = 8$. $\Sigma\, y^2 = 2(32 + 8) = 80$. Hence, from eq. (2) $r_0 = \dfrac{30{,}000 \times 6}{80 + 82.6} = 1110$. The distance z to rivet a is $\sqrt{4^2 + 3.5^2} = 5.3$ in., and hence $r_m = 1110 \times 5.3 = 5900$ lbs. The value

of r_d is $30{,}000/10 = 3000$ lbs. Vert. comp. of r_m is $5900 \times \frac{3.5}{5.3} = 3900$; hor. comp. $r_m = 5900 \times \frac{4.0}{5.3} = 4450$; vert. comp. $r_d = 3000 \times \sin 50° =$ 2300; hor. comp. $r_d = 3000 \times \cos 50° = 1930$. Total vert. comp. $= 3900 + 2300 = 6200$; total hor. comp. $= 4450 + 1930 = 6380$. Resultant stress $= \sqrt{(6200)^2 + (6380)^2} = 8900$ lbs.

Rivet b will have a moment stress equal to a, but in the opposite direction. Its total stress will be equal to

$$\sqrt{(3900 - 2300)^2 + (4450 - 1930)^2} = 2980 \text{ lbs.}$$

EXAMPLE.—2. Fig. 20 shows a bracket attached to a column and supporting a vertical load of 30,000 lbs. Dimensions as shown. Here we have $\Sigma x^2 = 2 \times 5 \times 4^2 = 160$; $\Sigma y^2 = 2 \times 3 \times 6^2 + 2 \times 2 \times 3^2 = 252$; $n = 13$; $r_0 = 30{,}000 \times 10/412 = 730$; $r_d = 30{,}000/13 = 2310$; z for rivet $a = \sqrt{4^2 + 6^2} = 7.2$ in. Moment stress on rivet $a = 730 \times 7.2 = 5260$ lbs. Vert. comp. $= 5260 \times \frac{4}{7.2} = 2910$; hor. comp. $= 5260 \times \frac{6}{7.2} = 4380$. Total vert. comp. $= 2910 + 2310 = 5220$. Resultant stress $= \sqrt{(5220)^2 + (4380)^2} = 6800$ lbs.

Centre of Motion.—In Fig. 21, if $O\ D$ is drawn perpendicular to R, the moment stress on any rivet on this line is in a direction parallel

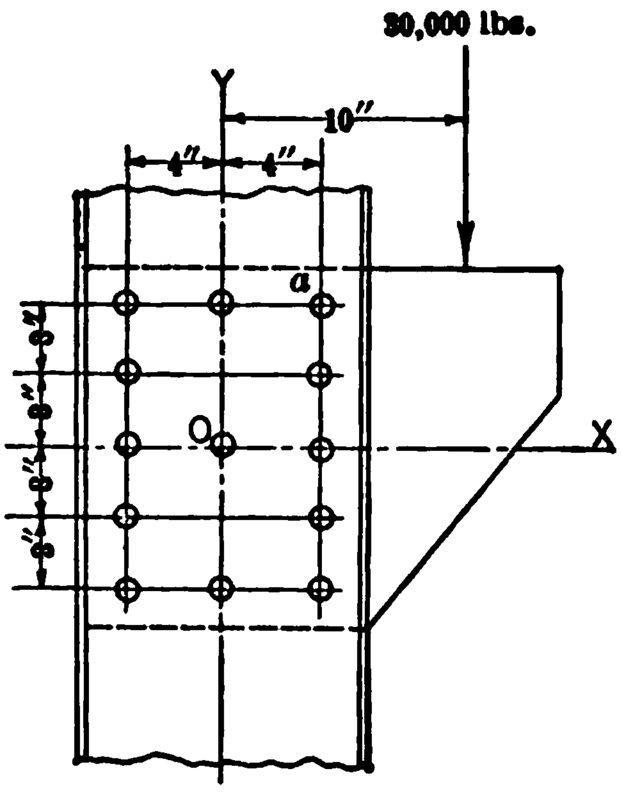


FIG. 20

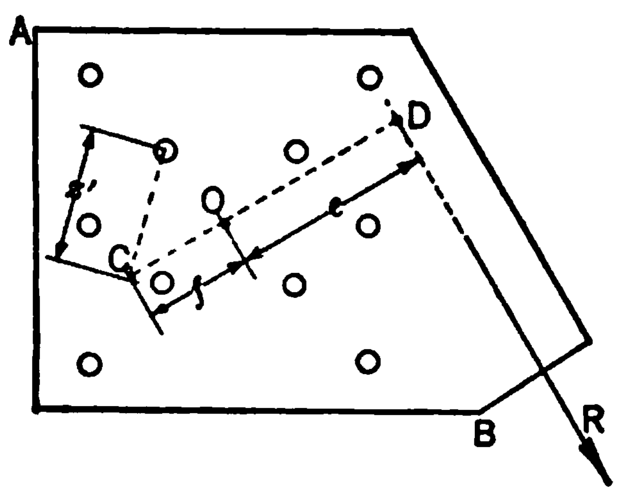


FIG. 21.

to R and the direct stress is in the same direction. On the side of O toward R, the resultant is the sum of the two stresses; on the opposite side of O it is the difference. There will be some point C,

therefore, where a rivet would, theoretically, have no stress; it would be the centre of motion of the plate. The distance f to this point can be found by equating r_m and r_d of eqs. (3) and (4) and solving for z. From this operation we get

$$f = \frac{\Sigma x^2 + \Sigma y^2}{ne} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The point C being the centre of motion, the most stressed rivet will be the one most remote from C, and the direction of its stress will be at right angles to the radius drawn to C. The *total* stress on any rivet will now be equal to the moment stress about C, which we may call r'_m, which is again proportional to the distance from C. As before, we have $r'_m = r'_0 z'$, where r'_0 is the stress on a rivet at unit distance from C. But it can be shown that $r'_0 = r_0$ of eq. (2), hence we have the total stress,

$$r'_m = r_0 z' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where r_0 is the moment stress on a rivet distant unity from the centre of turning, given by eq. (2), and z' is the distance from C to any rivet.*

The determination of the centre of turning is desirable in some cases, as the relative total stress on different rivets, as well as the di.ection cf such stre3s, can then be readily seen. The computations of maximum stress may also be slightly briefer than by the first method.

EXAMPLE.—Example 1, previously solved, will now be solved by determining the centre of turning. The value of $\Sigma x^2 + \Sigma y^2$ is 162.6 as before. Then from eq. (5) $f = 162.6/10 \times 6 = 2.71$ in. The centre of motion is 2.71 in. below O on line $D\,O$ produced. The distance to rivet a is found by measurement or calculation. The coordinates of point C are $y = 2.71$

* That r'_0 is equal to r_0 of eq. (2) can be proven as follows: The value of $\Sigma x^2 + \Sigma y^2$, referred to O as centre, is the polar moment of inertia of the rivets, taking each as unity. Call this I_0. Then with respect to centre C the polar moment of inertia is $I_c = I_0 + nf^2$, where n = number of rivets. The moment of resistance of the rivets about centre C is $r'_m z' = r'_0 z'^2 = r'_0 I_c = r'_0 (I_0 + nf^2)$. This moment is also equal to $R(e + f)$, hence $r'_0 = \frac{R(e+f)}{I_0 + nf^2}$. Substituting the value of $f = I_0/ne$ we get $r'_0 = \frac{Re}{I_0}$ which is the value of r_0 of eq. (2). (For further discussion of this method of analysis see *Eng. Rec.*, Nov. 7, 1914, p. 578.)

$\cos\theta = 1.74$, and $x = 2.71 \sin\theta = 2.07$. Then distance to $a = \sqrt{(4 + 1.74)^2 + (3.5 + 2.07)^2} = 8.0$ in. The value of r_0 is, as before, $\dfrac{Re}{\Sigma x^2 + \Sigma y^2} = 1110$. Then stress on $a = 1110 \times 8.0 = 8880$ lbs., practically the same as before.

92. Effect of Eccentric Connections on Stresses in Members.— In the preceding analysis the main member to which the joint plate is attached has been assumed as rigid, and the line of action of the

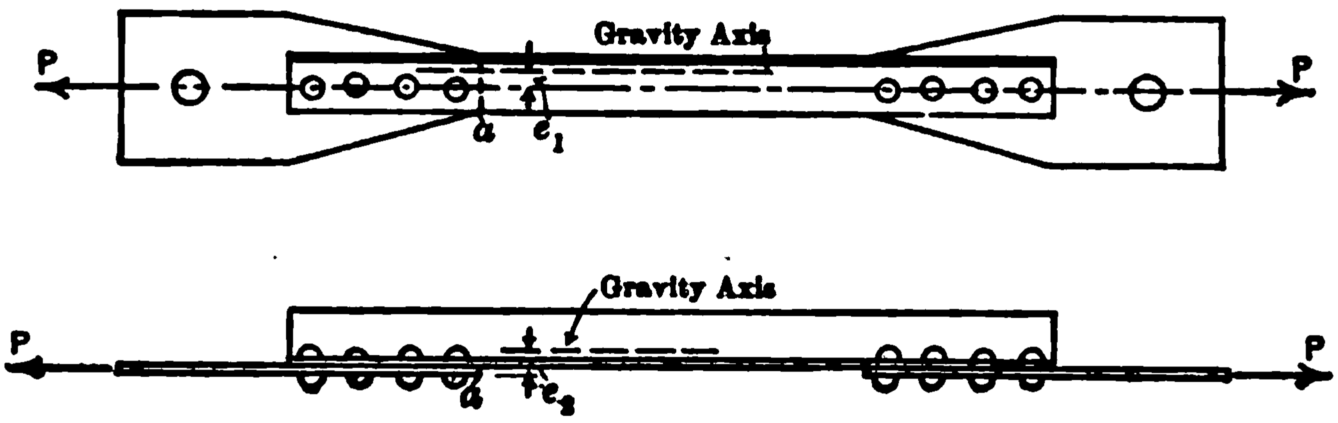


FIG. 22.

force R has been assumed as known. If all members at a joint are riveted, the moment due to eccentricity will be taken partly by each member and by the rivets in each. The most rigid member and its connections, however, resist the most moment, and if a relatively flexible member is attached eccentrically to a rigid one, as is the common case of eccentric connections, then the rigid member and its rivets may be assumed to carry the moment, as in the above examples.

The effect of an eccentric connection upon the stress in the member itself may be considerable, but whether an eccentric connection involves bending stresses depends upon the distortion of all the members connected. Take for example the common case of the angle connected by one leg. Suppose the end connections be made to pin plates as shown in Fig. 22, with pin holes in the line of rivets, and the force P applied through a pin. Then with respect to the gravity axis of the angle the horizontal moment would be Pe_1 and the vertical moment Pe_2, these tending to cause a greatly increased stress on the edge at a. The horizontal moment on the angle can be eliminated by moving the pin holes a distance e_1 into line with the gravity axis of the angles. If now the joint plate, instead of being attached to a pin so as to make a flexible joint, be attached to a large member

$A\ B$ as shown in Fig. 23, the conditions are changed. If the angle between the two members $A\ B$ and $C\ D$ remains fixed then there will be no horizontal bending moment in the angle. The effect of eccentricity cannot bend the angle unless a turning of the plate is permitted, which is assumed not to occur here. What happens is that the

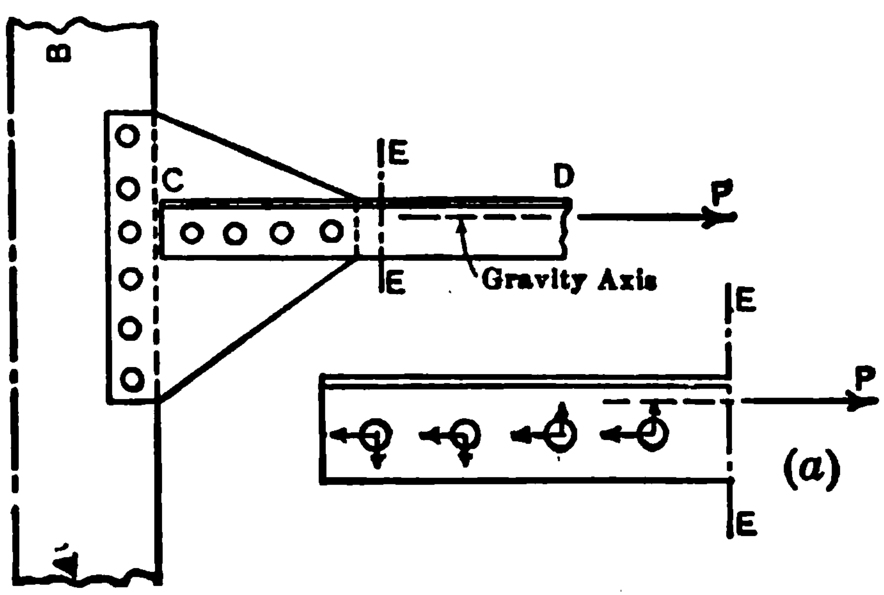


FIG. 23.

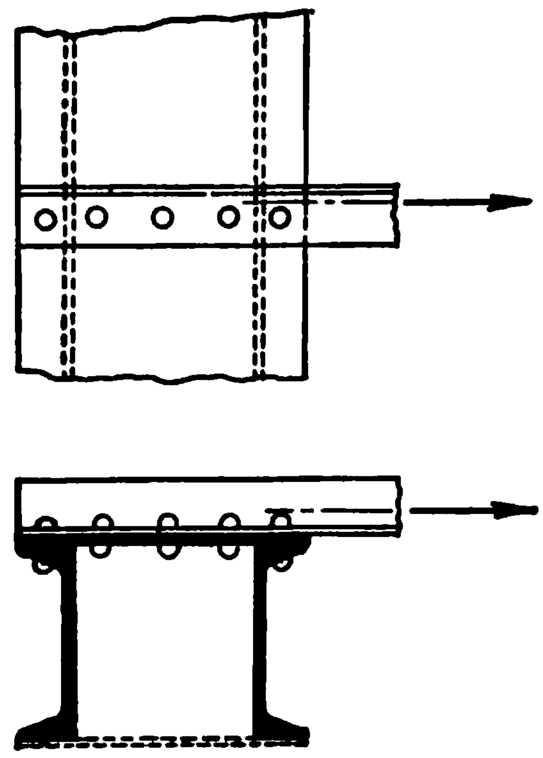


FIG. 24.

rivets in the angle take up the moment due to eccentric connections, as shown, and the stress in the angle at section $E\ E$ is uniform so far as horizontal bending is concerned. In a vertical plane such a joint plate is quite flexible and cannot therefore resist the moment due to eccentricity and a bending stress will result. If the connection is made to a member which is rigid in a vertical plane, also, as in Fig. 24, then the bending in that direction is also zero or very small.

The general principle involved is that there can be no bending stress without deformation, and if a member is held in a straight line by an attached member there can be no deformation and no bending stress. Moment due to eccentric connection is in such cases balanced by lateral forces in the connection itself so that the resultant action upon the connected member is central.

Tests of angles connected by one leg show results of 75 to 80 per cent of the strength of the material. This is increased 5 to 10 per cent by connecting both legs. Most of such tests are so made, however, as to allow free bending of the angle and do not simulate such conditions as represented in Figs. 23 and 24. Some tests in which the

line of application of the stress was placed in the gravity axis showed as good results as connecting both legs, and other tests, where the angle was rigidly supported against bending, showed no advantage from connecting both legs.* It should also be noted that connecting

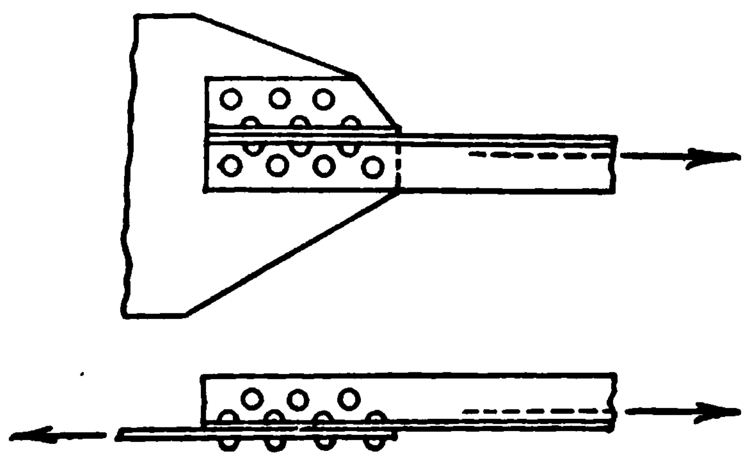


FIG. 25.

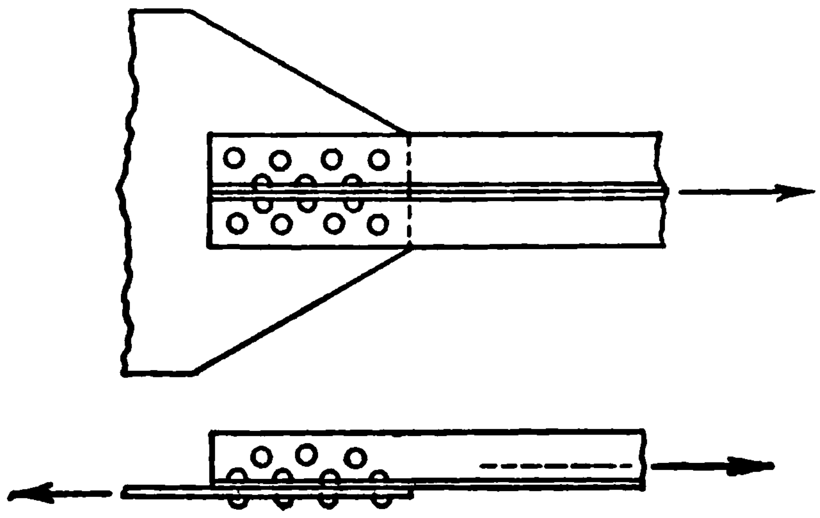


FIG. 26.

both legs does not necessarily give a central or supported connection. For example in Fig. 25 the connection of both legs does not eliminate bending movement in a vertical plane, which will reduce the strength of the angle at least 10 to 15 per cent. The same thing applies to two angles connected as in Fig. 26. The advantage of the lug in Fig. 25 is small, as the plate itself holds the members very rigidly against horizontal bending.

In the case of two or four angles connected by lattice, the end detail shown in Fig. 27 is poor. The joint plates A and B are relatively

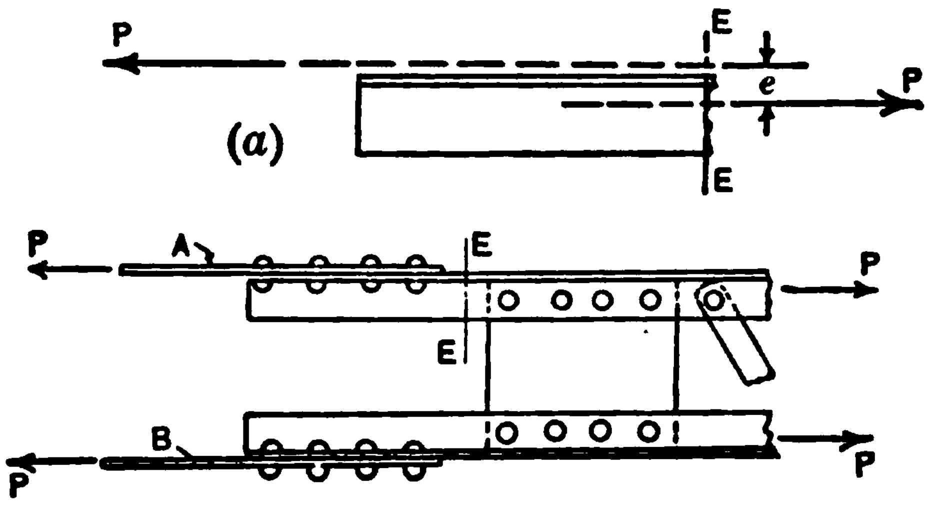


FIG. 27.

flexible and hence each angle is subjected to a moment equal to Pe at section E E. This is avoided by placing the batten or tie plate

* See paper by J. E. Greiner, Trans. Am. Soc. C. E., Vol. 38, 1897, p. 41; also paper by F. P. McKibben, *Eng. News*, July 5, 1906, p. 14.

near the end, as in Fig. 28, so it will support the angle. The forces acting at the end of the angle are then as represented in Fig. (*a*).

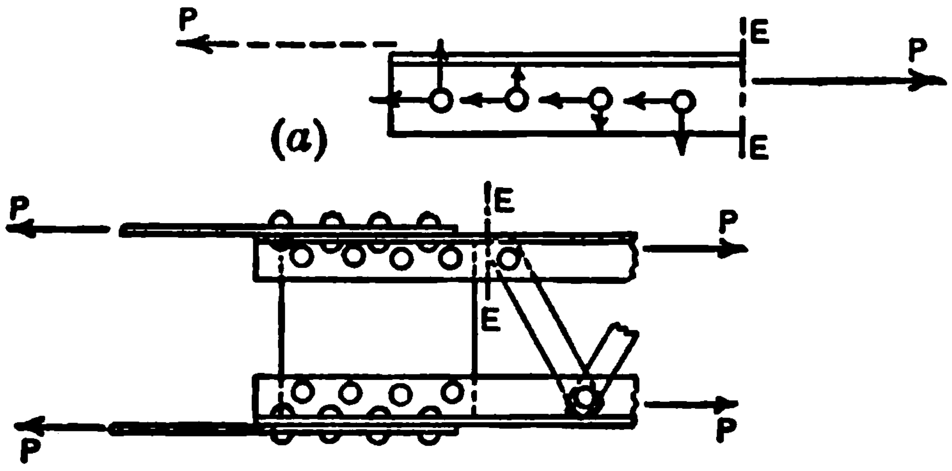


FIG. 28.

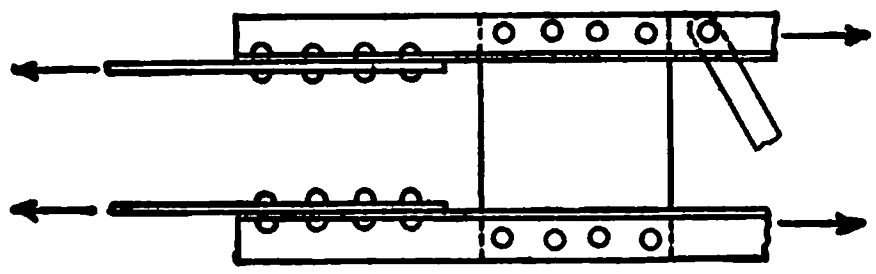


FIG. 29.

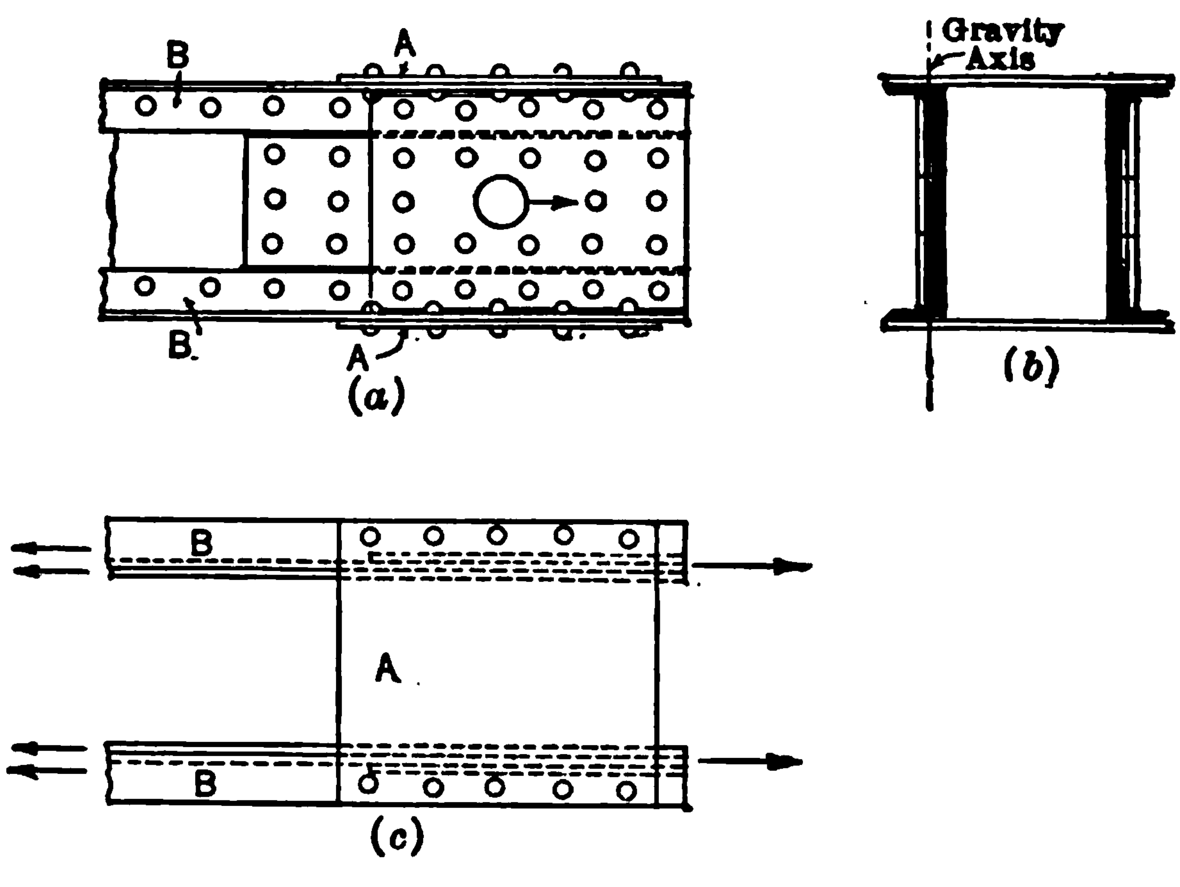


FIG. 30.

The action in Fig. 29 is the same as is Fig. 27 and cannot be so readily remedied without turning the flanges inward.

Fig. 30 shows an ordinary design of a pin-connected built member. The pins bear at the centre of the bearing area shown in Fig. (*b*). Very often this centre of bearing area is eccentric with respect to the gravity axis of the half member, thus causing some bending moment

in the half. To resist such moment, and any moment due to lateral force, the batten plates *A* are effective if placed over and extended somewhat to the right of the pin bearing. If placed to the left of the pin hole the moment must be carried by the resistance of the half member itself. The angles *B* receive their stress in one leg only, but as they are rigidly held to the other members they must act with them and will bend only if the entire half member bends. With batten plates as shown, all members will be held in line and the angles will be of full value.

Another common case of eccentric connection is the usual floor-beam attachment on one side of a vertical post. While the connection is in fact eccentric, if it is a rigid riveted joint the resulting bending moment in the post is a function of relative deflection of floor-beam and post, rather than of the eccentricity of the connection.

93. **Reduction of Section by Rivet Holes.**—In the case of the standard structural steel, rivet holes are generally made by punching, up to a plate thickness of 3/4 in. Above that thickness reaming or drilling is required. Punching of mild steel injures it to some extent, tests showing that the effect is slightly to reduce the ultimate strength and to increase the elastic limit. The ductility is reduced very considerably as compared to drilled or reamed holes. The effect of punching is greater the thicker the metal and harder the steel, and for metal thicker than 3/4 in., reaming or drilling is generally required. Some engineers require holes to be reamed for all thicknesses. Reamed or drilled holes show a somewhat higher ultimate strength and elastic limit than plain specimens. In deducting rivet holes no allowance is made for effect of punching, but the deduction is made for a hole 1/8 in. larger than the rivet. Where several rows of rivets are employed the determination of the true net section, or section of least strength, is not always a simple matter.

The necessary stagger of rivets to avoid diagonal rupture, and the calculation of net section through a line of staggered rivets, does not appear to have been well determined by experiment and various rules are employed for this purpose. The simplest method, and one which appears to be safe, is to determine rivet section by taking the least section across the member whether at right angles to the axis of the member or in a diagonal line. Thus in Fig. 31 the net section would

be either $A\ B$ or $A\ C\ D\ E$, whichever gives the least net area. This assumes that the tensile stress on a diagonal section, $C\ D$, is as great as on a right section. Theoretically this is not the case, but on the other hand the section is not symmetrical and the rivet holes cause

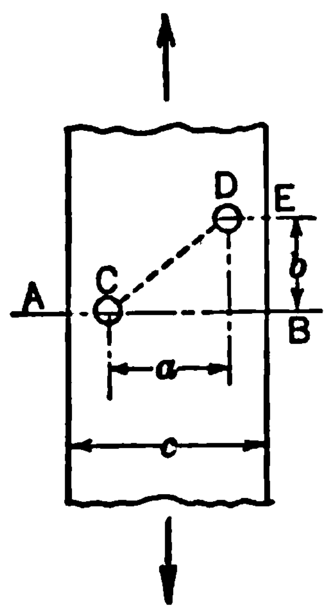


FIG. 31.

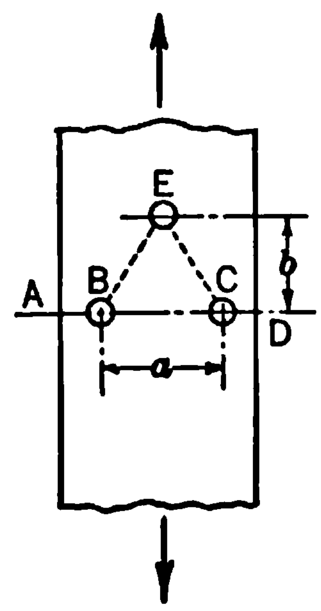


FIG. 32.

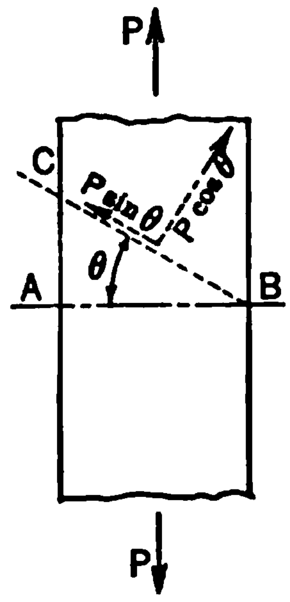


FIG. 33.

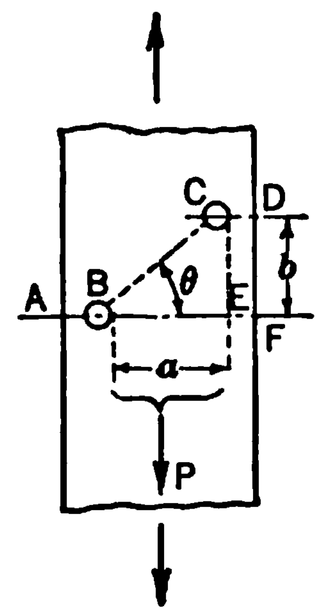


FIG. 34.

an irregular distribution of stress over the section so that this rule is probably not too liberal.

The necessary stagger b to make the net areas equal on this theory is readily found. If d = diameter of hole the net width on $A\ B$ is $c - d$. The net width on line $A\ C\ D\ E$ is $c - (a + d) + (\sqrt{a^2 + b^2} - d) = c - a - 2\,d + \sqrt{a^2 + b^2}$. For equal net widths we have $d = a + 2\,d - \sqrt{a^2 + b^2}$, whence

$$b = \sqrt{2\,a\,d + d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

With the arrangement shown in Fig. 32 the necessary stagger b to make net width on line $A\ B\ E\ C\ D$ equal to $A\ B\ C\ D$ is found to be

$$b = \tfrac{1}{2}\sqrt{2\,a\,d + d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Table III of Appendix B gives the required stagger of rivets to maintain net section as calculated by eq. (7).

A less stagger than provided above will result if the diagonal tension is calculated and provision made only for that stress. In a plate subjected to direct tension, Fig. 33, the greatest tension exists on a right section $A\ B$. On a diagonal section, $B\ C$, if the stresses are resolved into shear and direct stress, the total tension will be $P \cos \theta$, and the total shear will be $P \sin \theta$. In Fig. 34 the diagonal section

$B\,C$ will have to carry the same load as the right section $B\,E$, and to be of equal strength the unit tensile stress on $B\,C$ should not exceed that on $B\,E$. If P = total stress carried by strip a, then the unit stress on $B\,E = P/a\,t$. The component of this stress perpendicular to $B\,C$ is $P\cos\theta = Pa/\sqrt{a^2+b^2}$. The net area $B\,C$ is $(\sqrt{a^2+b^2}-d)t$ and the unit stress is $\dfrac{Pa}{\sqrt{a^2+b^2}\,(\sqrt{a^2+b^2}-d)t}$. Placing this equal to $P/a\,t$ and solving for b gives

$$b = \sqrt{\tfrac{1}{2}d^2 + d\sqrt{\tfrac{1}{4}d^2+a^2}}\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Since $\frac{1}{4}d^2$ is small as compared to a^2 we may write, approximately, $d\sqrt{\frac{1}{4}d^2+a^2} = d\,a$, whence

$$b = \sqrt{a\,d + \tfrac{1}{2}d^2} = 0.7\sqrt{2\,a\,d + d^2} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which is $\frac{7}{10}$ of the stagger required by eq. (7). In the case of angles and shapes the correct net section is found by considering the shape flattened out into a plate.

In designing tension splices it is desirable to so arrange the rivets that the deduction for rivet holes shall be as small as practicable. To this end the number of rivets in line at the end of a splice plate is reduced to one or two, and, if practicable, staggered with respect to rivets in other parts of the member. Thus in the splice shown in Fig. 35 (a), for net section in plate A, three rivet holes are deducted, in Fig. (b) two rivet holes. In Fig. (c) the net area on section E is found by deducting one hole, but on section F the stress is less than that on section E by the value of only the rivet a. If this value is less than the tension value of a strip of plate A equal to one rivet diameter in width, then the strength on section F, plus rivet a, is less than the strength on section E. The net section is then section F, plus the value of one rivet. Where the rivet value is single shear this condition may arise, but for double shear or bearing it is not likely to.

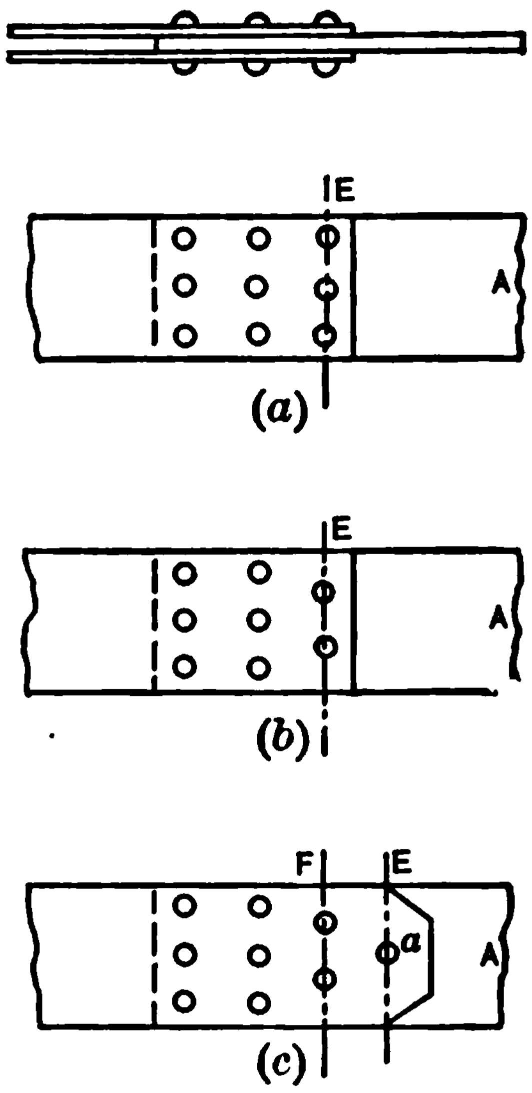


FIG. 35.

Similar calculations may be needed wherever the number in the second row is considerably greater than in the end row. In Figs. (*b*) and (*c*) the net section through the splice plates requires the deduction of three holes.

94. Practical Rules for Rivet Spacing.—Besides the question of net tensile section the rivets must not be spaced so near together in the line of stress as to cause the rivet to pull through the plate, or so near the edge or end of the plate as to give rise to danger of splitting out. In compression members, rivets connecting component parts must not be spaced so far apart as to allow the individual elements to buckle; and in all built-up members the parts must be well held together to prevent entrance of water and consequent rusting. If once started, rusting between plates will continue, and often to such an extent as to force the plates wide apart between rivets, bending them in a series of wave-like curves. The specifications of Appendix A give the following rules for spacing:

(39) The minimum distance between centres of rivet holes shall be three diameters of the rivet; but the distance shall preferably be not less than 3 in. for 7/8-in. rivets and 2 1/2 in. for 3/4-in. rivets. The maximum pitch in the line of stress for members composed of plates and shapes shall be 6 in. for 7/8-in. rivets and 5 in. for 3/4-in. rivets. For angles with two gauge lines and rivets staggered, the maximum shall be twice the above in each line. Where two or more plates are used in contact, rivets not more than 12 in. apart in either direction shall be used to hold the plates well together. In tension members composed of two angles in contact, a pitch of 12 in. will be allowed for riveting the angles together.

(40) The minimum distance from the centre of any rivet hole to a sheared edge shall be 1 1/2 in. for 7/8-in. rivets and 1 1/4 in. for 3/4-in. rivets, and to a rolled edge 1 1/4 in. and 1 1/8 in. respectively. The maximum distance from any edge shall be eight times the thickness of the plate but shall not exceed 6 in.

(43) The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivet for a length equal to one and one-half times the maximum width of the member.

95. Tables and Standards.—In Appendix B there are given for the convenience especially of the student, several tables, standard clearances, and conventional signs in general use. These are as follows:

I. *Shearing and Bearing Values of Rivets.*

II. *Size and Weight of Rivet Heads and Clearance for Driving.*

III. *Stagger of Rivets for net Section.*
IV. *Standard Gauge Lines in Angles.*
V. *Conventional Signs for Riveting.*
VI. *Standard Eye-Bars.*

96. Bolted Connections.—Bolts are ordinarily used only for temporary work as in fitting-up and erection. For this purpose rough bolts are used. Occasionally, permanent connections are made by the use of bolts, in which case the bolts are accurately finished to exact dimensions. The holes must also be drilled to an exact fit with the bolts. Such joints are more reliable in direct tension than riveted joints and it may be questioned whether they would not be in general more reliable for field connections where the rivets are very long.

CHAPTER VI

PLATE GIRDER BRIDGES

97. **A Plate Girder** is essentially a built-up I-beam in which the flanges are composed of an assemblage of shapes (generally plates and angles), which are connected by rivets to a solid web plate. Economy requires the material to be arranged as in the rolled I-beam, the flange material being concentrated as much as practicable at the

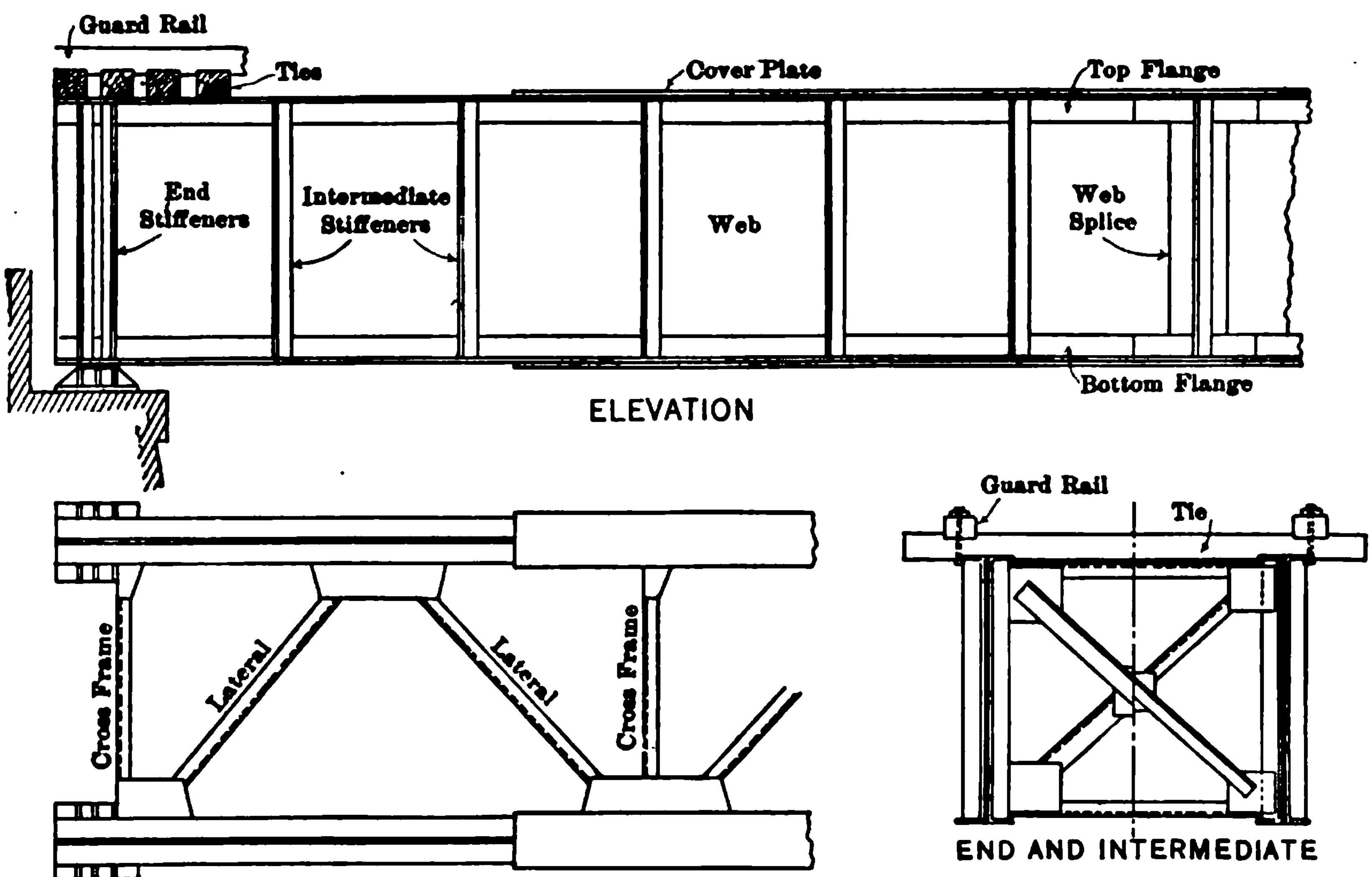


FIG. 1. Deck Bridge.

edges of the girder and the web made only so thick as necessary to carry the shearing stresses. In a beam of such cross-section, most of the bending moment is resisted by the flanges and most of the shear by the web.

98. *Plate Girders for Bridges.*—Plate girders are used for railroad bridges for span-lengths up to about 100 ft., and many have been built of lengths from 100 to 125 ft. For such long spans, the weight

of metal in the web is considerably more than that of the web members of a riveted truss, and for this reason the truss is likely to be the more economical. For equal weights, however, the plate girder will be the cheaper, owing to lower shop cost and therefore lower pound price.

Fig. 1 illustrates the usual arrangement of a deck bridge. The girder flanges are composed of two angles and one or more *cover-plates*.

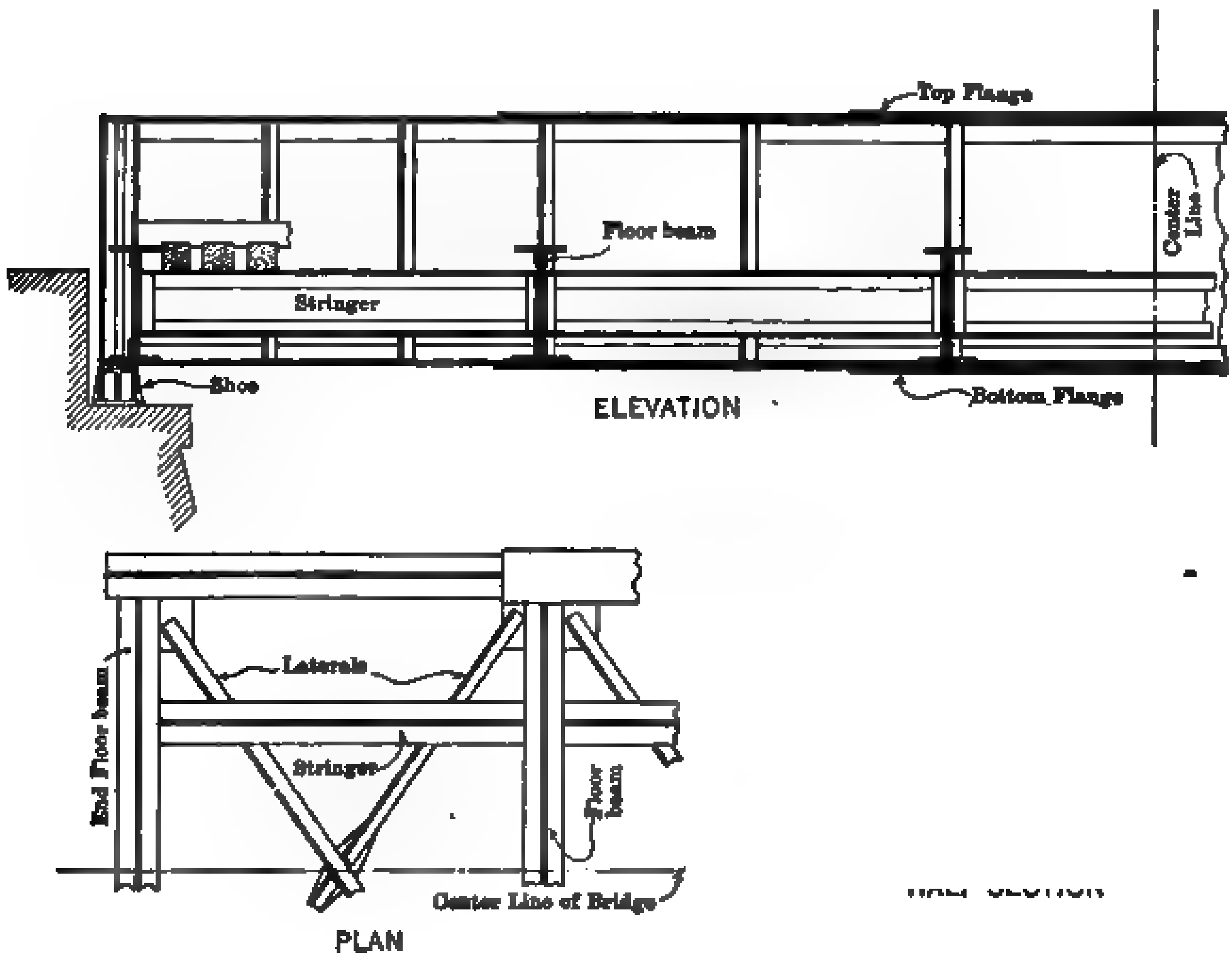


FIG. 2. Through Bridge.

The two girders are connected by transverse frames, called *cross-frames*, and by lateral bracing in the plane of the upper flanges, and frequently, also, in the plane of the lower flanges. The ties rest on the top flanges. (See Art. 110 for other forms of flanges.) Except in the case of very shallow girders, the web plate is stiffened at intervals by angles called *stiffeners*, as shown in Fig. 1. Other stiffeners are also provided at points of heavy concentrations, such as the end stiffeners over the abutment in Fig. 1.

Fig. 2 shows a common arrangement for a through bridge. The

track is supported on longitudinal *stringers*, usually of I-beams. These are framed into *floor-beams*, which are riveted to the webs of the main girders. The load upon the girders is therefore concentrated at certain points, which may be called *panel points*, as in a truss. Since

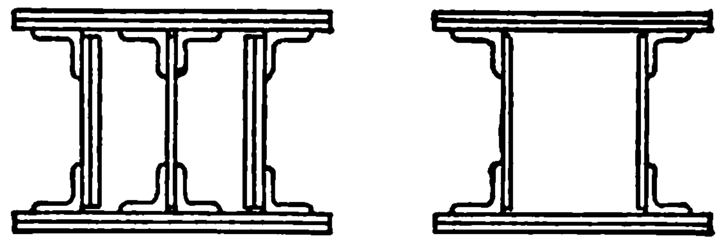


FIG. 3. Box Girders.

the floor-beams may readily be attached to the girders at frequent intervals, the panel length is, for economical reasons, made relatively short, generally from 10 to 15 ft.

In the through girder a lower lateral truss is used, and the upper flanges are braced laterally by means of *gusset* plates attached to the floor-beams.

Figs. 1 and 2 show the usual methods of supporting the track where the ordinary wooden floor is used. Art. 135 illustrates various other methods of supporting the track, including the use of continuous *solid floors* of steel or concrete.

99. *Plate Girders for Buildings.*—In building construction, limitations of headroom generally require the depth of the beam or girder to be made less than the economical depth, which frequently necessitates the use of a much greater web area than can be supplied by a single plate. Arrangements, such as shown in Fig. 3, are often used in which two or more webs are employed. Such girders are commonly called *box girders*. Here the webs carry a very considerable part of the bending moment. Heavy web reinforcement is often required in building construction to take care of column loads as illustrated in Fig. 4. Such reinforcement may consist of plates and angles or channels.

100. Moments and Shears.—In the design of a plate girder we need to know first the maximum moments and shears at various sections, calculated for both dead and live loads. These are determined by methods fully explained in Part I.

The dead load may be considered as uniform, and as continuously applied on the girder, thus giving parabolic moment curves and straight

line shear curves, as explained in Arts. 72 and 74, Part I. Locomotive concentrations are usually employed in live-load calculations. In the case of the deck bridge, or the through bridge having a solid or continuous floor, the separate wheel loads may be assumed to act directly upon the girders in calculating moments and shears therein.

For short spans it is sufficient to find the maximum live-load moment at or near the centre and the maximum shear at the end, and then to calculate values of moments at other sections on the assumption that the moment curve is a parabola, and values of shears

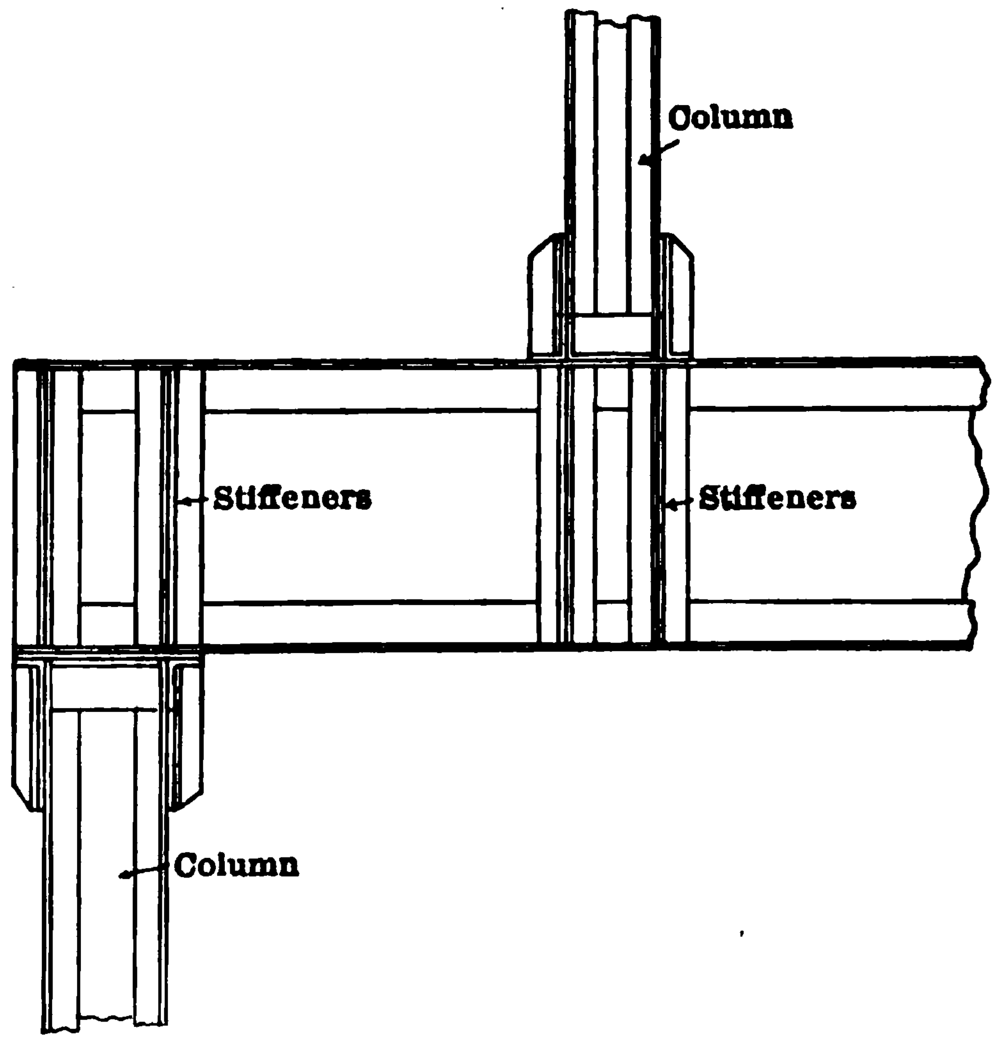


FIG. 4.

by the rule given in Part I, Art. 170, namely, that the maximum shear at the quarter point and centre are equal, respectively, to five-eighths and two-sevenths of the maximum end shear. The table of Art. 169, Part I, gives maximum moments at or near the centre, and maximum end shears for all spans up to 125 ft., for Cooper's E–50 loading. For long spans the moments and shears should be computed at intervals of 5 to 8 ft.

In the case of the through bridge of the type shown in Fig. 2 the live load is applied to the girders at *panel points*. The maximum moments should be found at these points, and the maximum shears in the several panels, as in truss analysis.

101. Internal Stresses in a Girder.—In order to design the various details of a girder intelligently it is necessary to have a clear understanding of the internal stresses which are caused in the various parts by the moments and shears in the beam considered as a whole. In analyzing these stresses, the girder will be considered to act as a solid beam of the same *gross* cross-section. That is, the rivets are assumed to connect the parts so effectively that they will act as a

(*a*) (*b*) (*c*) (*d*)

FIG. 5. Direct and Shearing Stresses in a Plate Girder.

unit, and, for the present, the reduction in section caused by the rivet holes will be neglected.

A study will be made of the stresses in a girder (Fig. 5) 48 ins. deep, web plate 48 x 3/8 ins., and flanges consisting of two 6 x 6 x 1/2-in. angles.

102. *Stresses on a Vertical Section.*—Consider any vertical section $A B$, Fig. 5 (*b*), and suppose that at this section the total resisting moment is M and the total vertical shear is V, corresponding to the external moment and shear. The intensity of the direct or horizontal stress at any point Q, distant y from the neutral axis is given by the formula

$$f_y = \frac{M y}{I} \quad \ldots \ldots \ldots \ldots \quad (1)$$

in which I is the moment of inertia of the section about the neutral

axis. For different values of y the intensity of stress varies directly as y and is represented by the straight line $C D$ of Fig. 5 (c). The stress on *extreme fibre* =

$$f_{max} = \frac{M c}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The intensity of the shearing stress at point Q is given by the equation

$$v_y = \frac{V m}{I b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which V = total vertical shear;
m = statical moment of area of section outside of below) point Q taken about the neutral axis, and
b = width of beam at point Q, = thickness of web in this case.

For the case at hand, Fig. (d) shows the distribution of vertical shear over the section, for an assumed maximum shear of 7,500

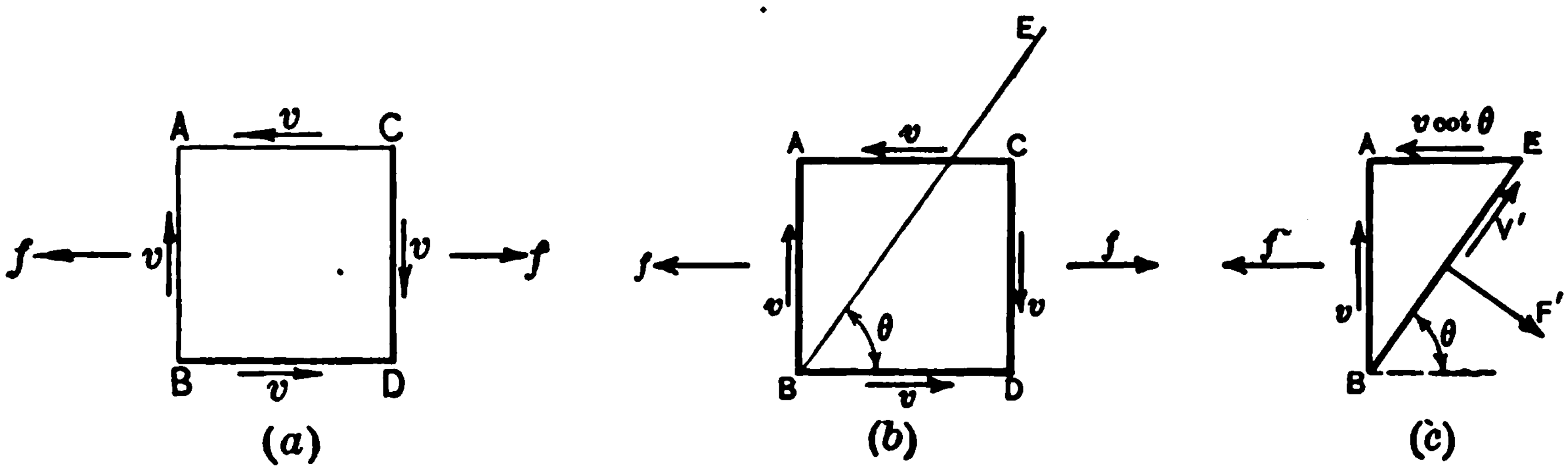


FIG. 6.

lbs. per sq. in. at the neutral axis. It will be seen that the shearing stress is nearly constant over the entire web, varying only from 7,500 to 6,250 lbs. per sq. in. or 16⅔ per cent from the maximum. In this case, to assume the total shear uniformly distributed over the web, would give a value for this shear of 6,430 lbs. per sq. in.

103. *The Horizontal Shear.*—Consider a square element of the web of unit dimensions (unit thickness also) at point Q, enlarged in Fig. 6. The faces $A B$ and $C D$ being taken near together, the forces acting on these faces may be assumed to be equal, and will consist of a tensile stress f and a shearing stress v. For equilibrium

of moments, there must exist shearing stresses on the faces $A\ C$ and $B\ D$, also equal to v. In general, the direct stresses acting on these faces are zero, that is, there is no compression or tension acting in a vertical direction; but in the vicinity of points of application of external loads these relations will be somewhat modified and there will exist more or less direct stress on the faces $A\ C$ and $B\ D$. The forces v acting on $A\ C$ and $B\ D$ represent the *horizontal* shearing stresses, and this analysis shows that the horizontal shear intensity is equal to the vertical.

In the design of a girder the horizontal shear must be provided for in any *horizontal* splice, as in the riveting between the flanges and web and between two adjacent elements of the flange, or along any other horizontal joint. The vertical shear must likewise be provided for in any *vertical* joint or splice, such as the usual web splice. And, finally, the direct stresses must be provided for in any splice where these stresses are involved. In short, to correctly design the riveting in any joint of a girder, the distribution of stress intensity, as above outlined, must be considered.

104. *Stresses on Inclined Sections.*—Consider again the unit element represented in Fig. 6. Pass any section $B\ E$ at angle θ with the horizontal, and consider the portion $A\ B\ E$, Fig. 6 (c). On the section $B\ E$ the resultant stress may be resolved into the shear V' and direct stress F'. The shear acting on $A\ E = v \cot \theta$. From $\Sigma x = 0$ and $\Sigma y = 0$, taken in directions parallel and perpendicular to the face $B\ E$, we derive the equations:

$$F' = f \sin \theta + v \cot \theta \sin \theta + v \cos \theta$$

$$= f \sin \theta + 2 v \cos \theta. \qquad \text{(4)}$$

$$V' = f \cos \theta + v \cot \theta \cos \theta - v \sin \theta$$

$$= f \cos \theta + \frac{v}{\sin \theta} (\cos^2 \theta - \sin^2 \theta) \qquad \text{(5)}$$

The stresses per unit area are:

$$f' = F' \sin \theta = f \sin^2 \theta + 2 v \cos \theta \sin \theta$$

$$= \tfrac{1}{2} f (1 - \cos 2 \theta) + v \sin 2 \theta \qquad \text{(6)}$$

$$v' = V' \sin \theta = f \cos \theta \sin \theta + v (\cos^2 \theta - \sin^2 \theta)$$

$$= \tfrac{1}{2} f \sin 2 \theta + v \cos 2 \theta \qquad \text{(7)}$$

For a maximum value of f', we find by differentiation,

$$\tan 2\theta = -\frac{2v}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and for maximum v',

$$\tan 2\theta = \frac{f}{2v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Substituting these values of θ in the equations for f' and v' we find the maximum values to be respectively

$$f'_{max} = \tfrac{1}{2}f + \sqrt{\tfrac{1}{4}f^2 + v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$v'_{max} = \sqrt{\tfrac{1}{4}f^2 + v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From this analysis it is seen that the plane on which the direct stress or shear is a maximum at any point is, in general, neither vertical nor horizontal, but is inclined at some angle θ, determined by the relation between f and v. Furthermore, it is to be noted from the values of $\tan 2\theta$ in eqs. (8) and (9), that the plane on which the direct stress is a maximum (or minimum) is at 45° inclination from that on which the shear is a maximum (or minimum). These maximum stresses and their directions at any point are readily determined when the horizontal direct stress f and the vertical shear v for the given point are known.

Eqs. (8) and (10) show that at the neutral axis where $f = 0$, $f'_{max} = v$, and the direction of this maximum direct stress is at 45° inclination ($\theta = 135°$). At other points the direction of the maximum direct stress varies, depending upon the relative values of f and v

From eq. (6) the value of f' at any point, for $\theta = 45°$ is

$$f'_{45°} = \tfrac{1}{2}f + v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

As an illustration of the relative magnitude of the inclined direct stresses the values of f'_{max} have been calculated for the section shown in Fig. 5 for an assumed value of extreme fibre stress $f =$ 14,000 lbs. per sq. in., and maximum shearing stress v at the neutral axis of 7,500 lbs. per sq. in. The results are given in Fig 7 (*c*). The variation of direct stress f is given by the line $C\,D$, and the maximum tensile and maximum compressive stresses by the curves $D\,H\,G$ and $C\,E\,F$, all measured from the axis $G\,F$. The directions of these maximum direct stresses are given in Fig 7 (*b*). Fig 7 (*d*) shows the intensities of the tensile and compressive stresses calculated for a

constant inclination of 45° from the horizontal at all depths. It is to be noted that the maximum stress in the web, Fig. 7 (*c*), reaches a value of 13,420 lbs. per sq. in. at a point close to the flange angle, where the horizontal direct stress is only 10,500 lbs. per sq. in.*

This illustration shows that under certain conditions it is quite possible for the maximum inclined stress in the web near the flange to be as great as or even greater than the allowable working stress on extreme fibre. Generally, in simply supported beams, the maximum

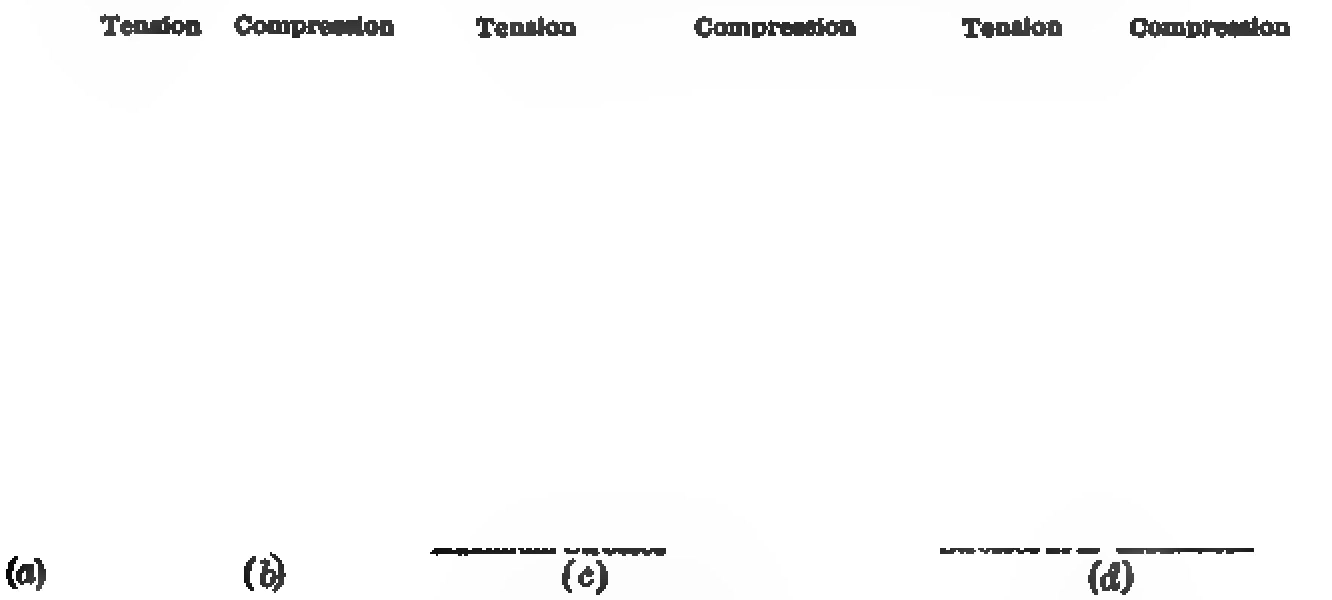


FIG. 7. Inclined Tensile and Compressive Stresses in a Plate Girder Web.

permissible fibre stress f will not occur at the same section as the maximum shearing stress v, but in cantilever and continuous girders this coincidence is likely to occur.

Generally speaking, it is not necessary in practice to calculate the inclined stresses as here done, but such stresses must be recognized

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* In dealing with stresses in two directions it is probably more accurate to assume that the elastic limit of the material is reached when the strain or distortion reaches a certain limit than when the stress reaches the elastic limit value as determined by tests under stress in one direction only. According to this maximum-strain theory the effect of a combination of shear and direct stress is measured by a value of f' determined by the formula

$$f' = \frac{1 - \mu}{2} \cdot f + (1 + \mu) \sqrt{\tfrac{1}{4} f^2 + v^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

in which μ = "Poisson's ratio" of lateral to longitudinal distortion under direct stress. For steel, μ is generally taken at ¼ giving

$$f' = \tfrac{3}{8} f + \tfrac{5}{4} \sqrt{\tfrac{1}{4} f^2 + v^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

Applying eq. (b) to the above case gives $f' = 14{,}140$ instead of 13,420, an increase of about 5.4 per cent.

in the design. The large compressive stresses in the upper half of the web tend to buckle or wrinkle it and require special attention in stiffening the same. As to the inclined tensile stresses, it is obvious that these should not materially exceed the allowable tensile working stress. Where a very heavy flange is attached to a deep, thin web there will be a line of weakness, or high stress, along the inner line of rivets, or inner edge of flange, which may require reinforcement by using angles of wider vertical legs, or by the use of wide plates beneath the angles, or by a thicker web. Still more severe conditions exist in the rolled I-beam with light web, as in this case the web is of

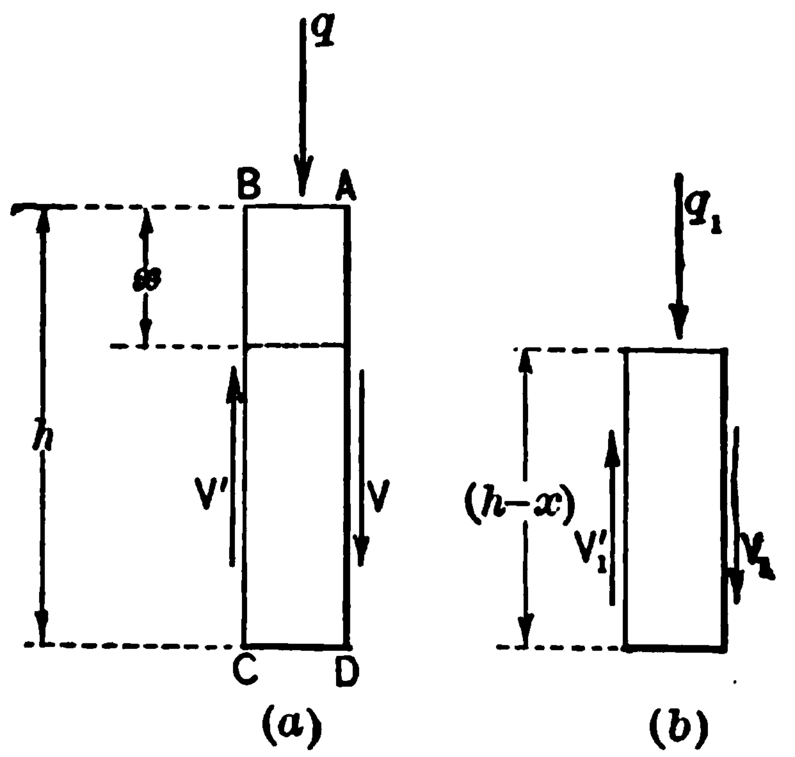


FIG. 8.

minimum thickness within a very short distance from the outside fibre. The value of f'_{max} may in such a case be as much as 15 or 20 per cent higher than the allowable direct fibre stress.

The maximum shearing stresses given by eq. (11) are at all points equal to the maximum stress f'_{max}, minus $\frac{1}{2} f$. With the usual units employed these stresses hardly require consideration. The maximum intensity is just inside the flange, where it is equal to 8,170 lbs. per sq. in. in the example given.

105. *Effect of Vertical Loads.*—In the preceding analysis the effect of external loads applied between the assumed vertical sections was neglected. Where heavy vertical loads rest upon the flanges the result is to add considerably to the maximum stresses as found above. If such loads are placed on the top flange the compressive stresses

are increased; if on the bottom flange, the tensile stresses are increased. Any heavy concentrated load should be transferred to the web by means of stiffening angles or other form of reinforcement, which will distribute the load over a considerable length of web.

The effect of a vertical load applied to the web at any point can be analyzed theoretically in the same manner as in Art. 104. Let $A\,B\,C\,D$, Fig. 8 (*a*), be a section of the web, one unit long (in the direction $B\,A$) and one unit thick, and suppose a vertical load be applied at the top equal to q per unit area. This load q is, in effect, the increment of shear on this element and is equal to $V' - V$. If we consider the stress q_1 on a horizontal surface at any distance x below the top, we have, Fig. 8 (*b*), $q_1 = V'_1 - V_1$. The direct compressive stress q, therefore, decreases from top to bottom in the ratio that the difference in the shears decreases on the portion below the section considered. In the case of a web plate the shearing stress is practically constant from top to bottom, hence we may assume that the vertical stress q_1 varies uniformly from top to bottom. If it is zero at the bottom then at the neutral axis it is one-half the value at the top.

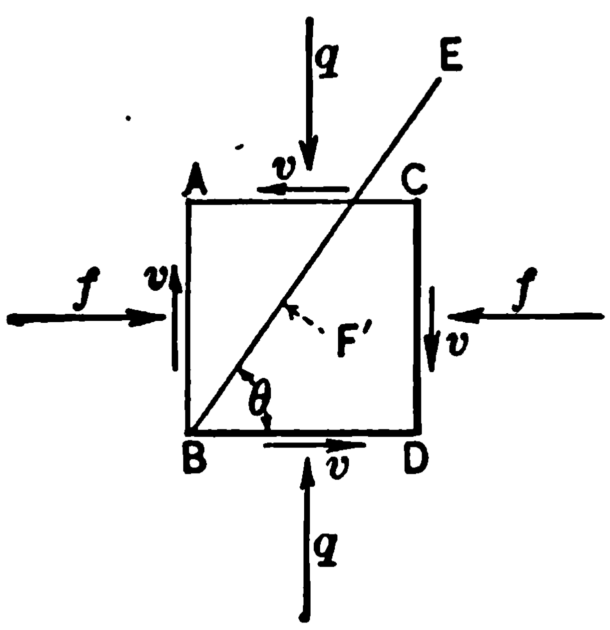


FIG. 9.

Having found the value of q at any desired point the stresses on the faces of any element will be as indicated in Fig. 9. The element is here taken above the neutral axis so that the direct stresses f are compressive.

An analysis for maximum direct stress f' at angle θ, as in Art. 104, gives the following values:

For maximum f'

$$\tan 2\,\theta = \frac{2\,v}{f - q} \quad \ldots\ldots\ldots\ldots \quad (13)$$

$$f'_{max} = \tfrac{1}{2}\,(f + q) + \sqrt{\tfrac{1}{4}(f - q)^2 + v^2} \quad \ldots\ldots \quad (14)$$

As an example, suppose that at a point near the upper flange of a girder, a heavy concentrated load causes a stress q = 5,000 lbs. per sq. in.; also that f = 12,000 and v = 7,000 lbs. per sq. in. From eq. (14) we then find that the maximum compressive stress is

$$f'_{max} = \tfrac{1}{2}(12{,}000 + 5{,}000) + \sqrt{\tfrac{1}{4}(12{,}000 - 5{,}000)^2 + 7{,}000^2}$$
$$= 16{,}300 \text{ lbs. per sq. in.}$$

The direction of this maximum stress is given by

$$\tan 2\,\theta = \frac{14{,}000}{7{,}000} = 2, \text{ whence}$$

$$2\,\theta = 243^\circ\ 26';\ \theta = 121^\circ\ 43'.$$

If the vertical stress q did not exist the value of f'_{max} would be 15,200 lbs. per sq. in., and the angle $\theta = 114^\circ\ 42'$.

106. The Flange Area.—Considered as a solid beam, the resisting moment of a plate girder is given by the formula for beams

$$M = \frac{f\,I}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

in which

f = stress in extreme fibre;
c = distance of extreme fibre from neutral axis;
I = moment of inertia of the section about the neutral axis.

Let A_f = area of each flange of the girder, assuming the areas equal;
A_w = web area;
h = distance between centres of gravity of flanges, usually called the "effective" depth;
h_1 = depth of web;
h_2 = maximum depth of girder = $2\,c$;
t = thickness of web;
I_f = moment of inertia of flange section about its gravity axis.

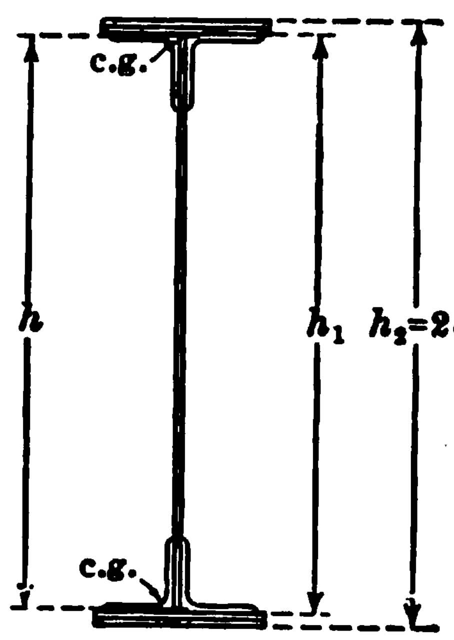


FIG. 10.

Then for the entire girder

$$I = 2\,A_f\left(\frac{h}{2}\right)^2 + 2\,I_f + \frac{t\,h_1^3}{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and

$$M = A_f f\,h \cdot \frac{h}{2\,c} + \frac{2\,I_f f}{c} + \frac{A_w f\,h_1}{6} \cdot \frac{h_1}{2\,c} \quad . \quad . \quad . \quad . \quad (17)$$

In the design of ordinary girders it is customary and sufficiently

accurate to take $h_2 = h_1 = h = 2c$, and to neglect the quantity I_f. This gives

$$M = A_f f h + \frac{A_w f h}{6} = \left(A_f + \frac{A_w}{6}\right) f h \quad . \quad . \quad . \quad . \quad (18)$$

From eq. (18) it is seen that the moment of resistance is found by adding to the area of one flange one-sixth the web area, and multiplying this sum by $f h$. The equivalent girder flange, $\left(A_f + \frac{A_w}{6}\right)$, may thus be treated exactly as the chord member of a horizontal-chord truss.

107. *The Tension Flange.*—In the design of a girder it is customary to determine the area of the tension flange on the basis of the allowable stress on net section, and then to make the compression flange of the same gross area. In arriving at the net section of the flange, rivet holes must be deducted, as explained in Art. 143. In the web, also, there will be deducted a vertical row of rivet holes on account of the rivets connecting a stiffener angle or a splice plate which may occur at or near the section considered. A 3-in. pitch of rivets, with 7/8-in. rivets (deducting 1-in. holes), reduces the web section one-third, and a 4-in. pitch one-quarter. The corresponding *equivalent* web area to be added to the flange will then be one-ninth and one-eighth, respectively, instead of one-sixth, as in eq. (18). As a 4-in. pitch will usually be sufficient, an allowance of one-eighth the web area as flange area will generally be sufficiently accurate, and is adopted as a rule in the specifications given in Appendix A. Hence we have, generally, for tension flanges

$$M = f\left(A'_f + \frac{A_w}{8}\right) h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$A'_f = \frac{M}{f h} - \frac{A_w}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

in which A_w refers to *gross* web area and A'_f to *net* flange area.

Under some specifications the web area is not to be considered in determining flange area, or, as generally stated, the flanges must be designed to carry all the moment. The web does, in fact, carry its due proportion of moment, except at splices where the web stress

has not been properly provided for. In that case the splice rivets will be overstressed and the web at that section will carry somewhat less than its share, so that under such conditions it is desirable to design the flange for the entire moment. A better and more exact method is to make due allowance for the web and then design the splices accordingly. This gives a stronger girder for the same expenditure.

108. *The Compression Flange.*—This is usually made of the same gross area as the tension flange, thus making the fibre stress on gross area materially less than the working tensile stress. This makes allowance in a rough way for the requirements of the upper flange as a compression member, or long column. As such a column it

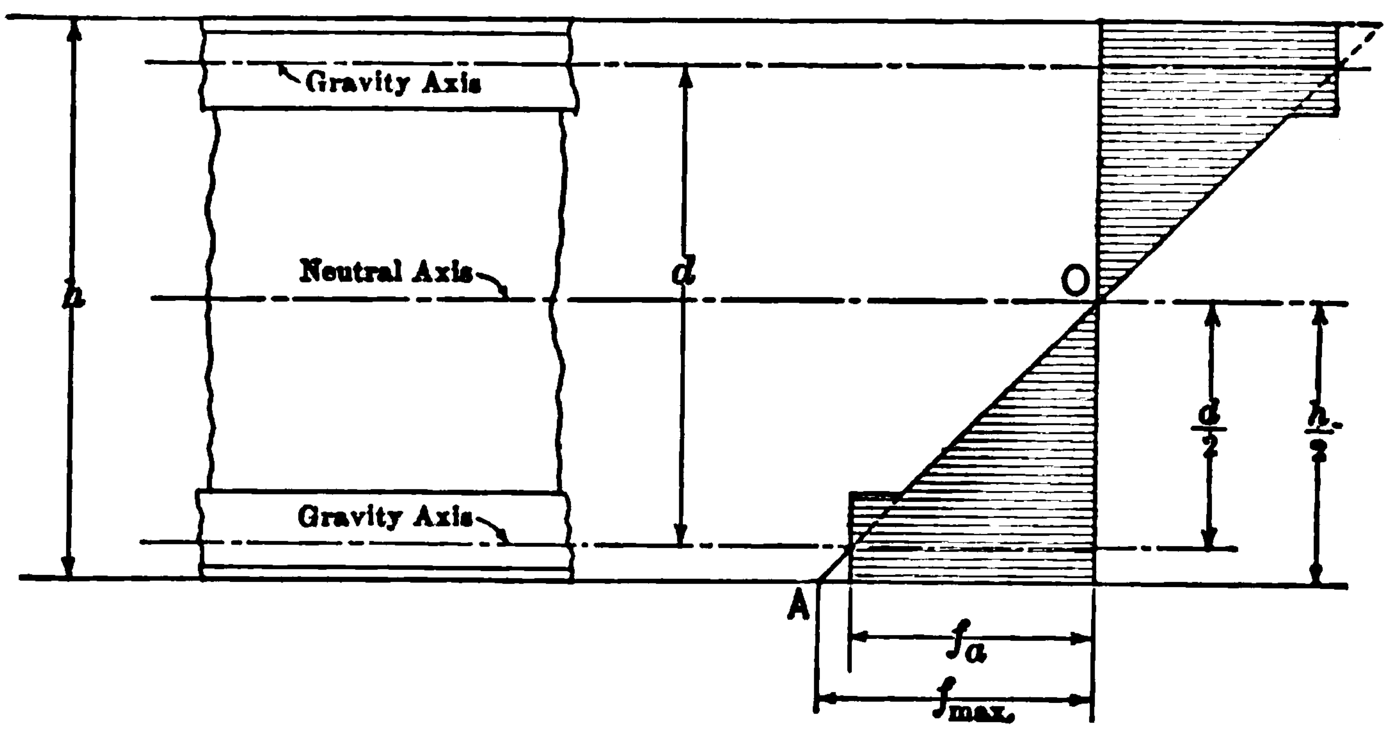


FIG. 11.

is fully supported vertically by the web, but for lateral support other provision must be made. In the deck bridge this support is furnished by the lateral system, and in the through bridge by knee braces or gusset plates made the full depth of the girder and attached to the stiffeners and floor beams.

In the specifications, Art. 30, a special column formula is given for determining the limits of stress on the compression flange, which is about equivalent to the general formula in Art. 16.

109. Determination of Flange Area by the Moment of Inertia Method.—The method of design given in the preceding articles is based on the approximate formulas of eq. (18). This method assumes that the fibre stress is uniform across the entire flange section

and that it is equal to the fibre stress at its centre of gravity. It also assumes that the fibre stress in the web plate at the centre of gravity of the flange section is the same as that in the flange, and that it is zero at the neutral axis of the girder. Fig. 11 shows the assumed variation in fibre stresses. The true variation in fibre stress is shown in Fig. 5 (*c*), Art. 101. If linear distribution of stress be assumed for the conditions shown in Fig. 11, taking the fibre stress at the centre of gravity of the flange section as a base, the line $O A$ represents approximately the true variation in stress. For the conditions shown, $f_{max} = f_a h/d$, where f_{max} = extreme fibre stress, f_a = average fibre stress, h = total depth and d = effective depth. If the centre of gravity of the flange section is located any considerable distance inside the extreme fibre, it can be seen that the true maximum fibre stress is appreciably greater than the average. In practice it will be found that for girder flanges of the type used in the design given in this chapter, the effective depth varies from 90 to 95 per cent of the total depth. The maximum fibre stress is therefore greater, by approximately 10 or 5 per cent respectively, than the average fibre stress assumed in the design. Where the effective depth is less than about 95 per cent of the total depth of the girder, it will be best to use a more exact method of calculation than the usual approximate one employed. For this purpose use is made of the general formula for flexure, $f = M c/I$, and the actual moment of inertia determined.

The application of this formula must be modified somewhat to take into account the conditions met with in built-up beams. On the tension side of such beams the section is reduced by the presence of rivets. If the effect of these rivet holes upon the position of the neutral axis is taken into account, it will be found that the neutral axis will change position at each line of rivets. Since the net sections on rivet lines take up only a small part of the total length of the girder, and since deflection and distortion are functions of the gross girder section, it is evident that the neutral axis and moment of inertia should be determined from the gross area of the girder section. A correction can then be made for increase in fibre stress at rivet lines due to reduction of sectional area.

The proper procedure is then to determine the center of gravity

and moment of inertia of the gross section of the girder, and from this information get the extreme fibre stress on gross section. The variation of such fibre stress will be as represented in Fig. 12 (*b*). Fig (*c*) represents a section through rivet holes which, on the tension side, must be deducted. The neutral axis will, as explained above, remain practically at the same position as in Fig. (*b*). Hence the fibre stresses on net area must be greater than on gross area in order that the total tension remain the same as the total compres-

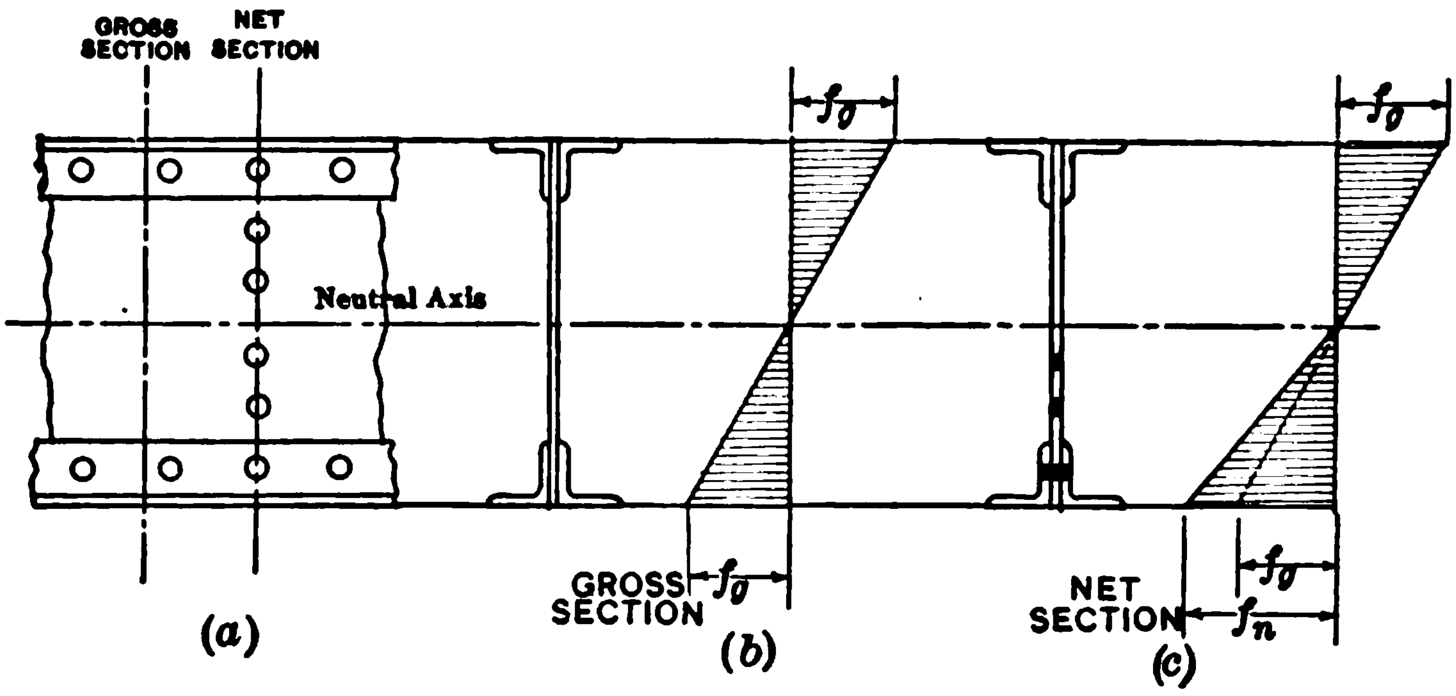


FIG. 12.

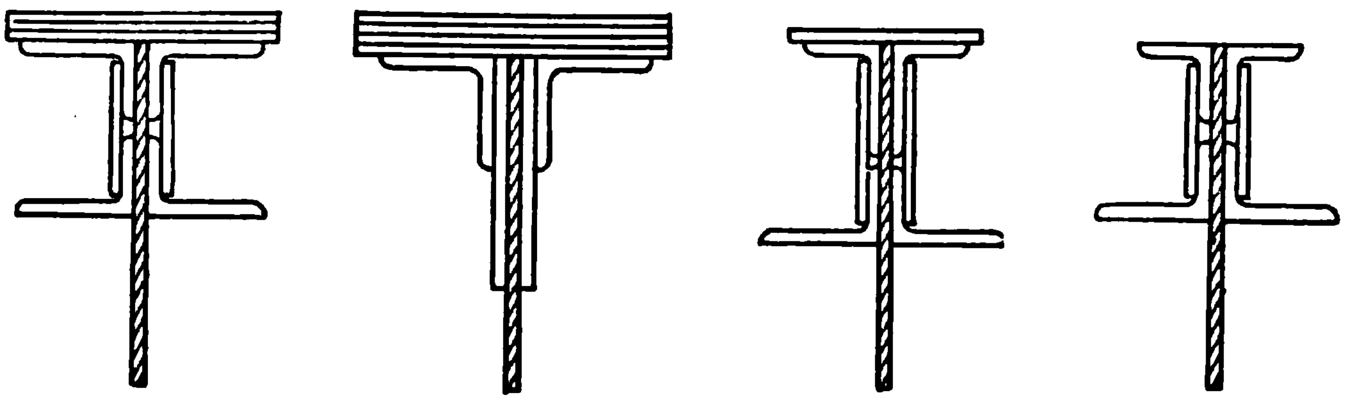
sion. In Fig. 12 (*c*) is shown the variation of fibre stress on net section, assuming a uniform variation.

Assuming the fibre stresses on the gross and net tension flange areas to be inversely proportional to the available flange area, we have

$$f_n = f_g \frac{F_g + \frac{1}{6}\text{Web area}}{F_n + \frac{1}{8}\text{Web area}} = f_g \frac{A_f + \frac{1}{6}A_w}{A'_f + \frac{1}{8}A_w} \quad . \quad . \quad . \quad (21)$$

where f_n is the extreme fibre stress on net flange area; f_g is extreme fibre stress on gross flange area, calculated with I and c for gross areas; and A_f and A'_f are, respectively, gross and net flange areas. In case the rivet spacing in the web is such that more or less than one-eighth of the web is available flange area, the denominator of the above equation should be modified accordingly. (For illustrative example, see Art. 144.)

110. Forms of Flanges.—A flange composed of two angles and cover plates is the most economical form as regards efficient use of material in resisting bending moment, but variations from this form are frequently made for practical reasons. Fig. 13 shows several such modifications. Figs. (*a*) and (*b*) show forms suitable for very heavy flanges where the use of two angles and cover plates alone



(*a*) (*b*) (*c*) (*d*)

FIG. 13.

would require excessively long rivets, or cause difficulty in properly riveting flange to web. In Fig. (*a*) four rows of rivets can be used between flange and web. In Fig. (*b*) the rivets attaching angles to web can be counted in double shear. Figs. (*c*) and (*d*) illustrate arrangements used where it is desired to keep the top surface of the girder at a uniform elevation for the convenience of floor details. The flange area is varied by omitting, first, the side plates and then the lower angles.

111. Lengths of Flange Plates.—In constructing a flange built up of angles and plates, or of several elements of any form, the maximum section is required only in the region of maximum moment. Toward the ends of the girder a less area will suffice. To determine the theoretical required lengths of the flange plates, or other flange elements which may be made of less length than the girder, the maximum bending moments must be calculated at frequent intervals (about five feet apart). The length over which any given area of flange is adequate is then determined by calculating the moment of resistance of such area and comparing with the bending moment. This comparison is best made graphically, as illustrated in the example of Art. 145. The theoretical length being found, the actual length is made enough greater to accommodate two or three rows of rivets beyond the theoretical length. Where the floor rests upon

the top flange it is desirable to extend one cover plate the entire length of the girder in order to strengthen the flange and flange rivets against the effects of the vertical load. (See Art. 78 of the specifications.)

Where the load can be considered as uniform, the lengths of

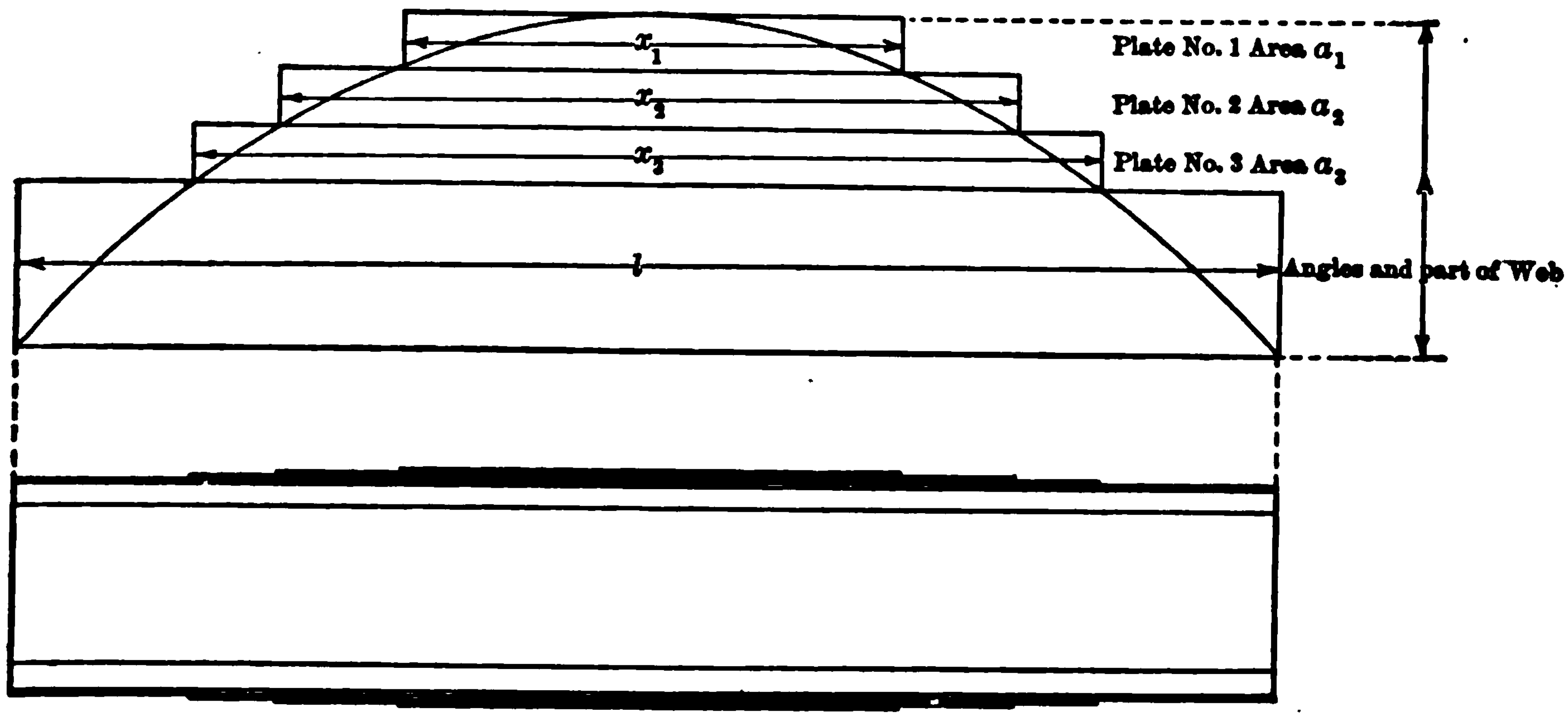


FIG. 14.

flange elements can be determined readily by use of the parabolic moment curve as follows (Fig. 14):

Let A = total flange area required, including that part of the web considered as flange.

$a_1, a_2, \ldots a_n$ = areas of the several plates or other elements, the subscript denoting the number of the plate from the *outside;*

$x_1, x_2 \ldots x_n$ = theoretical required lengths of the several plates or elements;

l = length of girder c. to c. of bearings.

Then, assuming the flange areas to vary by the parabolic law, with the moments, we have $\dfrac{a_1}{A} = \dfrac{x_1^2}{l^2}$, whence $x_1 = l\sqrt{\dfrac{a_1}{A}}$.

Likewise $x_2 = l\sqrt{\dfrac{a_1 + a_2}{A}}$, and for the nth plate

$$x_n = l\sqrt{\frac{a_1 + a_2 + \ldots a_n}{A}} \quad \ldots\ldots\ldots \quad (22)$$

112. The Web Plate.—The thickness of web plate is determined from the following requirements: Its section must be sufficient to resist the maximum shearing stress; its thickness must be sufficient to give the requisite bearing area for flange rivets to permit of proper rivet spacing; and it must be thick enough to avoid undue warping and buckling in manufacture and handling, or excessive use of stiffeners to prevent buckling under load. The question of durability may also be a factor. Except in the case of very heavy girders, the web is made of the same section throughout. Splices are required only by reason of limitations of manufacture of long plates, and this requirement will, in some cases, have a bearing on width and thickness used for maximum economy.

Standard practice requires a minimum thickness for railway bridges of 3/8 in.; for highway bridges of 5/16 in.; and 1/4 in. for building work where corrosion is not a factor. While specifications do not require the analysis of web stresses in accordance with Art. 104, those principles should be kept in mind, and in selecting working stresses some allowance may need to be made for the reasons there given.

In very deep girders a certain minimum ratio of thickness to depth is desirable to prevent undue warping and buckling in manufacture. In the specifications (Art. 29) the least ratio of thickness to unsupported depth is placed at 1/160, but considerably smaller ratios are used by many engineers.

Generally, the shearing unit stress is taken about two-thirds the tensile unit stress. In the specifications the unit shearing stress is 10,000 lbs. per sq. in. on gross-section as compared to 16,000 lbs. per sq. in. for the tensile unit stress. As a matter of fact, the web stresses which generally control its strength are the tensile and compressive stresses rather than the shearing stresses, and the basis of selection of the shearing unit stress should be the intensity of the direct stresses involved, and which are in part dependent upon the shearing stresses. From this standpoint, a vertical row of rivet holes in the web is of little effect in reducing its strength except as to tensile stress, and hence it is hardly necessary to deduct rivet holes for shear in the same manner as for tension. Still less do they affect the compressive strength of the web, its weakest point.

113. Rivets Joining Flange and Web.—The rivets connecting flange to web must be designed to carry the horizontal shear existing between web and flange, as shown in Art. 103; also any direct vertical load which may come upon the flange. The following methods of calculation consider the entire flange concentrated at its centre of gravity as is permissible in the usual design. The use of the more general formula is explained in Art. 117.

114. *Case I. Web Moment Neglected.*—Fig. 15. Consider a section of the web $q\,q'$ of width equal to the pitch of rivets, and in Fig.

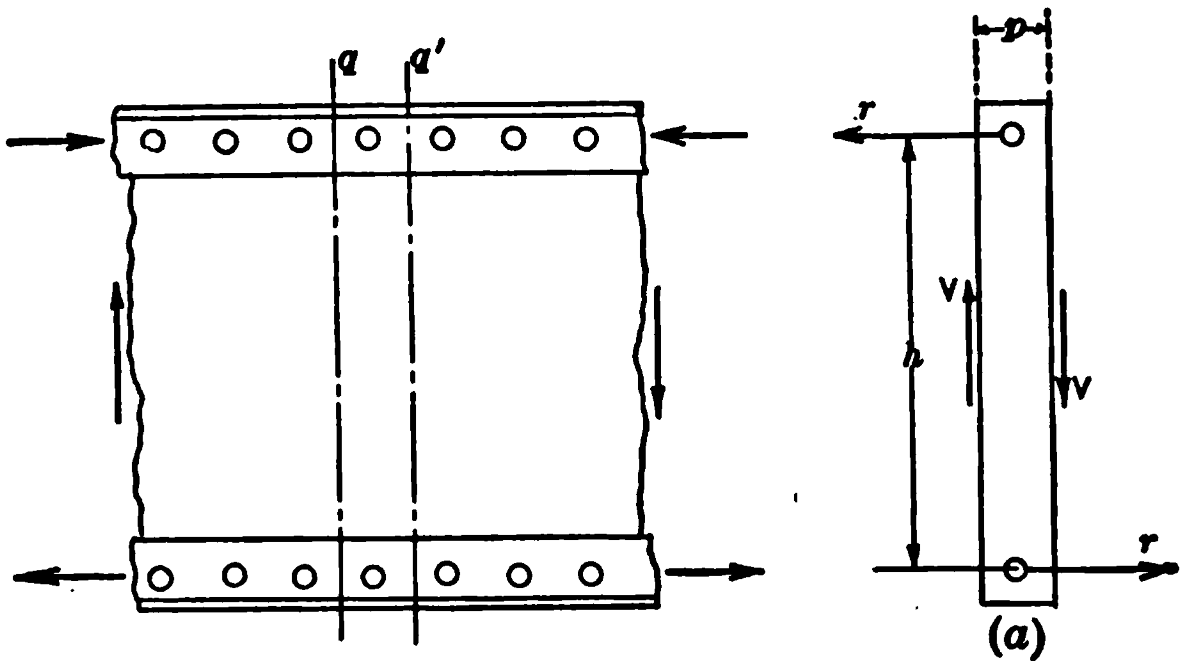


FIG. 15.

(*a*) represent this section with all forces acting. The vertical shear is V and the stress in the rivet is r (assumed to be the rivet value), this stress being that produced by the horizontal shear in the distance p. This horizontal shear over the length p is, in other terms, the increase of moment developed in the distance p, which is transmitted from flange to web, or *vice versa*, as we may view the matter. The web itself actually carries part of the moment, but it is assumed here that the flange carries all the moment.

An equation of moments gives $h\,r = V\,p$, whence

$$p = \frac{h\,r}{V} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

which is the proper formula for rivet pitch for shear only.

If there is, at the same time, a vertical load on the rivet due to a load applied directly to the flanges, the total rivet stress will be the resultant of this vertical load and the horizontal stress due to shear.

If w = load on flange per unit length, then $p\,w$ = vertical load on one rivet. The horizontal force acting is, from eq. (23), equal to Vp/h. The total stress is then

$$r = \sqrt{\left(\frac{V\,p}{h}\right)^2 + (p\,w)^2} \quad \ldots\ldots\ldots \quad (24)$$

whence

$$p = \frac{r}{\sqrt{\left(\frac{V}{h}\right)^2 + w^2}} \quad \ldots\ldots\ldots \quad (25)$$

115. *Case II. Web Moment Considered.*—If the resisting moment of the web is considered, then the increment of flange stress to be transmitted from flange to web will be proportionately reduced. If A'_f = net flange area and $\frac{1}{8}$ of the web is considered as flange, then the stress in the rivet due to shear only will be

$$r = \frac{V\,p}{h} \times \frac{A'_f}{A'_f + \frac{1}{8}A_w}$$

Equation (23) then becomes

$$p = \frac{hr}{V} \cdot \frac{A'_f + \frac{1}{8}A_w}{A'_f} \quad \ldots\ldots\ldots \quad (26)$$

For a vertical load on the flange, as in eq. (25)

$$p = \frac{r}{\sqrt{\left(\frac{V}{h} \cdot \frac{A'_f}{A'_f + \frac{1}{8}A_w}\right)^2 + w^2}} \quad \ldots\ldots \quad (27)$$

116. Rivets Joining Flange Elements.—The rivets connecting the various parts of the flange, such as flange plates to angles, are stressed in the same manner as the rivets connecting flange to web; that is, they serve to transmit the horizontal shear existing along the plane in question.

From eq. (3), Art. 102, this shear is seen to be proportional to the statical moment of the area of the flange *outside* the plane of stress, taken about the neutral axis of the beam. Inasmuch as the entire flange may usually be assumed as concentrated at its centre of grav-

ity, it follows that these shearing stresses between the several elements will be proportional to the sectional areas outside of the respective shearing planes. If, for example, the total cross-section of the flange plates is one-half the total flange area, then the shearing stress between plates and angles is one-half the stress between angles and web. Or, in general, if v_n = shearing stress per unit length, a_1 a_2, a_3, . . . a_n be the sectional areas of the various flange plates (or other elements connected consecutively), counting from the outside, and A_f is the total flange area, then

$$v_n = \frac{V}{h} \cdot \frac{a_1 + a_2 + \ldots a_n}{A_f + \frac{A_w}{8}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

With the usual arrangement of flange plates and angles, there will be at least two rows of rivets in single shear connecting plates and angles, and not more than two rows in double shear between flange and web, whose value will generally be determined by bearing on the web. With the same rivet spacing in the horizontal and the vertical legs, the strength of the horizontal connection will be ample with the usual proportions employed. With very thick webs and large flange plates this item may require consideration.

117. Rivet Spacing Determined from Moment of Inertia.—The horizontal shearing stress at any horizontal section is given by the general formula of eq. (3)

$$v = \frac{Vm}{Ib} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where V = total vertical shear in the girder; m = statical moment of the area *outside* of the plane in question about the neutral axis; I = moment of inertia of gross area and b = thickness of girder at section in question. The horizontal shear *per unit of length* of girder is

$$v_n = vb = \frac{Vm}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

a more convenient formula for the purpose in question.

This formula is of general applicability and may well be used

where the web is relatively thick, as in shallow box girders, and also in the case of very deep flanges where it cannot well be assumed that the flange is concentrated at its centre of gravity.

Take, for example, the flange section shown in Fig. 16. The shear per unit of length between any two consecutive elements may

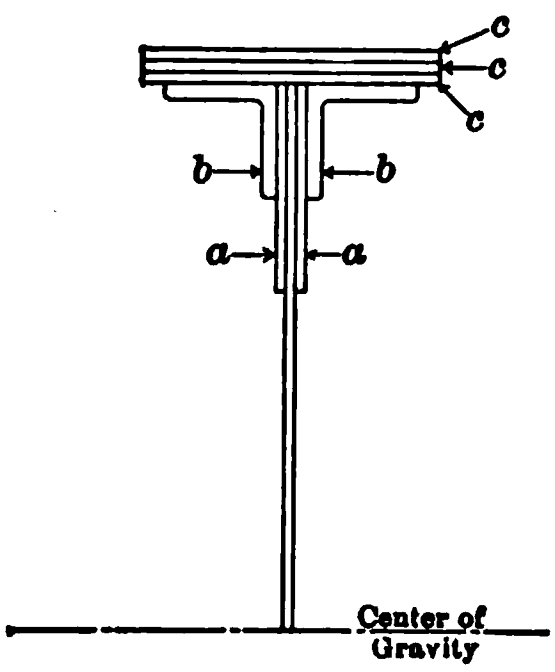


FIG. 16.

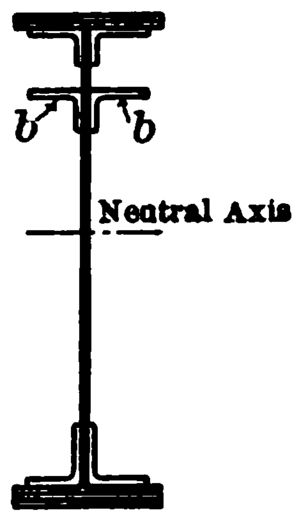


FIG. 17.

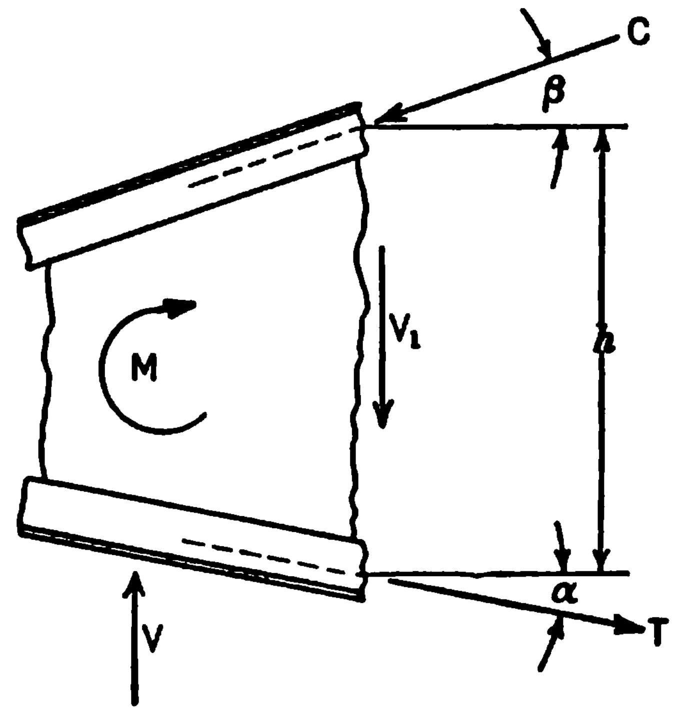


FIG. 18.

be found from eq. (30). Between the web and the side plates *a*, the value of *m* is the moment of the area of the entire flange (except the web) taken about the neutral axis. For shear between the side plates *a* and the angles *b*, the value of *m* is the moment of the angles *b* and the cover plates, and for the shear between the cover plates and angles it is the moment of the cover plates alone.

118. Girders Having Unsymmetrical Sections.—For special reasons the section of a plate girder is sometimes made unsymmetrical. In this case the centre of gravity of the section should be found, using for this purpose the *gross* area.* The resisting moment, or fibre stress, should then be determined from the general formula involving the moment of inertia of the total section. In this calculation it is preferable to use *gross* areas and corresponding fibre stresses, making the proper allowance for rivet holes in tension flange in determining the allowable stress on gross section.

In a case such as shown in Fig. 17, the riveting of angles *b* to the web will be determined by the general formulas of Art. 117. The

* The location of the neutral plane in a beam is a question of distortion, and therefore gross areas should be used, as explained in Art. 109

horizontal shear per unit length between angles b and web will be equal to $v_n = \frac{Vm_b}{I}$, where I is the total moment of inertia and m_b is the statical moment of the area of angles b about the neutral axis.

119. Girders with Inclined Flanges.—In certain girders used in building work, floor beams of bridge trusses, and in turntable girders, one or both flanges may be inclined to the horizontal. The analysis

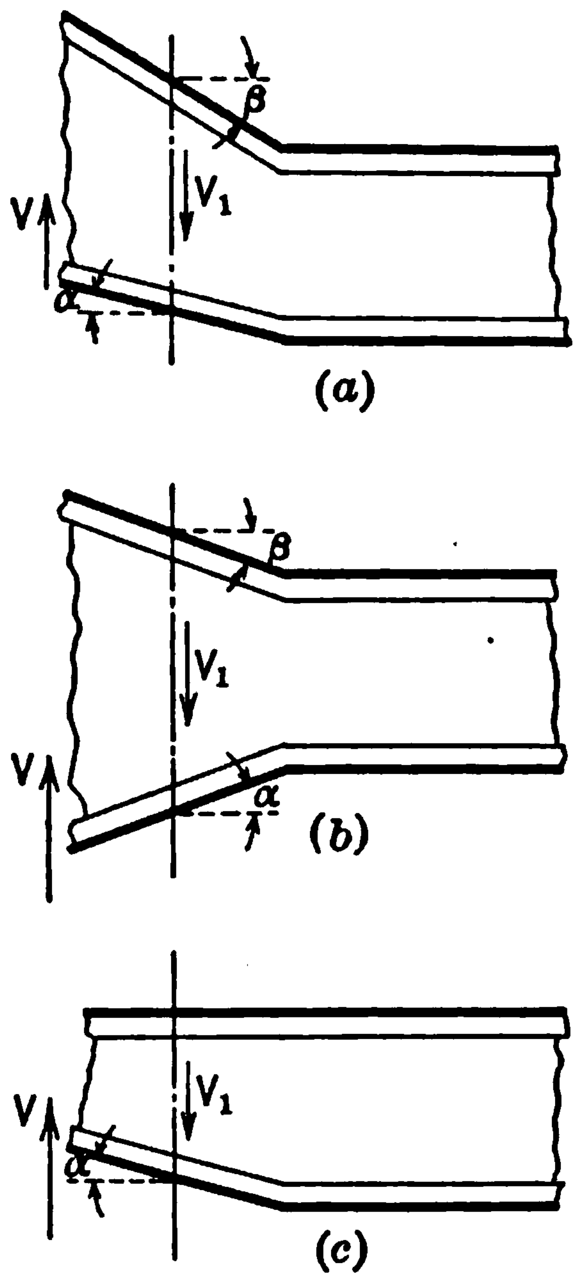


FIG. 19.

for such a case for flange and web stress and rivet spacing will be here given.

120. *Flange and Web Stresses.*—In Fig. 18 the bending moment is M and total vertical shear V. The shear carried by the web is V_1 and the flange stresses are C and T. Taking moments about centres of gravity of opposite flanges we have

$$C = \frac{M}{h \cos \beta} \text{ and } T = \frac{M}{h \cos \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

As in Art. 107 one-eighth of the web section may be taken as flange if the web is properly spliced.

For web stress V_1 we have

$$V_1 = V - C \sin \beta - T \sin \alpha \quad . \; . \; . \; . \; . \; (32)$$

whence from (31)

$$V_1 = V - \frac{M}{h} (\tan \alpha + \tan \beta) \; . \; . \; . \; . \; . \; . \; (33)$$

When the inclinations of the flanges are of opposite sign to that shown in Fig. 18, the shear V being still upward, the effect will be to change the sign of one or both of the tangents in eq. (32). Referring to Fig. 19, we have:

For Fig. (*a*)

$$V_1 = V - \frac{M}{h} (\tan \alpha - \tan \beta) \quad . \; . \; . \; . \; . \; (34)$$

For Fig. (*b*)

$$V_1 = V + \frac{M}{h} (\tan \alpha + \tan \beta) \; . \; . \; . \; . \; . \; . \; (35)$$

For Fig. (*c*)

$$V_1 = V - \frac{M}{h} \tan \alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (36)$$

121. *Rivet Spacing.*—The rivet spacing given by the formulas of Arts. 114 and 115 will no longer hold true, as the shear is modified by the vertical components of the flange stresses at the section in question. Fig. 20 (*a*) shows a girder with flanges inclined to the horizontal at given angles. The pitch p will be measured along the axis of the girder flange. Consider a section of the web plate, equal in width to the horizontal projection of the rivet pitch. This section is shown in Fig. (*b*) with all forces acting. In determining the rivet pitch in the lower flange a strip of web of width $p \cos \alpha$ will be considered. Taking moments about the top flange rivet gives $r\, h \cos \alpha = V_1\, p \cos a$, from which

$$p = \frac{r\, h}{V_1} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (37)$$

where r is the value of a rivet and h is the vertical distance between rivet lines, which for purposes of calculation may be taken as equal to the effective depth of the girder.

From eqs. (33) and (37)

$$p = \frac{r\,h}{V - \frac{M}{h}(\tan\alpha + \tan\beta)} \qquad \ldots\ldots \quad (38)$$

Eq. (38) assumes that all of the bending moment is taken by the flange angles. In case one-eighth of the web area is considered

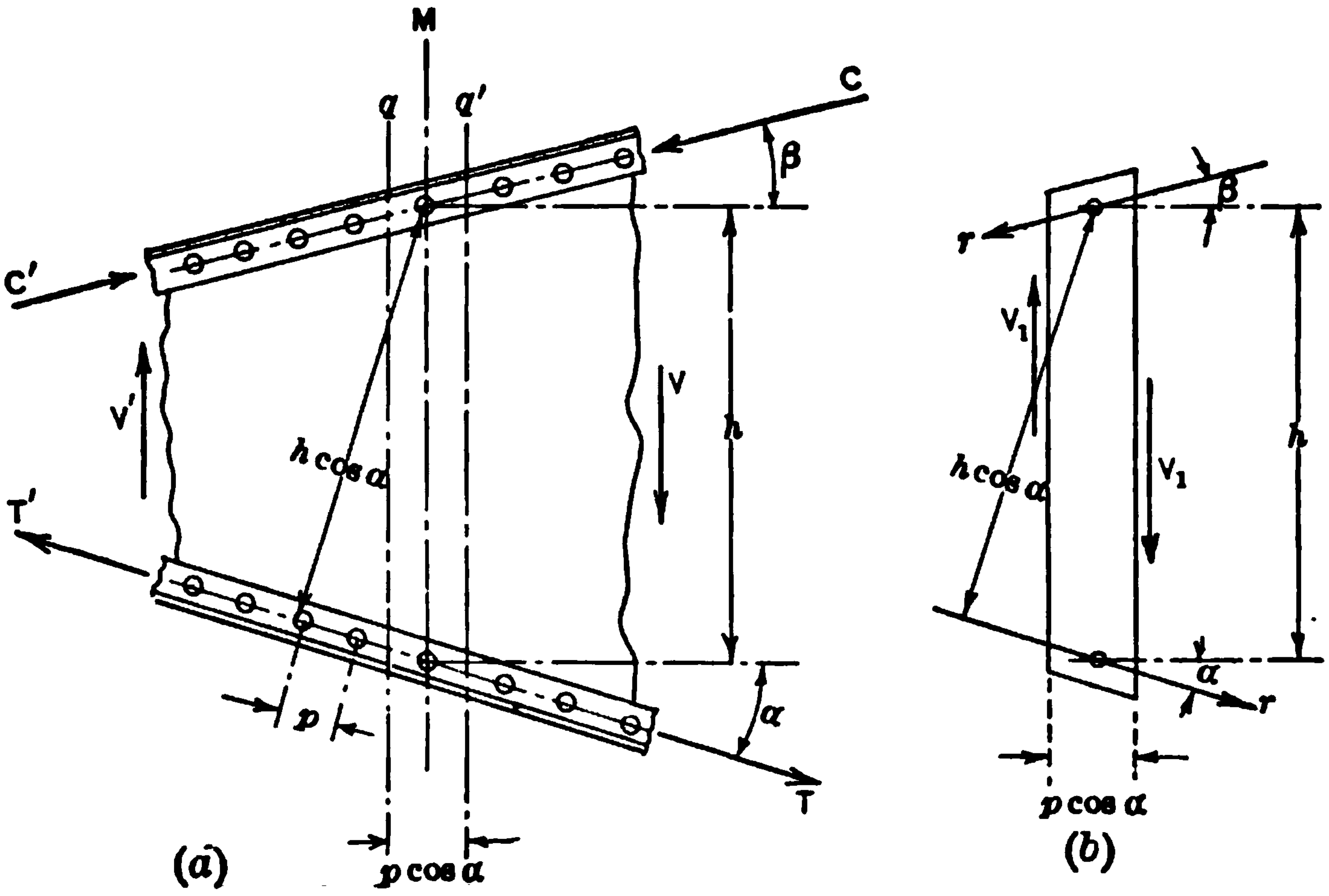


FIG. 20.

as flange section, the moment carried by the flange proper is $M \dfrac{A'_f}{A'_f + \dfrac{A_w}{8}}$, where A'_f = net flange area and A_w = web area.

Proceeding as in Art. 115 we get

$$p = \frac{r\,h}{V - \frac{M}{h}(\tan\alpha + \tan\beta)} \cdot \frac{A'_f + \frac{A_w}{8}}{A'_f} \qquad \ldots \quad (39)$$

The top flange spacing is determined by a similar method. Con-

sidering a strip of web of width $p \cos \beta$, the resulting equations are exactly the same as eqs. (38) and (39). This shows that the rivet pitch measured along the top and bottom flanges is to be the same. In order to make the rivet pitch such that top and bottom flange rivets will lie in the same vertical plane, it is only necessary to calculate the pitch for the flange which has the greater inclination to the horizontal. The rivets in the other flange can then be placed in the same vertical plane.

When the inclinations of the flanges differ from that shown in Fig. 20 (a), the rivet pitch can be determined from eq. (38) by making the proper changes in the values of tan α and tan β. If the flanges are both horizontal, tan α and tan β equal zero, and the equations of Arts. 114 and 115 follow at once. For the conditions shown in Fig. 19 (a) we have

$$p = \frac{r\,h}{V - \frac{M}{h}(\tan \alpha - \tan \beta)} \qquad \text{. . . . . . . (40)}$$

In the same way, for Fig (b), we have

$$p = \frac{r\,h}{V + \frac{M}{h}(\tan \alpha + \tan \beta)} \qquad \text{. . . . . . . (41)}$$

and for Fig. (c)

$$p = \frac{r\,h}{V - \frac{M}{h}\tan \alpha} \qquad \text{. . . . . (42)}$$

122. Web Splices.—To splice a web plate properly, both shearing and bending stresses should be provided for. The splice should be designed to transmit as directly and efficiently as possible all stresses which actually occur in the web. In simple terms, the splice should be made equivalent at all points to the web net section.

In determining rivet values in such a splice it may be assumed that vertical or shearing stresses will stress the rivets equally, and that horizontal or bending stresses will stress the rivets in proportion to their distances from the neutral plane. Rivets near the neutral axis are therefore not very effective in carrying horizontal stress, but their resistance is, in fact, proportional to the stress to be carried.

A rational splice, designed to transmit the stresses as directly as possible, should provide splice plates of suitable section, extending the entire depth of the girder, with a uniform spacing of rivets, in one or more rows, from top to bottom. The main splice plates generally

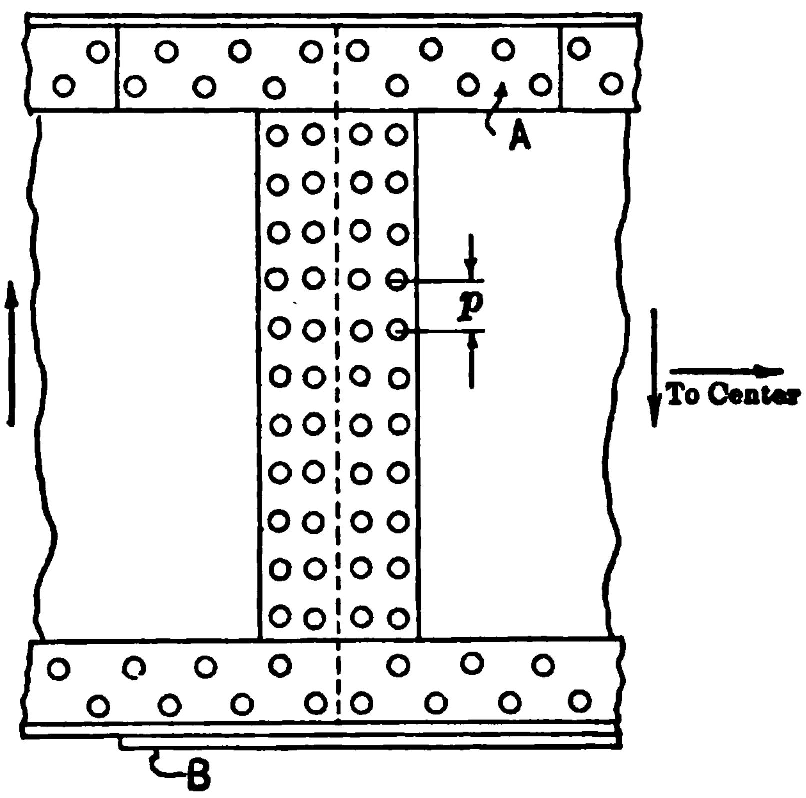


FIG. 21.

extend only from flange to flange, as in Fig. 21. The stresses in the web underneath the flanges are then taken care of by splice plates, as at (A), or by extending a cover plate beyond the required length, as at (B), so as to serve as a splice plate. Generally the splice can be so located that the excess area of flange at the section can be depended upon to furnish the splice for that part of the web covered by the flange.

123. *Splice with Uniform Rivet Spacing.*—In the following calculation the proper uniform pitch is determined for a web splice which shall be equivalent at all points, in both shear and moment, to the web section actually covered by the splice plates. If these plates do not reach the full depth of the web, additional provision is required for such portions not covered by them.

Let p = vertical pitch of rivets;

r = rivet value, generally the bearing value on the web plate;

r_h = horizontal component of rivet stress;

r_v = vertical component of rivet stress;

f = permissible fibre stress on *gross* area of flange, assuming one-sixth of the web area as flange area;

V = total vertical shear on the section;
h = depth of girder, assumed equal to the depth of web;
t = thickness of web.

Assume, first, that one row of rivets is used. Then, since the stress on the extreme fibre of the web is equal to that in the flange, $= f$, the horizontal stress on one rivet placed at the extreme edge of the girder would be

$$r_h = f\,p\,t \quad \text{(43)}$$

and

$$r_v = \frac{V\,p}{h} \quad \text{(44)}$$

and hence

$$r = \sqrt{r^2_h + r^2_v} = \sqrt{(f\,p\,t)^2 + \left(\frac{V\,p}{h}\right)^2} \quad \text{(45)}$$

$$= p\sqrt{(f\,t)^2 + \left(\frac{V}{h}\right)^2} \quad \text{(46)}$$

whence

$$p = \frac{r}{\sqrt{(f\,t)^2 + \left(\frac{V}{h}\right)^2}} \quad \text{(47)}$$

If the shear is small it may be neglected, giving

$$p = \frac{r}{f\,t} \quad \text{(48)}$$

a formula which may be used for approximate results in any case.

If the value of p thus found is less than the permissible spacing, or reduces the net section of the web too greatly (4-in. spacing makes one-eighth of the web available as net flange), two or three rows of rivets must be used, the pitch being increased accordingly. The value of f here used is determined by reducing the specified working stress in proportion to the reduction of gross flange section by the rivet holes. If the flange at the given section contains excess material, the actual fibre stress f will be less than the allowable stress, but the above method of calculation results in a splice of the same resisting moment as the web.

EXAMPLE.—Suppose a web plate is to be spliced under the following conditions: $V = 140{,}000$ lbs.; $t = 3/8$ in.; $h = 60$ in.; flange section is composed of two $6 \times 6 \times 1/2$-in. angles and one $14 \times 1/2$-in. plate. Suppose the allowable fibre stress on net section = 16,000 lbs. per sq. in. Rivets, 7/8 in.; $r = 7{,}900$ lbs.

The gross flange area is

Angles	11.5 sq. in.
Plate	7.0 " "
1/6 web	3.75 " "
Total	22.25 " "

The net flange area is

Angles	10.5 sq. in.
Plate	6.0 " "
1/8 web (approximate)	2.9 " "
Total	19.4 " "

The fibre stress on gross section at this point, when the flange is fully stressed is, therefore, $16{,}000 \times \frac{19.4}{22.25} = 14{,}000$ lbs. per sq. in. $= f$.

Then from eq. (47)

$$p = \frac{7{,}900}{\sqrt{(14{,}000 \times 3/8)^2 + \left(\frac{140{,}000}{60}\right)^2}} = 1.38 \text{ in.}$$

Therefore, if two rows are used, pitch = 2.76 in., and if three rows it would be 4.14 in. To retain 1/8 of the web as available flange, three rows would be needed. If two rows were used at $2\frac{3}{4}$-in. pitch, then the web section would be reduced $\frac{1}{2.75}$ part, and its equivalent flange value would be $\frac{1}{6} \times \frac{1.75}{2.75} = 0.10$; that is, $^1/_{10}$ the web would then be available as flange.

If the shear be neglected, the value of p becomes 1.5 in.; or 3 in. for two rows, a solution not greatly in error. In practice, a web splice will generally be located where the shear is relatively much less than here assumed, so that a very close estimate can be made without considering the shear at all, or from eq. (48).

To splice the web beneath the flange it may be assumed that the stress to be transmitted $= f\,t\,h_f$, where h_f is the depth of flange. The area of splice plates must be at least equal to the web section involved, and the number of rivets must be sufficient to carry the stress.

Where side plates are used, as A, Fig. 21, the rivets perform also the function of flange connection to the web. If the splice is located to the left of the centre so that the shear considered is positive, then the flange stress will be greater on the right of the splice than on the left. The flange increment on the right of the splice will then go partly to the splice plate, relieving the rivets in the web, while on the left these rivets will receive extra duty. On the right of the splice, therefore, the splice plate should extend far enough to include sufficient rivets of the flange to take over the stress carried by the splice, in double shear; while on the left (towards the end of the girder), there will be required *additional* rivets sufficient to carry the stress back into the web, the rivet value being its bearing value on the web. The splice plate must be of a length sufficient to take in these extra rivets, thus transmitting its stress into the flange angles over such a length as not to overstress the rivets. In short, the following rule may be used: Towards centre of girder no extra rivets are required, extend splice plates to include enough rivets to take the stress; towards end of girder, run splice plate to include enough *extra* rivets to take its stress in addition to regular flange increment.

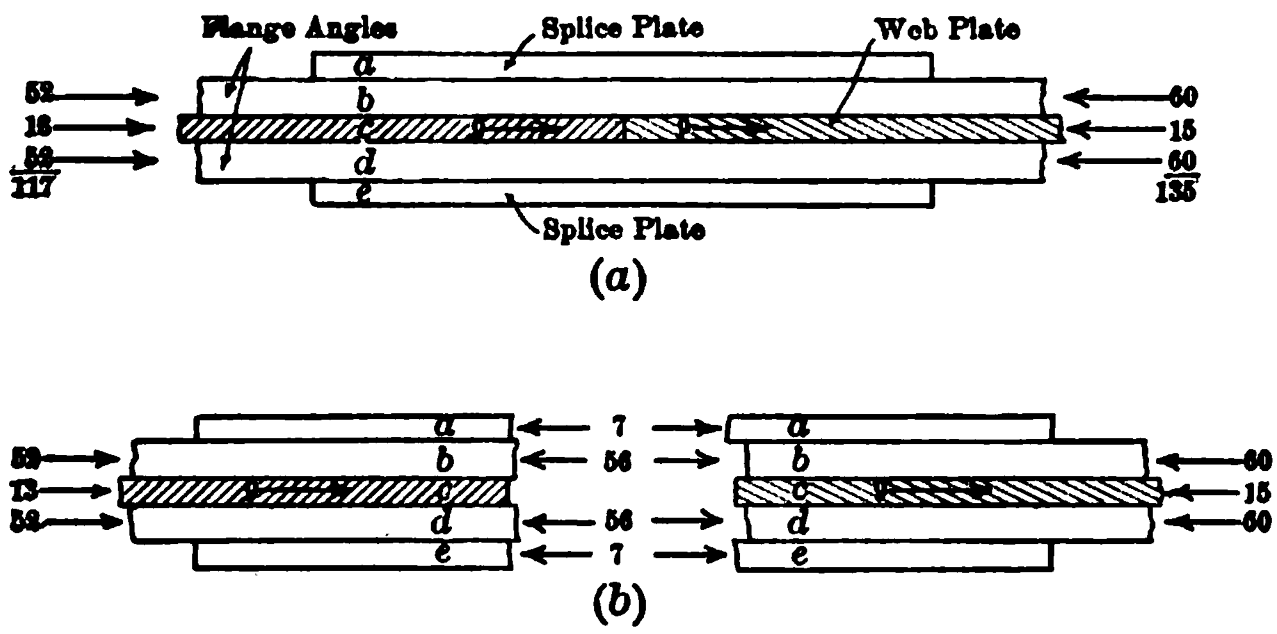


FIG. 22.

The conditions are illustrated by the splice shown in Fig. 22. Fig. (a) is a sectional plan showing the web, vertical legs of flange angles and the splice plates. The total compressive stress on the right is assumed to be 135,000 lbs. distributed as shown, and on the left it is 117,000 lbs., the difference of 18,000 lbs. being given over to the web in this distance, one-half each side of the joint. The

compressive stress to be transmitted by the web splice is 14,000 lbs. Fig. (*b*) shows the two halves of the splice separated. On the right side the shear between plates *a* and b = 7,000 lbs., and shear *b* to c = 3,000 lbs. The rivets are determined by the shear of 7,000 lbs. On the left, shear *a* to b = 7,000 lbs., and shear *b* to c = 11,000 lbs. The number of rivets in this half must therefore be sufficient to take 11,000 lbs. in single shear, or 22,000 lbs. in bearing on the web. If

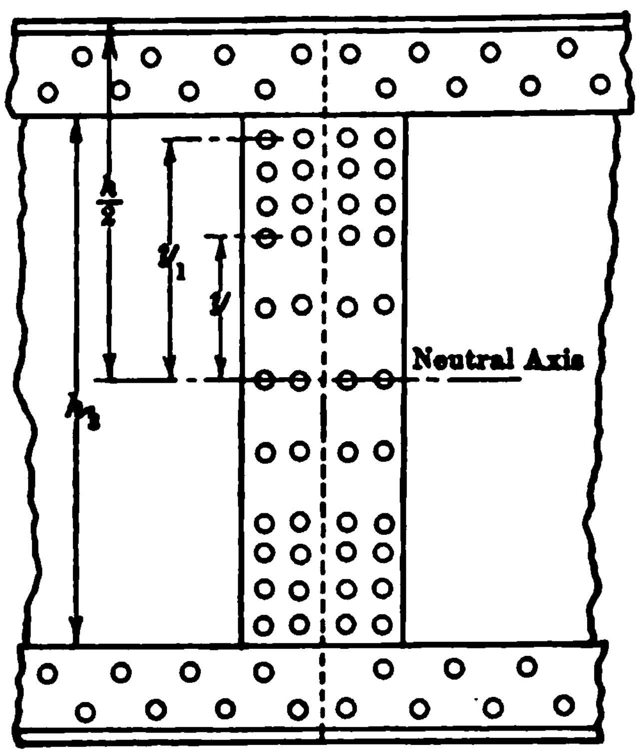


FIG. 23.

no splice were made here the duty of these rivets would be 9,000 lbs. in bearing on the web, the effect of the splice being to require an additional duty of 14,000 lbs.* Hence the rule as above stated.

Where the splice is made by the use of a cover plate the same principles apply. The plate must be extended far enough to permit extra rivets in the vertical legs of the flange angles to transmit the stress in question back into the web.

124. *Splice with Variable Rivet Spacing.*— (Fig. 23.) Many designers use a variable rivet spacing in the splice, placing the rivets closer near the edges, where they are more effective in carrying moment, than at the centre where their effectiveness is small. This is incorrect in principle, as already indicated. However, as the moment stresses in the centre third of the web are small the practical results are satisfactory. The calculation of the rivets in such an arrangement

* The slight discrepancy between 22,000 lbs. and the sum of 9,000 and 14,000 is due to the small flange increment (15,000–13,000) taken care of in the web itself.

is not so simple as for a uniform spacing as it requires a detailed analysis of the resisting moment of the rivets as actually spaced.

The simplest way to proceed in this case is to determine the actual moment carried by that part of the web between the flanges, and to equate this moment with the resisting moment of the rivets.

Let f = permissible stress on extreme fibre, calculated as before on the gross section;
h = total depth of girder;
h_s = height of portion of web considered as spliced by the main splice plate;
n = number of rivets on one side of the splice;
M_w = moment of resistance of portion of web considered.

Then

$$M_w = f \cdot \frac{h_s}{h} \cdot \frac{t\, h_s^2}{6} = \frac{f\, h_s^3\, t}{6\, h} \quad \ldots \ldots \ldots \ldots \quad (49)$$

The stress on any rivet is proportional to its distance from the neutral axis. Let y = the distance to any rivet, and r_h = horizontal stress on a rivet, distant y_1 from the neutral axis. Then the horizontal stress on any rivet $= r_h \dfrac{y}{y_1}$, and its moment of resistance $= r_h \dfrac{y}{y_1} \times y$. For all the rivets, the total moment of resistance =

$$M_r = \Sigma \frac{r_h\, y^2}{y_1} = \frac{r_h}{y_1} \Sigma\, y^2 \quad \ldots \ldots \ldots \quad (50)$$

Equating (49) and (50), and solving for r_h, we get

$$r_h = \frac{f\, h_s^3\, t}{6\, \Sigma\, y^2} \cdot \frac{y_1}{h} \quad \ldots \ldots \ldots \ldots \quad (51)$$

The vertical component is

$$r_v = \frac{V}{n} \quad \ldots \ldots \ldots \ldots \quad (52)$$

where V = vertical shear carried by the part of the web considered. Finally, the total maximum stress on a rivet is

$$r = \sqrt{r_h^2 + r_v^2} = \sqrt{\left(\frac{f\, h_s^3\, t\, y_1}{6\, h\, \Sigma\, y^2}\right)^2 + \left(\frac{V}{n}\right)^2} \quad \ldots \quad (53)$$

If the shear is small, we may use eq. (51).

The value of r_h in eqs. (51) and (53) is the horizontal component of the stress on the extreme rivet in the splice plate. In order not to overstress the rivets in the flange the value of r_h should not exceed $r\frac{y_1}{h}$, where r = rivet value. Hence the pitch must not exceed the value given by the equation $r\frac{y_1}{h} = \frac{f h_3^3 t}{6\Sigma y^2}\cdot\frac{y_1}{h}$, whence

$$\Sigma y^2 = \frac{f h_3^3 t}{6 r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (54)$$

Where the moment is large, eq. (54) is likely to control; where the shear is large eq. (53) will probably control.

The same method may be used where the splice is of the form

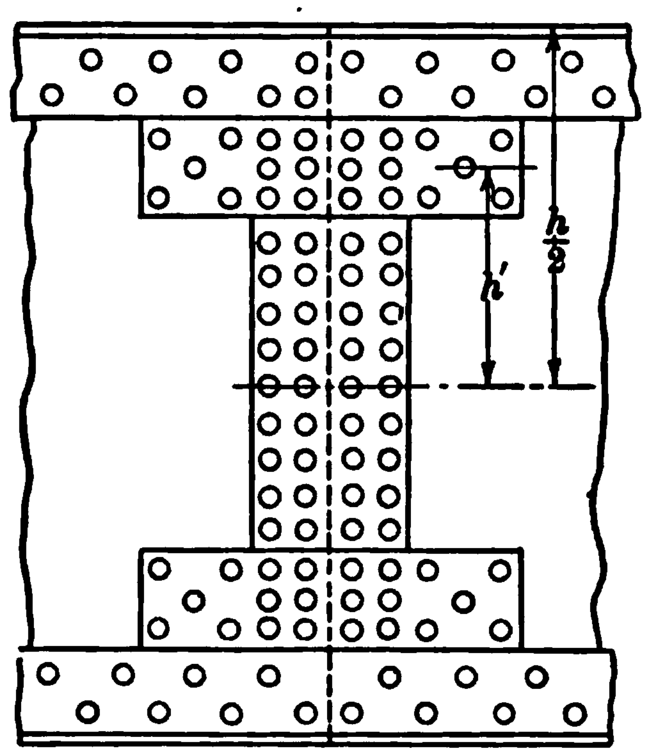


FIG. 24.

shown in Fig. 24, the long horizontal plates being intended to provide for the moment in the web.

In practice, the spacing near the flanges is made about 3 ins. and that at the centre about 6 ins., using a double row of rivets. The solution by eq. (53) requires a preliminary design to be assumed and then tested as to the maximum rivet stress. Such a design can be made very closely by neglecting the shearing stress and calculating from the moments alone, using eq. (51).

The web underneath the flange is preferably spliced as shown in Art. 123, although the entire bending moment carried by the web is

sometimes assumed to be spliced by the main web splice alone. In this case the value of M_w in eq. (49) becomes equal to $\frac{1}{6} f t h^2$.

125. **Flange Splices.**—Splicing of flanges should generally be avoided. It is possible, and generally advisable, to secure material of full length, but for very long spans this becomes expensive and inconvenient and a splicing of the angles and one of the cover plates may be desirable. Field splices of the entire flange are sometimes made in the case of long girders, but such a splice is difficult to make and a riveted truss will generally be preferred in such cases.

Where angles only are to be spliced, cover angles should be used, if practicable, of section equal to the angle spliced, otherwise an angle and plate should be employed. The point of splice should be selected where there is some excess area and only one angle or plate should be spliced at the same section. Generally the two angles are spliced on opposite sides of the centre. Some engineers require a flange splice to be calculated to give 25 per cent excess strength. Note the requirement of indirect transmission of stress in Art. 57 of the Specifications. All flange splices should be fully riveted, no reliance being placed upon abutting ends.

In calculating the rivets of a flange splice it should be observed that where stress is transmitted indirectly to the splice plate or angle through one or more intermediate plates, the shearing stresses on the rivets due to the splice will be of the same sign as the shear already existing on such rivets from other causes on one side of the splice, and of opposite sign on the other side. Where the splice is fully made by means of a splice plate immediately in contact with the member spliced, then no such effect occurs. These principles should be kept in mind in designing any indirect connection.

126. *Splice of Flange Angle.*—If the net section of cover or splice angle is equivalent to the angle to be spliced, the number of rivets each side of the splice must be equal, in single shear, to the full strength of the angle spliced. The rivets in the flange required for other purposes are available for this purpose also, and by making the splice angle of sufficient length to include the required number of rivets no extra rivets will be needed. However, it is desirable to use close spacing in the splice whether or not this is required for other purposes, thus reducing the length of splice to a minimum.

Where the necessary area cannot readily be obtained by means of a single splice angle, the deficiency is best made up by means of a plate on the vertical leg of the opposite angle. Often the cover

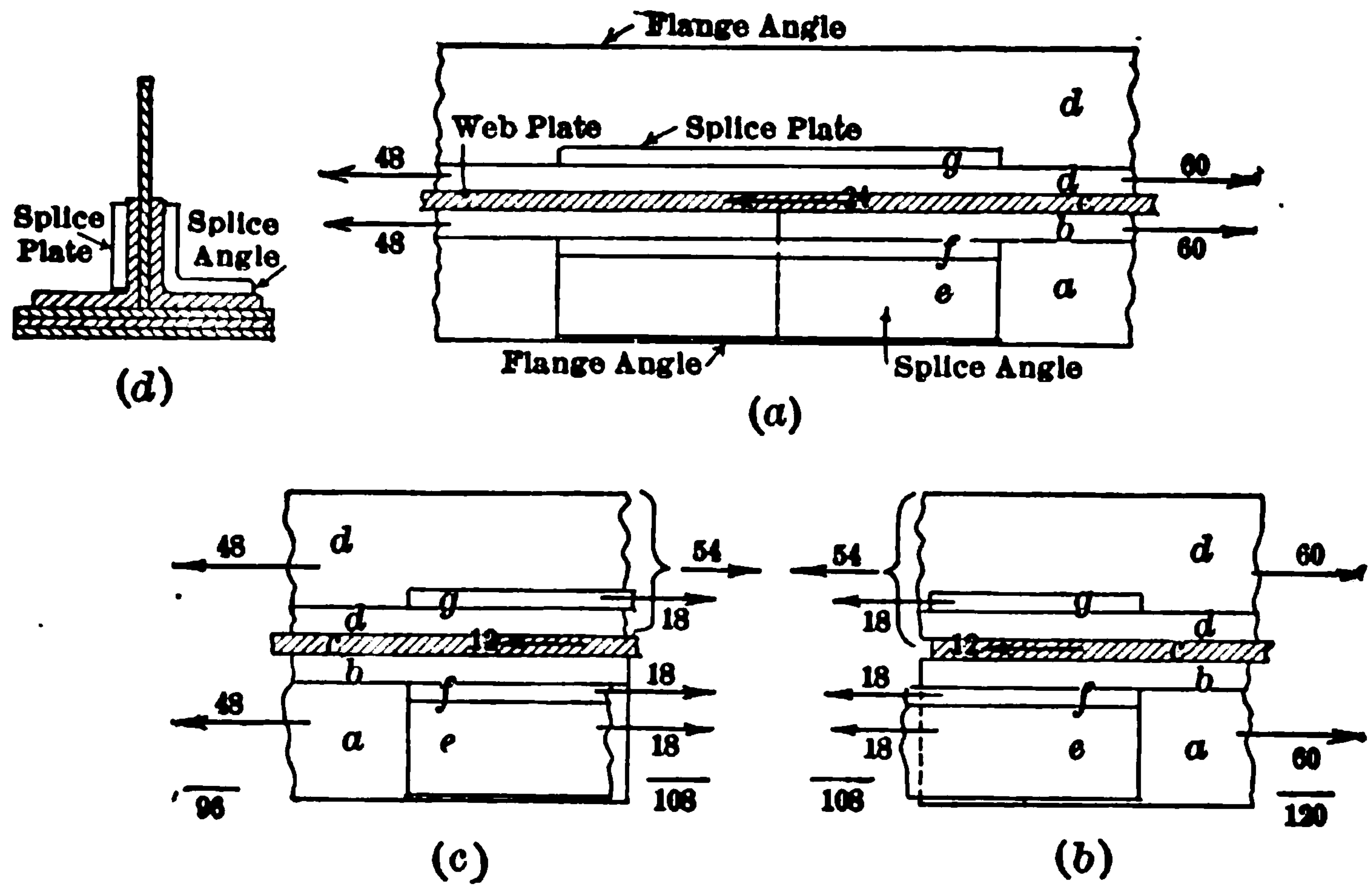


FIG. 25.

plate also supplies some excess area. This is an example of indirect splice and will be considered in detail.

Fig. 25 represents a splice in an angle of a tension flange. Fig. (*a*) shows the splice in sectional plan, the web being cut just above the angles; *a b* is the angle spliced. Figs. (*b*) and (*c*) show the two halves of the splice on either side of a section taken at the joint in the angle *a b*.

Suppose that on the right of the splice, Fig. (*a*), the stress in each angle is 60,000 lbs., and that at the left it is 48,000, the difference of 12,000 lbs. being given over to the web in the length of the splice. Fig. (*b*) shows the right half of the splice. Suppose the splice angle *e f* takes two-thirds of the stress in the angle *a b* and the plate *g* one-third. The stresses in the various elements, at the centre of the splice, will then be as shown on the left of Fig. (*b*). The web taking over 6,000 lbs., in this distance, the total stress to be transmitted by means of the splice will be 60,000 − 6,000 = 54,000 lbs. The splice angle will take 36,000 lbs., 18,000 in each leg *e* and *f*, and the splice plate *g* also 18,000 lbs.

The shears between the several surfaces in Fig. (*b*) will be as follows, considering the shear as positive where the lower portion tends to move towards the left with respect to the upper portion:

e to a, $+ 18{,}000$ lbs.
f to b, $+ 18{,}000$ lbs.
b to c, $+ 36{,}000 - 60{,}000 = - 24{,}000$ lbs.
c to d, $- 24{,}000 + 12{,}000 = - 12{,}000$ lbs.
d to g, $- 12{,}000 - 60{,}000 + 54{,}000 = - 18{,}000$ lbs.

The maximum shear is therefore between b and c and is equal to 24,000 lbs., which is the stress 60,000 lbs. less 36,000 lbs. transferred to the splice angle. The number of rivets in the half splice must therefore have a value of 24,000 lbs. in single shear on surface b to c. If no splice were made here the shear on b to c in this length would be $12{,}000/2 = 6{,}000$ lbs. The 24,000 lbs. now existing is therefore equal to the web increment of 6,000 lbs. previously existing, plus the 18,000 lbs. taken by plate g.

On the left of the splice, Fig. (*c*), the shears are as follows:

e to a, $- 18{,}000$ lbs.
f to b, $- 18{,}000$ lbs.
b to c, $48{,}000 - 36{,}000 = 12{,}000$ lbs.
c to d, $12{,}000 + 12{,}000 = 24{,}000$ lbs.
d to g, $24{,}000 + 48{,}000 - 54{,}000 = 18{,}000$ lbs.

The shear c to d is in this case increased by the splicing from 6,000 to 24,000 in the same manner as the shear b to c in the right half.

The rivet requirement here determined will usually make necessary a closer rivet spacing than otherwise needed, but this does not follow of necessity, as the rivets required for flange and web are not generally determined by single shear, but by bearing value on web, or by a maximum or convenient spacing.

In the design of a flange splice good practice requires that the strength of the splice must equal that of the member spliced, and that an excess number of rivets should be used in the indirect part of the connection. This would require the shear d to g to be calculated at $1\frac{2}{3}$ times the values given above (following the rule of Art. 57 of the Specifications).

In some cases the bearing value of the rivets in the angle leg *b*, Fig. (*b*), would require consideration. In the example the total bearing stress on these rivets is 60,000 − 18,000 = 42,000 lbs.

127. *Splice of Cover Plate.*—In splicing an outside cover plate the splice plate is made of the same section as the plate, and the entire

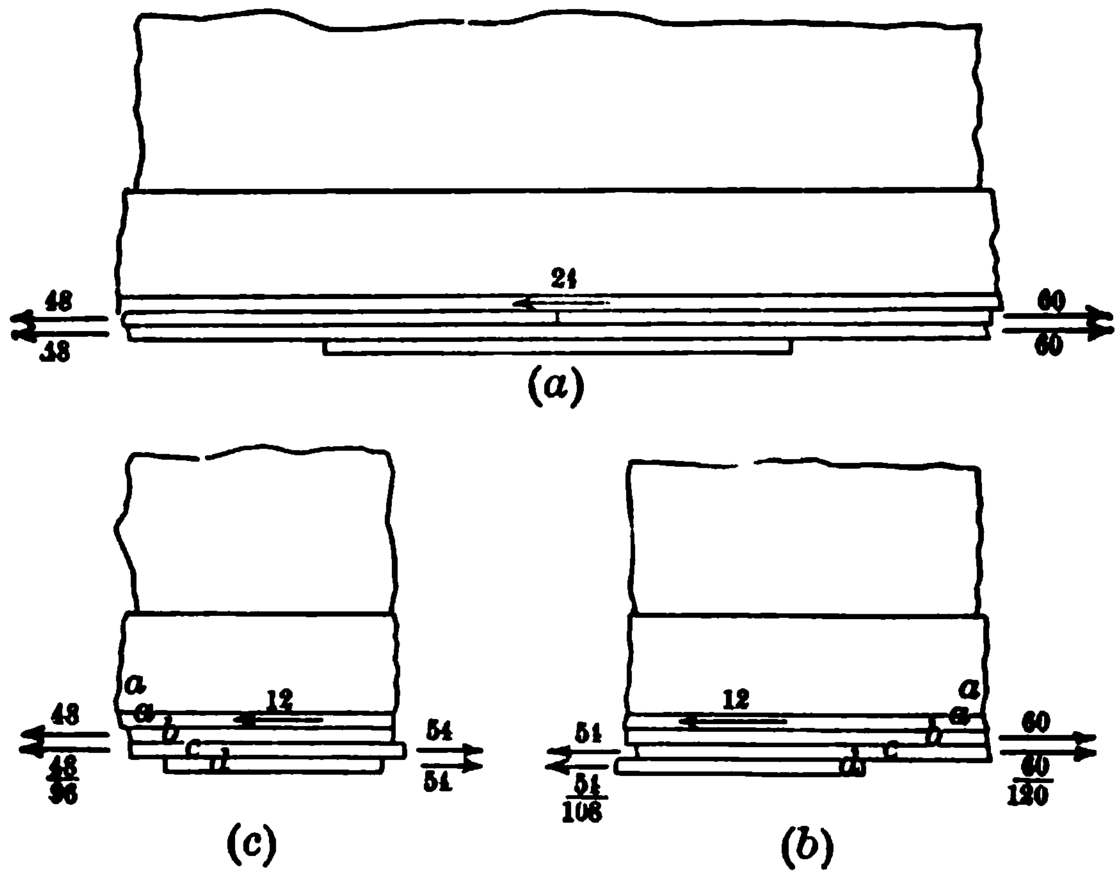


FIG. 26.

stress is assumed to be transferred to it, thus requiring rivets sufficient to take the load in single shear. All rivets are counted at full value as the splice does not materially modify the previously existing stresses.

Where an inside plate is spliced by means of a cover plate the transmission is indirect and the shearing stresses between the several elements of the flange are modified. Let Fig. 26 (*a*) represent a spliced flange with stresses as shown. Fig. (*b*) is the right half. The increment transmitted from the plates to the angles in this distance is 12,000 lbs. The shears in Fig. (*b*) are

a to *b*, − 12,000 lbs.
b to *c*, − 12,000 + 60,000 = + 48,000 lbs.
c to *d*, + 48,000 + (60,000 − 54,000) = + 54,000 lbs.

In Fig (*c*) they are:

a to *b*, − 12,000 lbs.
b to *c*, − 12,000 − 48,000 = − 60,000 lbs.

c to d, $-$ 54,000 lbs.

If the splice were not present the shear would be, in each half,

a to $b = -$ 12,000 lbs.

b to $c = -$ 6,000 lbs.

The maximum shear is therefore in Fig. (c) on surface b to c, a value of 60,000 lbs. This is seen to be equal to the full stress in the plate b of Fig. (b). If the splice were therefore designed to be of equal strength to the plate cut (at least 60,000 lbs.), in accordance with usual practice, it would fully meet all requirements of shearing stresses. In general, therefore, in cover-plate splices, the riveting should be designed to take the full value of the plate cut, and the shearing stresses due to transmission of flange increment may be neglected.

Where a field splice must be made, splices of cover plates should be arranged as shown in Fig. 27, the distance between splice points

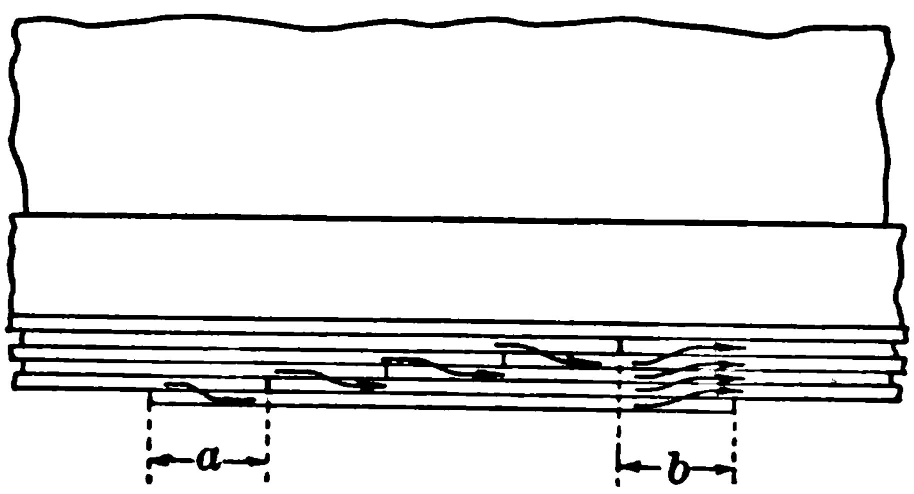


FIG. 27.

of consecutive plates being sufficient to include rivets enough to transmit in single shear the value of the plate immediately inside. The cover plate must overlap in both directions all of the splice points and be equal in strength to the largest plate cut. Fig. 27 shows by the arrows the assumed transmission of stress from plate to plate. The value of the rivets in the lengths a and b must equal the strength of the splice plate.

128. Web Stiffeners.—To assist the web plate in resisting the inclined compressive stresses, as explained in Art. 104, and to prevent buckling under these stresses, stiffeners, usually in the form of angles, are riveted to the web, generally in a vertical position. Similar members are also used to assist in distributing local concentrated loads into the web. The latter form is rather a reinforcing angle, but the name "stiffener" is employed.

129. *The Intermediate Web Stiffener.*—The design of the first type of stiffener is closely connected with the determination of web thickness, and the two should be considered together. From Fig. 7 it is seen that the direction of maximum compressive stress varies from an inclination of 45° at the neutral axis to a considerably flatter angle at the edge of the flange. It is also observed that, simultaneously with the compressive stresses, there exist tensile stresses acting at right angles to the compressive stresses, and, at the neutral axis, numerically equal thereto.

The general effect of this combination of stresses is a buckling tendency in one direction, accompanied by a straightening tendency at right angles. The strength of the web against buckling is therefore greater than that of an unsupported column of equal proportions, and many observations go to show that the web will sustain very high compressive stresses without buckling if concentrated loads are properly taken care of, or if the external loads are applied along the lower edge, thus reinforcing the tensile stresses.

Experiments conducted in 1907 by one of the authors* on the stresses in web plates and vertical stiffeners confirm these theoretical conclusions. It was found that, under working conditions, the axial stress in intermediate stiffeners not subject to local loading was very small, but that local loads, such as end reactions, caused compressive or tensile stresses in the stiffeners in accordance with the direction and point of application of such loads. It was also found that the deformation of the unstiffened web along a vertical line, under ordinary shearing stresses, was very small.

Experiment as well as theory thus shows that the function of the stiffener is to stiffen the web against buckling under the inclined compressive stresses. If thoroughly stiffened the web will be effective up to the permissible stress in shear or tension. The size and spacing of stiffeners necessary to give to the web its requisite strength is difficult to estimate. The principle followed in such analysis is to determine the buckling strength of the web between stiffeners by the application of some empirical column formula, based on the column action of an inclined strip of the web of a length equal to the

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*Jour. West. Soc. Engrs., Dec., 1907.

diagonal distance between stiffeners or flange angles. Thus in Fig. 28 the distance d or d' is the least unsupported width of the web, and is a measure of the unsupported length of a diagonal compressive strip along a 45° line. The formula for spacing of stiffeners, or for

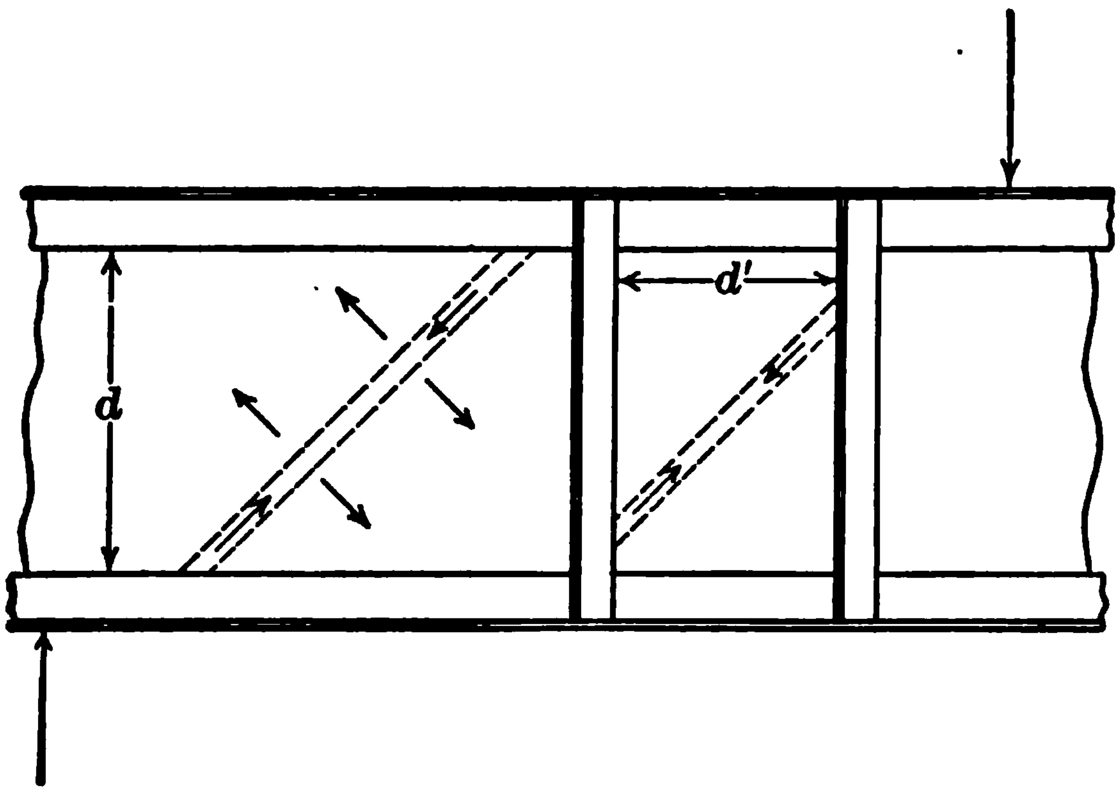

FIG. 28.

strength of web, is usually of a form similar to the straight-line column formula (see Art. 16 of the Specifications), but the coefficient of the term involving l/r is made much smaller than in the usual column formula on account of the fact that in the web the column element in question is greatly supported by adjacent material and by the tensile stresses acting transversely.

The provision for intermediate stiffeners of Art. 79 of the Specifications, Appendix A, requires stiffeners where the web thickness is less than 1/60 of the unsupported distance between flange angles. The distance between stiffeners is then given by the formula:

$$d' = \frac{t}{40}(12{,}000 - v) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where t = thickness of web and v = shearing stress per sq. in., but such spacing must not exceed 6 ft.

This formula, solved for v, gives

$$v = 12{,}000 - 40\frac{d'}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

which is the stress permitted for a given spacing d'. The specifications also limit the maximum shearing stress to 10,000 lbs. Eq. (56) is in the form of the straight-line column formula in which t replaces r. Taking the length of a diagonal strip as $1.41\,d'$ and $r = 0.29\,t$ the above formula becomes $v = 12{,}000 - 8.2\frac{l}{r}$. Comparing this with the column formula of Art. 16 of the Specifications $(16{,}000 - 70\frac{l}{r})$ it is noted that the term involving $\frac{l}{r}$ is relatively small, thus taking into account the supported condition of the web strip. Other formulas in use make this term larger, the coefficient of $\frac{d'}{t}$ of eq. (56) ranging from 75 to 100.

In the experiments above referred to, the elastic limit strength of the web of the experimental girder (web 24 x 1/8-in.) was reached at a stress of about 10,000 lbs. per sq. in. As the ratio $\frac{d}{t}$ in this case was about 100, eq. (56) would give a working stress of $12{,}000 - 4{,}000 = 8{,}000$ lbs. per sq. in. While the formula is not intended to apply to such extreme cases the authors believe that to make the formula of wider application it is desirable to use higher values in both terms. A formula $v = 14{,}000 - 80\frac{d'}{t}$ or $d' = \frac{t}{80}(14{,}000 - v)$ would permit about the same working stress, or stiffener spacing, in the ordinary case, but would make a more rapid reduction for very thin webs, which is a desirable condition.

The size of intermediate stiffener need not be great, as its function of preventing wrinkling of the web will be performed by a comparatively small shape. In railroad practice the minimum size of 3½ x 3½ x 3/8-in. angle is used up to a depth of web of 5 or 6 ft., and 5 x 3½ x 3/8-in. angle for greater depths. The riveting must be sufficient to hold the stiffener securely to the web. A 6-in. spacing is generally used. Where the stiffeners do not receive any consider-

able load direct from the flange, a tight fit between stiffener and flange is unnecessary.

Inclined stiffeners have been employed in some cases, the stiffener being sloped in the direction of the compressive stresses. In such position it assists directly in carrying these stresses and is doubtless more efficient in preventing buckling than a vertical stiffener. However, vertical stiffeners are much more convenient to use and if properly spaced are entirely adequate for the purpose.

130. *Supporting Stiffeners.*—The function of these stiffeners is to transmit local loads into the web, and therefore they must be designed to carry direct stress. At the end of the girder the entire reaction is transferred through the horizontal legs of the flange angles to the stiffeners and thence to the web, as the latter is not usually in full contact with the sole plate. These stiffeners should therefore be well fitted against the flange angles; they should be straight, and therefore placed on fillers, and they should be designed to take the full external load. A pair of stiffeners is generally designed as a long column, although, as the stress is gradually transferred to the web, it does not act as an unsupported column of a length equal to the stiffener length. A column length of one-half the total length of the stiffener may be assumed. The rivets connecting supporting stiffeners to the web must be sufficient to transfer the total load in question.

131. Lateral Bracing.—For spans up to 50 or 60 ft. the lateral bracing of a deck girder usually consists of an upper lateral system and transverse bracing at the ends and at intermediate points, as shown in Fig. 1. This arrangement furnishes a theoretically complete lateral bracing, but requires the lateral forces acting along the lower part of the girder to be transferred to the abutment in a less direct manner than when a lower lateral system is used. For longer spans, both an upper and a lower lateral system are employed, but the use of the lower lateral system, except for the longest spans, is not altogether general. The panel length should be about equal to the spacing c. to c. of girders, giving a 45° inclination to the diagonals, which gives maximum lateral rigidity (see Part II, Art. 272).

The type of bracing generally used in deck girders is the single Warren system, as shown in Fig. 1. Such a system is, however, not in general the best for lateral bracing, on account of the secondary

stresses it causes in the flanges, as shown in Part II, Art. 344. Due to the longitudinal deformation of the flanges (compression above and extension below) a single set of diagonal members tends to deflect the flanges laterally, and in opposite directions at successive panel points, thus producing very considerable lateral bending stresses. Such secondary stresses will be almost entirely avoided by using a double system and transverse members at each joint. Where no lower lateral system is employed, transverse frames should be used to stiffen the lower flange at intervals of 12 to 15 ft. Where a lower lateral system is also used the intermediate cross-frame is of little importance.

Lateral bracing for girders should be made of angles or channels, and riveted to flange angles rather than to small hitch angles attached to the web. Concentric rivet grouping is of more importance than intersection of gravity lines of lateral members, but on large structures the latter is also important. End cross-frames are designed on the theory that one-half the horizontal shear is carried by each diagonal, or the minimum section is used.

For through girders (Fig. 2) the top flange is braced by means of gusset plates attached to the floor system. This is not an ideal bracing, as the deflection of the floor beams causes a lateral movement of the top flange. Unequal loads on the floor beams thus result in unequal lateral deflection of the flange, causing some secondary stress. Deep floor beams are, in this respect, advantageous. In the floor laterals a double set of diagonals is generally used.

For double-track deck girders (four girders being used), the lateral bracing may well extend over the entire set of girders thus giving much greater rigidity than if braced separately. Diagonal bracing in a vertical plane should, however, not be used between girders of different tracks, as these should be allowed to deflect independently.

132. Spacing of Girders.—To make the lateral system effective requires girders of a single track bridge to be spaced not less than one-twelfth to one-tenth of the span-length between centres of girders. A common standard for spans less than 60 to 70 ft. is 7 ft., increasing to 9 or 10 ft. for 100- to 110-ft. spans. A spacing of 10 ft. requires very heavy ties, which influences materially the relative economy of the plate girder.

133. End Bearings.—In designing the end stiffeners and end

bearings of a girder consideration must be given to, (1) reinforcement of web by the stiffeners in order to provide for the heavy reaction stresses, (2) proper distribution of the load on the masonry, (3) longitudinal motion due to temperature changes and change of length of lower flange due to stress, and (4) a change in angle of the end of the girder due to deflection.

The load from the girder is transferred to the masonry through a *sole plate* or *shoe* attached to the girder flange and resting upon a *bed plate* or *wall plate* anchored to the masonry. For spans up to

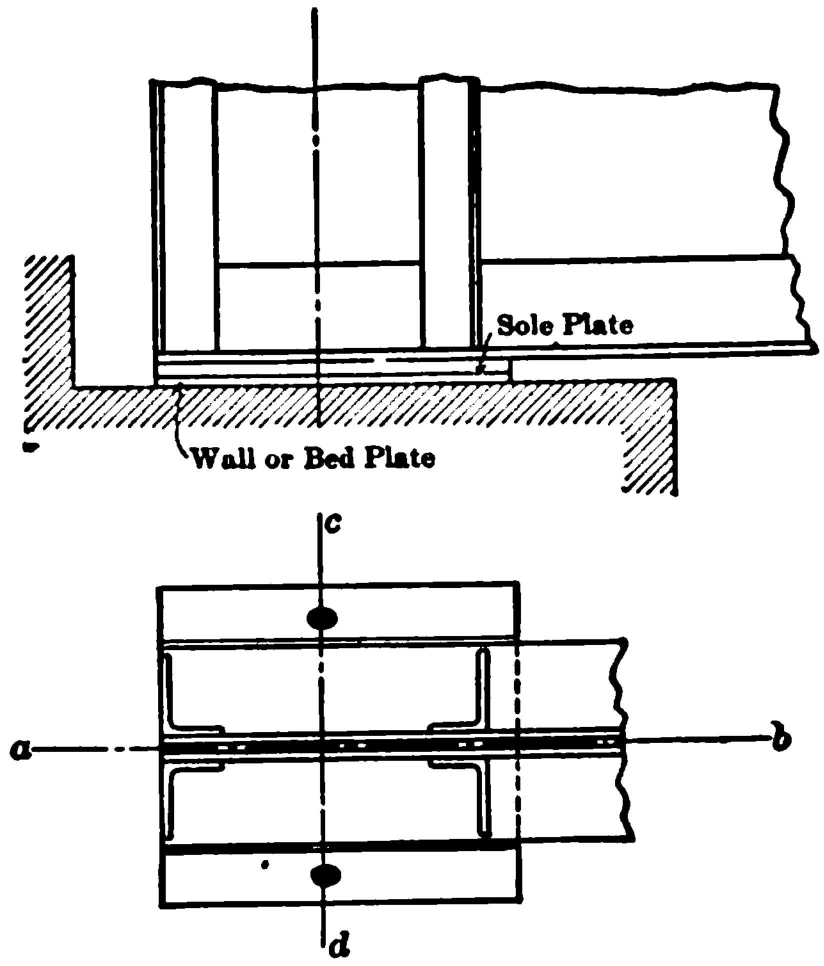

FIG. 29.

60 or 80 ft., expansion is usually provided for by allowing one end of the girder to slide upon the wall plate, the surfaces in contact being finished smooth; for longer spans a roller bearing is placed between the shoe and the wall plate.

The size of wall plate is determined by the load and the allowable pressure on the masonry. The main object of this plate being to distribute the load as uniformly as possible over the required area of masonry it should be relatively rigid. For short spans the wall plate usually consists of a structural steel plate $\frac{3}{4}$ to 1 in. thick, on which rests the sole plate of about the same size (Fig. 29). At the fixed end the girder is anchored to the masonry; at the expansion end the holes in the sole plate are slotted to permit of the necessary movement.

Such an arrangement of bearing is often used for long spans, but it is not well adapted to give an even distribution of load. It is too flexible to distribute the load properly in a lateral direction *c d*, and so rigid in the direction *a b* that excessive pressures are caused along the inner edge towards *b*, due to the deflection of the girder. For adequate rigidity of bed plate a greater depth is necessary, and a deep

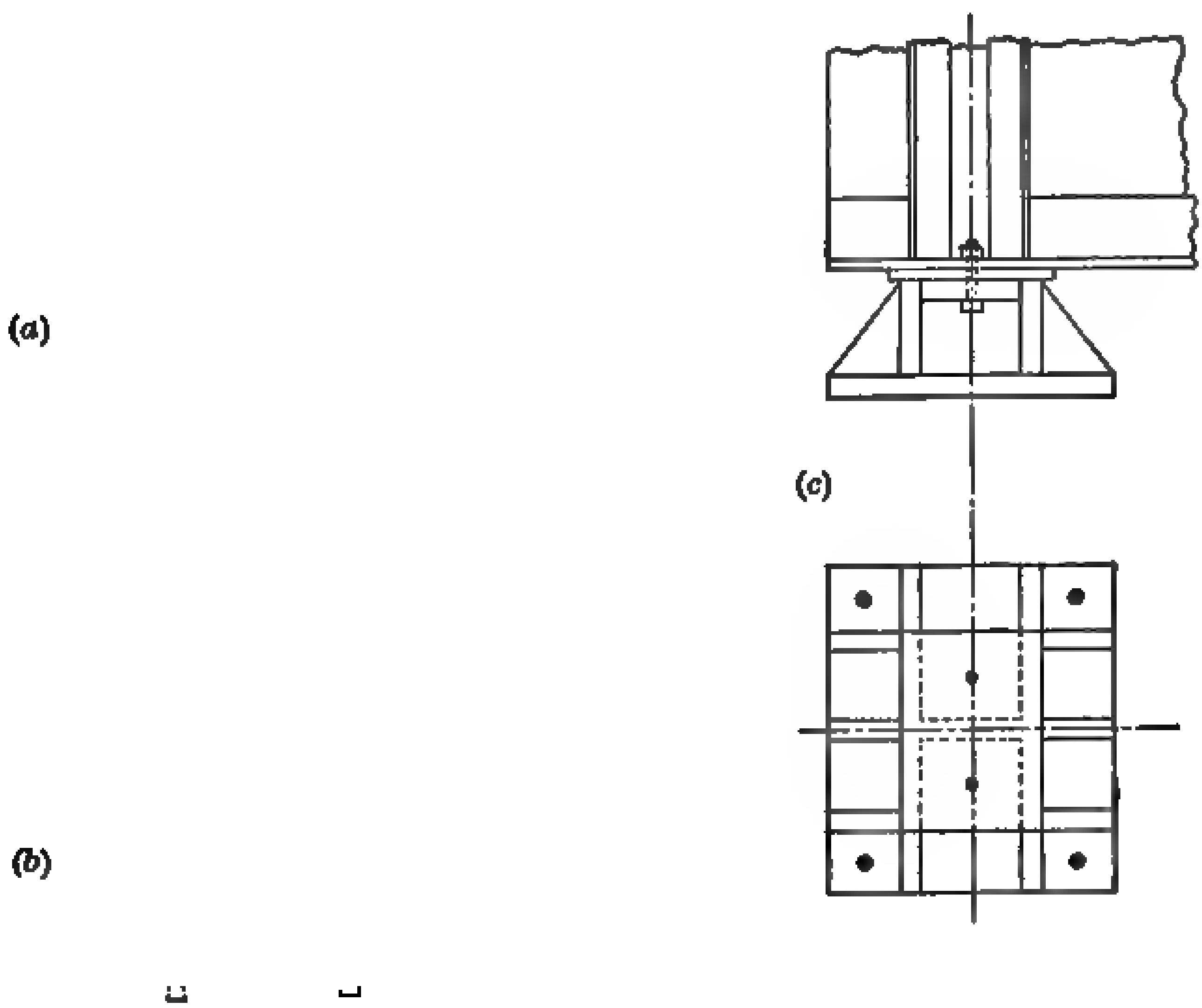

FIG. 30.

cast steel bed plate, or a built-up bolster of structural steel, should be used. Fig. 30 shows arrangements adapted to all spans up to a size where rollers are required. The area of contact between sole plate and bed plate should be made as short in a longitudinal direction as convenient, in order to avoid the objectionable effects of deflection as noted above. At the same time the length of sole plate should be sufficient to permit a convenient arrangement of reinforcing stiffeners above. The design (*b*) is in this respect preferable to (*a*). Design

(*b*) also permits the end of the girder to be brought closer to the masonry than (*a*), and is therefore more convenient for support of the floor. The smaller the top area of the bed plate, compared to its bottom area, the deeper it should be for the desired necessary rigidity. Its strength may be estimated by calculating the moment in the cantilever end *c d* on the assumption that the load is applied near the inner edge of the bearing of the sole plate at *e*. Fig. (*c*) shows a detailed design of a cast steel bed plate.

For spans longer than about 75 ft. (some place the limit at 60 ft.) a roller bearing should be provided at the expansion end, and the

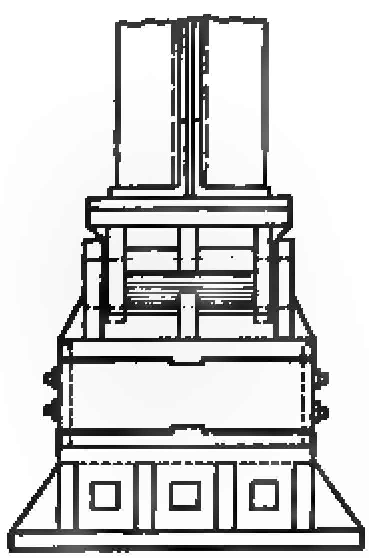

FIG. 31.

shoes at both ends should be hinged so as to insure an even distribution of pressure on the masonry in a direction parallel to the girder. This is of greater importance at the roller end, but the change of angle at the ends of a long girder due to deflection will produce very unequal pressures on the masonry and severe stresses in the girder if such a hinge is not provided.

Fig. 31 illustrates a typical design of end bearings with hinges and rollers. Segmental rollers should generally be employed in preference to the cylindrical form, as they permit the use of large diameters without using unduly large sole plates. To prevent such rollers from overturning or displacement they should be geared to the upper and lower plates as shown in Fig. 31. Lateral movement is usually prevented at the roller end by means of projections on the plates fitting into grooves in the rollers, as in Fig. 31.

Where a pin joint is used, as in Fig. 31, the upper shoe must be designed to distribute the concentrated pressure from the pin over a sufficient area of the girder so that it is conveniently taken care of by the reinforcing stiffeners above. A casting is usually employed for this purpose. The pin is designed for bearing, bending and shear, as in the case of pin-connected structures (Chapter VII). A simple form of hinged bolster for the roller ends, used successfully by the A. T. & S. F. Ry, is illustrated in Fig. 32. It concentrates the load upon a narrow fin of metal whose area is calculated on the basis of the usual bearing pressures. Experience shows that the action of

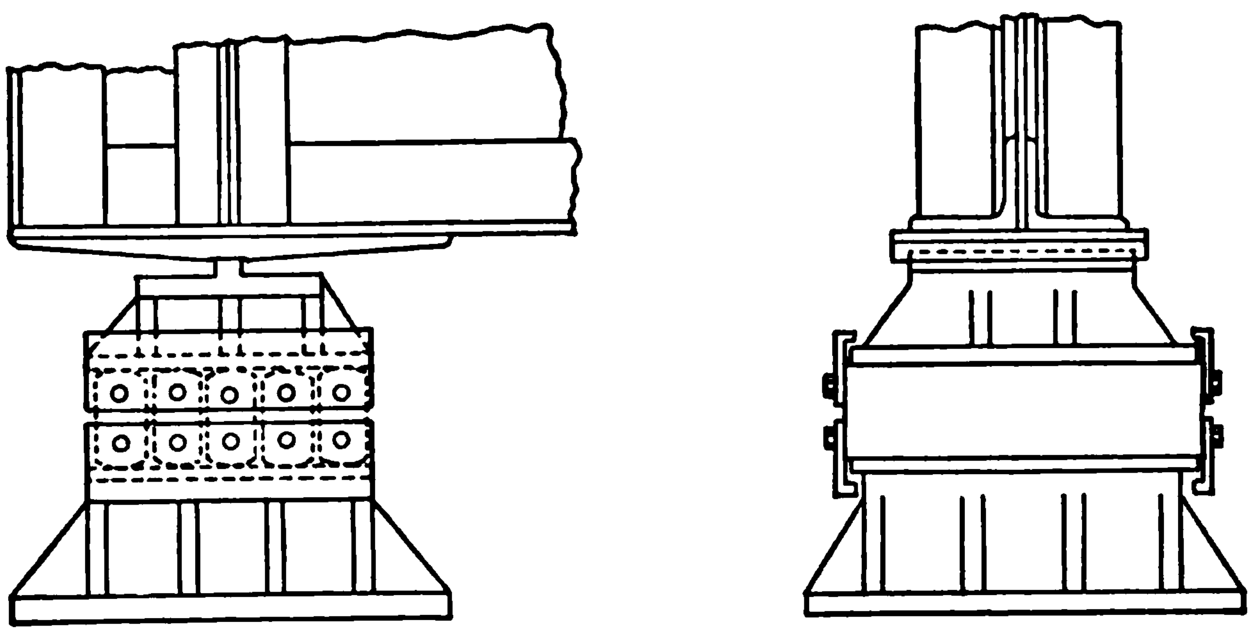

FIG. 32.

the girder tends to round off this fin to a slight degree, thus producing in effect, a natural pin bearing. Phosphor bronze sliding plates for expansion ends of long girders have been used successfully by the C. M. & St. P. Ry.

So far as temperature changes are concerned a sliding shoe is adequate, as only the dead load is to be moved. For long spans the change of length under live load becomes considerable and the friction of a slide bearing causes considerable stress or movement in the piers and abutments. In a 100-ft. span with an average live-load stress of 5,000 lbs. per sq. in. the movement is $\frac{1,200 \times 5,000}{30,000,000} = 0.2$ in. This movement develops a frictional resistance of about $0.2 \times 5,000 \times 100/4 = 25,000$ lbs. at each bearing due to live load alone. Where rollers are not provided the result is a considerable movement of the piers and abutments under the forces developed. A roller bearing

in good condition relieves the masonry of all such stress. Actual observation of a large number of structures having well-designed rollers indicates adequate freedom of motion under live loads.

The end stiffeners should be arranged with reference to the point or points of maximum pressure on the flanges. In fixed bearings such as shown in Figs. 29 and 30, the greatest pressures are likely to occur at the edges of the sole plate, hence the arrangement of stiffeners there shown. In the pin- or centre-bearing shoes the maximum pressure will be at the centre of bearing, hence the stiffeners should be placed at or near the centre as shown in Figs. 31 and 32. In through bridges the stiffeners must be located with reference also to the end floor beams.

134. **Through Girders.**—Where headroom is insufficient for a deck girder, a through girder must be employed with the floor sup-

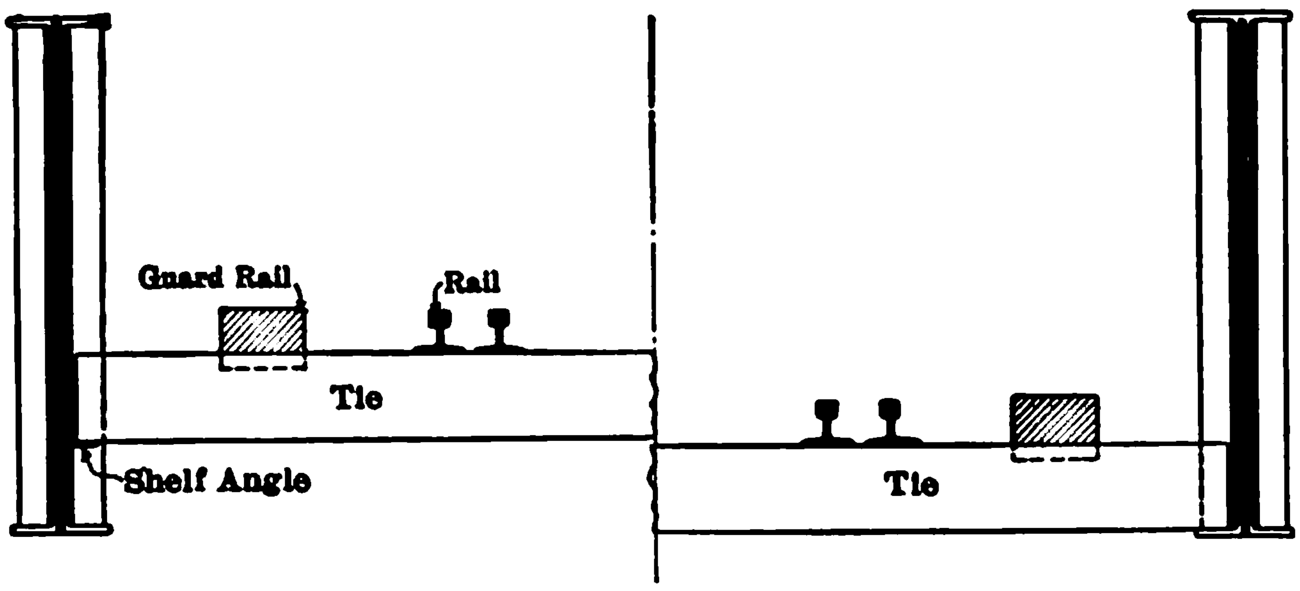

Fig. 33.

ported at some point below the top flange. Where wooden ties are used the most common type of floor consists of stringers and floor beams, as shown in Fig. 2, the panel length being generally from 10 to 15 ft. In this case the live load is to be applied at panel points in the calculations as explained in Art. 100. The top flange is braced laterally by gusset plates extending from the floor beam to the top flange. Shallow beams should be avoided, especially in double track girders, as the large deflection of such beams causes lateral bending of the upper flange.

Stiffeners in through girders are designed as in deck girders, but the tendency of web buckling is materially less as the loads are applied near the lower flange.

Fig. 33 shows a floor formed by resting the ties directly upon shelf angles, or upon the lower flange angles. This design is adapted only to light loads and relatively short ties, on account of the heavy bending moments in the ties. If shelf angles are used they should be supported at frequent intervals by short vertical supporting angles, as the eccentric load on the shelf angle causes considerable bending stresses therein as well as tensile stresses in the rivets. If supported on the lower flange, these angles should not be less than 5/8 in. thick and the rivets connecting flange to web should be spaced the minimum pitch throughout.

The difficulty of avoiding heavy bending stresses in such designs is shown by the calculation for an E-50 load. Calculating the tie load in accordance with the Specifications (Art. 5) and assuming ties

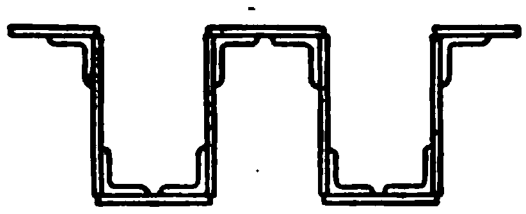

FIG. 34.

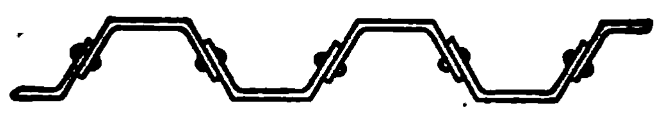

FIG. 35.

spaced 12 ins. apart, c. to c., and that the centre of bearing is only 1 in. from the root of the angle, the resulting fibre stress in the angle is 15,000 lbs. per sq. in. for a 5/8-in. angle. Cover plates assist somewhat, but heavy loads cannot safely be carried in this manner. Where no steel floor is used the gusset plates supporting top flange must be attached to cross-struts placed between ties.

For shallow floors where headroom is limited, closely spaced I-beams are often used, with the rail laid upon wooden or steel stringers resting upon the I-beams. (See Fig. 37.) The maximum load on each unit is determined by the methods of Art. 300, Part II.

Lateral bracing in through girders should be double diagonals of angles or other stiff shapes, and attached to joint plates connected to lower flange and to floor beams or cross-struts.

135. Solid or Continuous Foors for ballasted tracks are made of steel, wood or reinforced concrete. Fig. 34 illustrates the common type of trough flooring. By varying depth and thickness it can be

adapted to any span and load. Fig. 35 illustrates a type frequently used but less well adapted to carry heavy bending moments. It is also not a convenient form to rivet to a girder web and is therefore arranged to rest upon the flange or a shelf angle. I-beams are also used, covered by a steel plate or filled around by concrete. To protect the steel from corrosion a heavy coat of asphalt or other water-

FIG. 36.

proof material is placed below the ballast. Figs. 36 and 37 illustrate floors without ballast and Figs. 38 and 39 common types of ballasted floors.

Solid floors are also formed by using creosoted timber or reinforced concrete slabs resting upon transverse I-beams, or, in case of deck bridges, directly upon the top flange.

136. The Economical Depth.—The depth of a plate girder, when not fixed by available headroom, adjustment to old masonry, etc.,

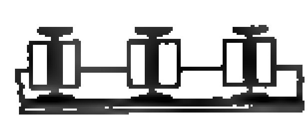

FIG. 37.

varies in practice from about one-eighth of the span length for short spans to about one-twelfth for very long spans. The chief consideration which determines the depth of a girder is that of minimum weight, but economy of construction depends to some extent on other elements than weight, such as increased cost of very wide plates,

limiting depth for shipping and increased cost of handling deep girders, so that it is generally economical to use a depth less than that giving the minimum weight. Inasmuch as a relatively large change in depth from that giving the exact minimum weight affects the weight but little, the best depth will be considerably less than such least-weight depth.

On account of various practical considerations the thickness of

Fig. 38.

web plate is varied but little for a wide range in depth of girder. In railroad practice, for example, the minimum thickness of $\frac{3}{8}$ in. is used by some designers for depths as great as 7 or 8 ft. and $\frac{7}{16}$ in. thickness for depths from 6 to 10 ft. It will therefore be useful to determine the theoretical least-weight depth for a girder with a given web thickness.

Fig. 39.

1st. *If the moment of resistance of the web is neglected.*

Let M = centre moment in girder due to live and dead loads, and impact if any;

h = depth of web, assumed also equal to the effective depth of the girder;

f = allowable fibre stress on gross section of flange;

t = thickness of web;
l = length of girder in feet;
W = total weight of girder.

That part of the total weight which varies with variation in depth is made up of, (*a*) the flanges, (*b*) the web plate, and (*c*) the splice plates, stiffeners and fillers. The variation in the cross-bracing may be neglected. The average cross-section of the flanges, where several cover plates are used, may be taken as equal to eight-tenths of their cross-section at the centre point. The average cross-section of two flanges would then be equal to $2 \times 0.8 \times M/fd$, and the weight in pounds would be closely equal to $1.6 \frac{M}{fd} \cdot \frac{10}{3} \cdot l$. The weight of the web is likewise equal to $t\,h. \frac{10}{3}. l$. The weight of stiffeners, etc., depends largely upon the specifications under which the design is made, but estimates show that these items will weigh usually from 50 to 70 per cent of the web. For small changes in depth their weight may be assumed to vary directly with the depth, in the same manner as the web, hence such weight may be included in the weight of the web in a simple manner by multiplying the latter by 1.5 to 1.7. We have then, approximately

$$W = 1.6 \frac{M}{fh} . \frac{10}{3} . l + 1.6\, t\,h. \frac{10}{3} . l \quad . \; . \quad . \; . \; . \quad (57)$$

Differentiating and solving for h we have, for a minimum value of W

$$h = \sqrt{\frac{M}{f\,t}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (58)$$

2d. *If the moment of resistance of the web is not neglected.*

Assuming that one-eighth of the area of the web is taken as flange area, top and bottom, this amount should be subtracted from the weight given in eq. (57), giving

$$W = 1.6 \frac{M}{fh} \cdot \frac{10}{3} . l + \left(1.6 - \frac{1}{4}\right) t\,h . \frac{10}{3} . l \quad . \; . \; . \quad (59)$$

From which we derive. as before, for a minimum value of W

$$h = 1.1 \sqrt{\frac{M}{f\,t}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (60)$$

In the case of short girders, where the flanges are of uniform

section throughout, the weight of the flanges would be $2\frac{M}{fh}\cdot\frac{10}{3}\cdot l$, and eqs. (58) and (60) would become, respectively, $1.12\sqrt{\frac{M}{ft}}$ and $1.22\sqrt{\frac{M}{ft}}$.

The effect of a given variation in depth from the exact value which gives the minimum weight, can be determined by calculating the value of dW/W for a given value of dh/h. It can be shown that for a given proportionate change in depth, dh/h, from the least-weight depth, the proportionate change in weight is closely equal to $\frac{1}{2}(dh/h)^2$. That is, if the depth is increased or decreased 10 per cent, the weight is increased approximately $\frac{1}{2}(10/100)^2 = 0.5$ per cent; for a change in depth of 20 per cent the change in weight is about $\frac{1}{2}(20/100)^2 = 2$ per cent, etc.* This shows the small relative effect of large changes in depth below or above the exact least-weight depth, and indicates that practical considerations of manufacture may well modify very considerably the theoretical depth. The actual depths used in practice are usually 15 or 20 per cent less than the theoretical least-weight depths, the resulting slight increase in weight of 1 or 2 per cent being offset by advantages gained in other ways.

In the application of the preceding principles a thickness must first be assumed and the corresponding economical depth then determined. The resulting proportions must then be tested for shearing strength, adequate provision for riveting, and any requirements there may be in the specifications regarding minimum ratio of thickness to depth. To meet these requirements a different thickness may have to be assumed or a considerable change made in the depth.

Where the ratio of thickness to depth is limited, as in the specifications of Appendix A, Art. 29, such provision, together with the requirements for shear (Art. 18), are likely to determine the web

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* The exact relation is represented by the differential equation

$$\frac{dy}{y} = \frac{(dx)^2}{2x(x+dx)}.$$

dimensions. To meet these two requirements of the specifications we may proceed as follows:

Let V = maximum total shear;

h_f = depth of vertical leg of flange angle;

$h - 2\,h_f$ = unsupported depth of web.

Then by Art. 18 of the Specifications

$h\,t = V/10{,}000$ and by Art. 29, $h - 2\,h_f = 160\,t$.

Eliminating t we find

$$h = h_f + \sqrt{\frac{V}{62.5} + h_f^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Then $t = h/160$, but not less than 3/8 in.

The depth resulting from the application of this method may be considerably less than the economical depth as given by the preceding method, but a greater depth cannot be used without using a thicker plate which will not be economical unless the discrepancy is very large. If the result gives a thickness less than 3/8 in., then 3/8 in. must be used and the proper depth may be found by the first method.

THE DESIGN OF A SINGLE-TRACK DECK PLATE GIRDER RAILWAY BRIDGE

137. **General Data.**—The principles set forth in the preceding articles will now be applied to the design of a railway deck plate girder of 70-ft. span over all, the live load to be Cooper's E-60 loading. The main features of the design will be governed by the specifications of the American Railway Engineering Association, 1910, as given in Appendix A.

Span Length.—The span length, centre to centre of bearings, to be used in the calculation will depend upon the arrangement of the end details and the size of the base plates. According to Art. 60 of the specifications, spans of less than 80 ft. may be arranged to slide on smooth bearings. For this design the arrangement shown in Fig. 30 (*c*), Art. 133 will be adopted. This requires the centre of the bearings to be placed about 1 ft. from the ends of the girder. The distance centre to centre of bearings will then be 68 ft. This dimen-

sion must be checked up when the end bearings are designed, and revision made if necessary.

Spacing of Girders.—The girders will be spaced 7 ft. centre to centre.

SPECIFICATIONS.—(3) The width centre to centre of girders and trusses shall in no case be less than one-twentieth of the effective span, nor less than is necessary to prevent overturning under the assumed lateral loading.

As the first condition would require a width of only $^{68}/_{20} = 3.4$ ft. between girders, it can be seen that the proper spacing is governed by other considerations. These relate to the width required for an effective lateral system, with certain minimum and maximum limits. A spacing about equal to depth of girder gives good porportions for both the lateral system and the cross-frames, and since the depth is generally made from $^1/_{10}$ to $^1/_{12}$ the span length this leads to a spacing in practice of from $^1/_{10}$ to $^1/_{12}$ the span, with a minimum of 6½ ft. and a maximum spacing of 10 or 12 ft. For a spacing greater than the upper limit, the ties become very large and heavy. Their deflection is also large, which tends to concentrate the load near the edges of the cover plates, thus causing large bending moments in the inside flange angles. On the other hand, if the girders are spaced less than 6½ ft. centre to centre, the loads come almost directly over the girders, causing heavy impact stresses from eccentric wheels and rough track due to the rigidity of the floor. A longer tie serves as a cushion and thus tends to reduce this impact effect.

Material.—The material will be taken as structural steel, with a minimum thickness of ⅜ in., as specified by the following:

SPECIFICATIONS.—(1) The material in the superstructure shall be structural steel, except the rivets, and as may be otherwise specified.

(85) The steel shall be made by the open-hearth process.

(38) The minimum thickness of material shall be ⅜ in., except for fillers.

The rivets will be taken as ⅞ in. in diameter. This is the size of rivet usually adopted for railway bridges of moderate size. The minimum angle which can be used is 3½ × 3½ in., in order to meet the requirements of Art. 41, Specifications.

(41) The diameter of the rivets in any angle carrying calculated stress shall not exceed one-quarter the width of the leg in which they are driven.

Loads.—The live load will be taken as Cooper's E-60, described in Arts. 7 and 8 Specifications, and Art. 169, Part I. The wind load and other lateral forces are as specified in Art. 10, Specifications.

138. The Wooden Floor.—The design of the wooden floor system is governed by the following:

SPECIFICATIONS.—(5) Wooden tie floors shall be secured to the stringers and shall be proportioned to carry the maximum wheel load, with 100 per cent impact, distributed over three ties, with fibre stress not to exceed 2,000 lbs. per sq. in. Ties shall not be less than 10 ft. in length. They shall be spaced with not more than 6-in. openings; and shall be secured against bunching.

In this case the maximum wheel load is given by the special loading of Art. 7, Specifications. For E-60, each load is 75,000 lbs.

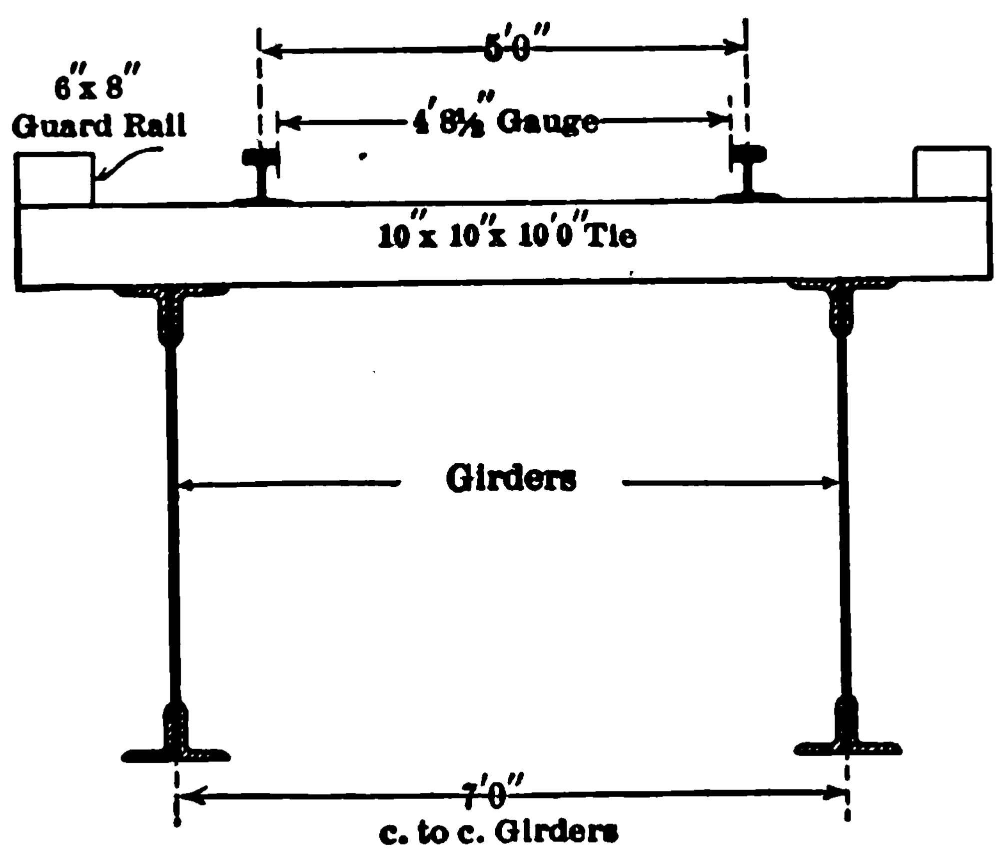


FIG. 40.

per axle, spaced 7 ft. apart. Assuming the loads to be applied to the ties at points 5 ft. apart and the girders to be spaced 7 ft. apart as shown in Fig. 40, the maximum bending moment on each tie due to live load and 100 per cent impact is

$$M = \frac{75,000}{3}\left(\frac{7-5}{2}\right) 12 = 300,000 \text{ in.-lbs.} \quad . \quad . \quad .$$

The moment due to the weight of the floor is small and is usually neglected. If necessary, this moment can be determined approximately by assuming the floor to weigh 450 lbs. per ft. Each tie will carry about 1¼ ft. of track. This load is usually assumed as concentrated at the rails. The resulting bending moment in the tie is then $M = \frac{562.5}{2}\left(\frac{7-5}{2}\right) 12 = 3{,}375$ in.-lbs., or only a little over 1 per cent of the live load moment.

With a fibre stress of 2,000 lbs. per sq. in. (Art. 5, Specifications) the required section modulus of the tie is

$$\frac{b\,h^2}{6} = 300{,}000/2{,}000 = 150.$$

For a 10 × 10-in. tie, we have $b\,h^2/6 = 166\frac{2}{3}$, which meets the re-requirements. The ties will be spaced 6 ins. apart, or 16 ins. centre to centre. A 6 × 8-in. wooden guard rail will be placed at the ends of the ties, notched 1 in. over the ties to prevent bunching. It is usual to fasten every fifth tie to the girder flanges by means of ¾-in. hook bolts.

139. The Dead Load.—

SPECIFICATIONS.—(6) The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weigh 4½ lbs. per foot board measure; ballast 100 lbs. per cu. ft.; reinforced concrete 150 lbs. per cu. ft.; and rails and fastenings, 150 lbs. per linear ft. of track.

At 4½ lbs. per ft. B. M., a 10 × 10-in. tie 10 ft. long weighs $10 \times \frac{5}{6} \times 10 \times 4\frac{1}{2} = 375$ lbs. For ties spaced 6 ins. clear, or 16 ins. centre to centre, the weight per foot of bridge is $375 \times \frac{12}{16} = 280$ lbs. Two 6 × 8-in. guard rails weigh 36 lbs. per ft. of bridge. The total weight of timber in the bridge floor is then $280 + 36 = 316$ lbs. per ft. of bridge. With track and fastenings at 150 lbs. per ft. the total weight of the track and floor is 466 lbs. per ft. of bridge. The weight of the steel in the girder is estimated by the formula of Art. 9, modified as stated for E–60 loading.* We then have: $w = 1.1\,(12.5\,l + 100) = 1.1\,(12.5 \times 68.0 + 100) = 1{,}040$ lbs. per ft. of

*The formula is $w = k\,(12.5l + 100)$.
For $E40$, $k = 0.9$; $E50$, $k = 1.0$; $E60$, $k = 1.1$.

bridge. In this formula l is to be taken as the distance centre to centre of bearings in feet.

The total dead load, floor, track, and girders, is $466 + 1{,}040 = 1{,}506$ lbs. per ft. of bridge. The value used in the following calculations is 1,500 lbs. per ft. of bridge.

140. Maximum Moments and Shears.—The live-load moments and shears are to be determined for Cooper's E–60 loading (see Arts. 7 and 8, Specifications). These moments and shears are calculated by the methods given in Chapter V of Part I. The dead-load moments and shears are calculated for the load determined in the preceding article by the methods given in Chapter IV of Part I. The length of the girder to be used in the above calculations is the distance centre to centre of bearings, or 68 ft.

The maximum live-load moment for each girder, calculated by the methods given in Arts. 125 and 126, Part I, is found to be 2,435,300 ft.-lbs. This moment occurs under wheel 13 when that wheel is placed 0.07 ft. to the left of the centre of the girder. The impact effect is determined by Art. 9, Specifications.

(9) The dynamic increment of the live load shall be added to the maximum computed live-load stress and shall be determined by the formula

$$I = L\frac{300}{l + 300}$$

where I = impact or dynamic increment to be added to live-load stress;
L = computed maximum live-load stress;
l = loaded length of track in feet producing the maximum stress in the member.

For maximum moment practically the entire span is covered with the live load. The value of l in the formula can then be taken as 68 ft. and we have $I = L\left(\frac{300}{68 + 300}\right) = 0.815\,L$. The allowance for impact is therefore $0.815 \times 2{,}435{,}300 = 1{,}984{,}800$ ft.-lbs. The dead-load moment at the same point is practically equal to that at the centre of the girder, where the value is 433,500 ft.-lbs. per girder. The total maximum moment is found by combining the above values, which gives 4,853,600 ft.-lbs. or 58,243,200 in.-lbs. as the moment to be carried by each girder.

The maximum live-load end shear occurs when wheel 2 is placed over the centre of bearings at the left end of the span, wheels 2 to 13 being on the span. The value of the shear is 161,750 lbs. The impact coefficient is determined for the span fully loaded, or $I = 0.815\ L$. The allowance for impact is, therefore, $0.815 \times 161{,}750 = 131{,}820$ lbs. The dead-load end shear is $\frac{1}{2} \times 1{,}500 \times 34 = 25{,}500$ lbs. The total end shear for one girder is then $161{,}750 + 131{,}820 + 25{,}500 = 319{,}070$ lbs.

The maximum total moments and shears were also calculated at 5-ft. points along the girder. The resulting values are given in the following table. In this table the impact coefficient for moments at all points is taken for $l = 68$ ft. in the formula. The impact coefficients for shears are different for each point. The length l in the impact formula is taken as the distance from the first wheel on the structure to the right end of the span.

TABLE OF DEAD, LIVE, AND IMPACT MOMENTS

Points	End	a	b	c	d	e	f	g
D. L. M......	0	96.0	199.1	283.5	349.1	396.0	424.1	433.5
L. L. M......	0	579.2	1,153.1	1,623.4	1,987.0	2,225.4	2,378.0	2,435.2
Imp. M......	0	472.0	939.8	1,323.1	1,619.4	1,813.8	1,938.1	1,984.7
Total ft.-lbs...	0	1,147.2	2,292.0	3,230.0	3,955.5	4,435.2	4,740.2	4,853.4
Total inch-lbs.	0	13,766.4	27,504.0	38,760.0	47,466.0	53,222.4	56,882.4	58,240.8

Moments in thousands of foot-pounds and thousands of inch-pounds.

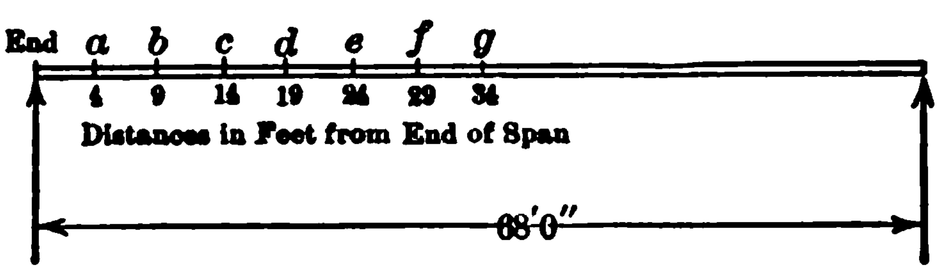


TABLE OF DEAD, LIVE, AND IMPACT SHEARS

Points	End	a	b	c	d	e	f	g
D. L. Shear...	25.50	22.50	18.75	15.00	11.25	7.50	3.75	0
L. L. Shear...	161.75	144.79	125.39	107.30	90.53	74.64	59.27	45.34
Imp. Shear...	131.82	118.00	102.45	88.96	76.05	63.59	51.27	39.76
Total Shear...	319.07	285.29	246.59	211.26	177.83	145.73	114.29	85.10

Shears in thousands of pounds.

141. General Conditions Governing Design of Plate Girders.—

SPECIFICATIONS.—(29) Plate girders shall be proportioned either by the moment of inertia of their net section, or by assuming that the flanges are concentrated at their centres of gravity, in which case one-eighth of the gross section of the web, if properly spliced, may be used as flange section. The thickness of web-plates shall be not less than $^1/_{160}$ of the unsupported distance between flanges.

(38) The minimum thickness of metal shall be ⅜ in., except for fillers.

(30) The gross section of the compression flanges of plate girders shall be not less than the gross section of the tension flanges; nor shall the stress per square inch in the compression flange of any beam or girder exceed $16{,}000 - 200\dfrac{l}{b}$, when the flange consists of angles only, or if cover consists of flat plates, ... where l = unsupported distance and b = width of flange.

(118) (in part) Material more than ¾ in. thick shall be sub-punched and reamed or drilled from the solid.

The make-up of the girder section will be determined in accordance with the above articles from the specifications and the general discussion given in Art. 110.

The flange stresses will be determined by assuming that the flange areas are concentrated at their centres of gravity, and that ⅛ of the gross area of the web plate is effective in resisting bending moment, this also being assumed as concentrated at the centre of gravity of the flanges. The shear will be assumed as taken entirely by the web plate.

Each flange will consist of two angles and enough cover plates to provide the required area. The thickness of these plates and angles is limited to ¾ in. (Art. 118, Specifications) in order to avoid drilling the rivet holes from solid material, or sub-punching and reaming. In girders of the length used in this design, the flanges usually consist of 6 × 6-in. angles and 14-in. cover plates. The area to be provided by cover plates is limited by some specifications to one-half the gross flange area, although some designers place as much as two-thirds of the flange area in cover plates. When this percentage is exceeded with 6 × 6-in. angles, it is best to use angles with longer legs, 8 × 8 ins., and 18-in. cover plates, or a section of the form shown in Fig. 13 (*b*) of Art. 110.

142. Depth and Thickness of Web Plate.—The proper depth of girder and thickness of web plate to be used for a given girder will depend upon the web area required to carry the end shear, and upon the required economical depth at the centre for moment. In some cases, practical considerations, such as shop practice or clearance conditions to be met in the field and in transportation, will govern the depth to be used. For this case, we will assume that shear and moment conditions only are to be considered.

The depth and thickness of the web plate necessary to carry the end shear are determined by Arts. 18 and 29, Specifications.

(18) (in part) Shearing: Plate girder webs; gross section, 10,000 lbs. per sq. in.

(29) (in part) The thickness of web-plates shall be not less than $^1/_{160}$ of the unsupported distance between flange angles (but not less than $3/8$ in.).

For an end shear of 319,070 lbs., as calculated in Art. 140, we find from eq. (61) of Art. 136, assuming 6 × 6-in. flange angles,

$$h = h_f + \sqrt{\frac{V}{62.5} + h_f^2} = 6 + \sqrt{\frac{319{,}070}{62.5} + 36} = 6 + 71.7 = 77.7$$

ins. as the depth of girder, and $t = \dfrac{h - 2\,h_f}{160} = \dfrac{77.7 - 12}{160} = 0.411$ in., or $7/16$ in. as the thickness of web plate. The gross area provided by this plate is $77.7 \times 7/16 = 34.0$ sq. ins. Area required for end shear is $319{,}070/10{,}000 = 31.9$ sq. ins.

The economical depth will now be found by the application of eq. (60) Art. 136, assuming a $7/16$-in. web. The centre moment is 58,243,200 in.-lbs. The fibre stress for net section is 16,000 lbs. per sq. in., and as the reduction for rivet holes amounts to approximately one-eighth of the gross section, the fibre stress for gross sectior is approximately $7/8 \times 16{,}000 = 14{,}000$ lbs. per sq. in. From eq. (60)

$$h = 1.1\sqrt{\frac{M}{ft}} = 1.1\sqrt{\frac{58{,}243{,}200}{14{,}000 \times {}^7/_{16}}} = 107.2 \text{ ins.}$$

Reducing this by about 20 per cent gives a depth of 86 ins., which would be about the correct depth to use were it not for Art. 29 of the Specifications, which allows a depth for a $7/16$-in. plate of $7/16 \times 160 + 12 = 82$ ins. This is 23 per cent less than the least weight depth

and, according to Art. 136, the resulting increase in weight will be about $0.23^2/2 = 2.6$ per cent, a relatively small amount. In order to use a greater depth, it would be necessary to use a ½-in. web, which would add much more to the weight than the above percentage.

An 80 × 7/16 in. web plate will be used and the flange angles placed 80½ ins. back to back to allow for clearance. Area of web = 80 × 7/16 = 35 sq. in.

143. The Flanges.—The total maximum moment near the centre of the girder is 58,243,200 in.-lbs., as given in Art. 140. The flange section will be made up of two 6 × 6-in. angles, and cover plates 14 ins. wide. The web plate will be taken as 80 × 7/16 ins., as determined in Art. 142, and the flange angles will be placed 80½ ins. back to back.

In making up a trial section, it will be assumed that the centre of gravity of the flange area is at the back of the angles. The effective depth is then 80.5 ins. Dividing the bending moment by the assumed effective depth, the flange stress is found to be 58,243,200/80.5 = 723,500 lbs.

SPECIFICATIONS.—(15) Axial tension on net section, 16,000 pounds per square inch.

The net area required for the lower, or tension, flange is 723,500/16,000 = 45.22 sq. ins. As stated in Art. 141 and Art. 29 of the Specifications, one-eighth of the web area is considered as effective in resisting bending moment. We then have 80 × 7/16 × ⅛ = 4.38 sq. ins. as the web area which may be considered as part of the flanges. The area to be made up by plates and angles is then 45.22 − 4.38 = 40.84 sq. ins. Two 6 × 6 × ¾-in. angles will be tried, gross area = 2 × 8.44 = 16.88 sq. ins. These are the heaviest 6 × 6-in. angles which can be used under Art. 118, Specifications, which limits the thickness of material to ¾ in.

The area to be deducted for rivet holes depends upon the arrangement of the flange rivets. On the general drawing, two rows of rivets are shown in the vertical legs of the angles, and a single row in each horizontal leg. The rivets in the vertical legs are placed in alternate rows and the distance between rivets is such that only one of these rivets need be considered in obtaining net section, as shown

by the discussion in Art. 93. In the horizontal legs, the rivet spacing is much greater than that in the vertical legs, as shown in Art. 116. If a uniform pitch of rivets is maintained in the horizontal legs, it will be found difficult to arrange the rivets so that only one rivet hole need be deducted from the whole section area. It is therefore usual to allow for one rivet hole in each angle leg in calculating net areas.

For ⅞-in. rivets, the diameter of the rivet hole is to be taken as 1 in. (Specifications, Art. 26.) Assuming two rivets on the same section of each angle, as shown in Fig. 41, the net area of two 6 × 6 × ¾-in. angles is $2\,(8.44 - 2 \times 0.75) = 13.88$ sq. ins.

The cover plates must provide $40.84 - 13.88 = 26.96$ sq. ins. Assuming 14-in. plates with two rivet holes deducted from the section,

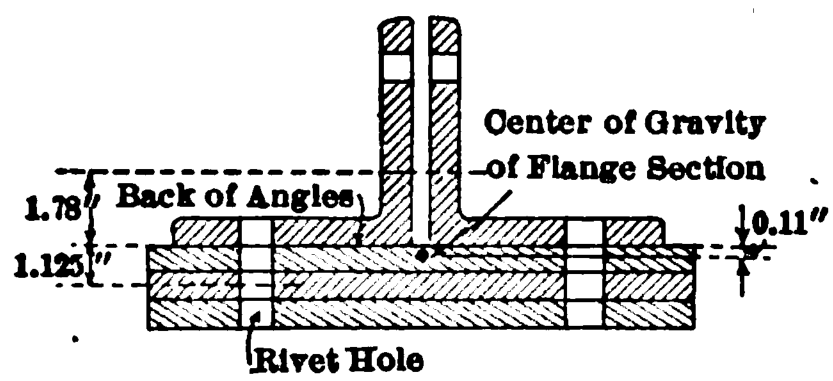


FIG. 41.

the net width of each plate is 12 ins. The thickness of plates required is $26.96 / 12 = 2.24$ ins. Remembering that the material is not to exceed ¾ in., we find that three ¾-in. plates will furnish the required thickness.

The total area for the assumed flange section is then as given in the following table:

Item	Gross Area, Sq. In.	Rivet Holes, Sq. In.	Net Area, Sq. In.
2 angles 6 × 6 × ¾ inches	16.88	3.00	13.88
3 cover plates 14 × ¾ inches	31.50	4.50	27.00
	48.38	7.5	40.88
⅛ web area = $\frac{1}{8} \times \frac{7}{16} \times 80$	.....	...	4.38
Total net area available	.....	...	45.26

It is now necessary to revise the above calculation, using the true effective depth of the assumed section given above. From the tables

in the handbooks, the gravity axis of a 6 × 6 × ¾-in. angle is found to be 1.78 ins. from the back of the angle. Taking moments about the back of the angle, using gross areas, and distances as shown in Fig. 41, the centre of gravity of the entire flange section is found to be at a point outside the backs of the angles a distance

$$x = \frac{31.5 \times 1.125 - 16.88 \times 1.78}{48.38} = 0.11 \text{ in.}$$

It is to be noted that if the moment of the angles exceeds that of the cover plates, the centre of gravity will be located inside the backs of the angles. The true effective depth is found to be 80.5 + 2 × 0.11 = 80.72 ins. Using this new effective depth, the revised flange stress and flange area required are as follows:

$$\text{Flange stress} = \frac{58,243,200}{80.72} = 721,500 \text{ lbs.}$$

$$\text{Flange area} = \frac{721,500}{16,000} = 45.09 \text{ sq. in.}$$

The flange area provided by the assumed section is therefore sufficient.

The flange section selected is the largest which can be made by using 6 × 6-in. angles and 14-in. cover plates. All material is ¾-in. thick, the maximum allowed by Art. 118, Specifications, and further addition of cover plates to these angles is not advisable because we now have $\frac{27.0}{40.88} = 66.3$ per cent of the flange area in cover plates. A larger moment can be taken care of by increasing the depth of the girders or by using 8 × 8-in. flange angles and 18-in. cover plates.

144. Calculation of Flange Area from Moment of Inertia.— To illustrate the method of calculation discussed in Art. 109, the flanges of this girder will now be calculated by this method.

Since the tension and compression flanges of these girders are alike, the neutral axis is located at the centre of the web-plate. The moment of inertia of the gross girder section about this axis is 176,600 ins^4., and the distance from the neutral axis to the extreme fibres of the section is 42.5 ins. Then $f_g = 58,243,200 \times 42.5/176,600 = 14,060$ lbs. per sq. in. The gross flange area = 48.38 sq. ins.; net flange area = 40.88 sq. ins.; web area = 35 sq. ins. Then from eq. (21) of Art. 109, $f_n = 14,060 \times 54.21/45.26 = 16,850$ lbs. per sq. in. The fibre stress on the extreme fibre of the tension flange thus exceeds the allowable by 5.3 per cent.

The foregoing calculations show that the girder section designed by the approximate methods is satisfactory. The total depth of the girder is 85 ins. and the effective depth 80.72 ins., giving an excess of total over effective depth of 5.3 per cent, or about the same as the excess of fibre stress. The effective depth is 95 per cent. of the total depth.

In order to bring out further the difference between the approximate and more exact methods of design, let it be assumed that, due to clearance conditions, the web plate of the girder designed in the preceding articles is limited to a depth of 48 ins.; flange angles to be placed 48½ ins. back to back. The maximum moment is 58,243,200 in.-lbs., and end shear is 319,070 lbs. Fig. 42 shows a girder section designed by the approximate method. The web area required is 31.9 sq. in. A 48 × $^{11}/_{16}$ in. web furnishes 33 sq. ins. The true effective depth is 46.02 ins., giving a flange stress of 1,268,000 lbs. and a flange area of 79.2 sq. ins. One-eighth of the web area is 4.1 sq. ins., so that the flanges must provide 75.1 sq. ins. The flange section shown in Fig. 42 has a gross area of 86.56 sq. ins., and a net area of 76.31 sq. ins., after deducting two rivet holes from each angle and each plate. This

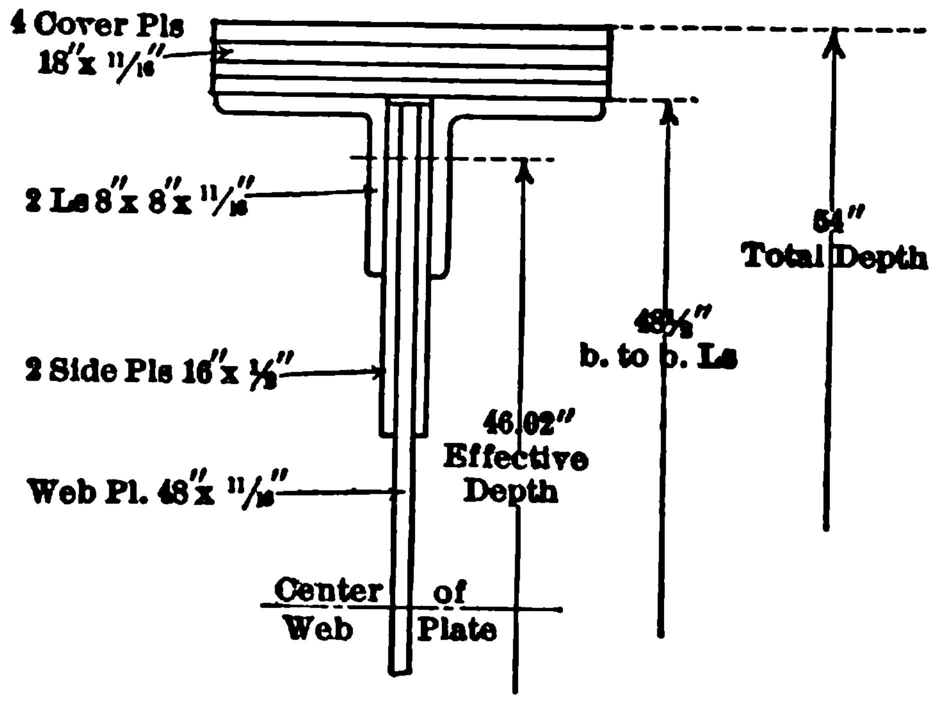


FIG. 42.

design is therefore satisfactory, according to the approximate theory. Note that the effective depth is 85.2 per cent of the total depth.

Applying the more exact method of calculation to the girder section of Fig. 42, neutral axis at the center of the web, it will be found that the moment of inertia of the gross girder section is 101,400 ins^4., and the distance to the extreme fibre is 27 ins. The extreme fibre stress on the gross section is 58,243,200 × 27/101,400 = 15,500 lbs. per sq. in. With the gross and net flange areas and the web area given above, we have f_n = 15,500 × 92.06/80.41 = 17,750 lbs. per sq. in. The extreme fibre stress on the tension flange is 1,750/16,000 = 10.9 per cent greater than the allowable.

As the extreme fibre stress is beyond allowable limits, addition must be made to the flange section. The additional area should be placed where it will have the greatest effect in increasing the moment of inertia of the section. Adding another cover plate to the section shown in Fig. 42 gives an extreme fibre stress on net area of 15,500 lbs. per sq. in. The revised

section has a gross flange area of 98.86 sq. ins. and a net area of 87.31 sq. ins., or an addition of 11 sq. ins. to the net flange area.

These calculations show that for the type of flange shown in Fig. 42, exact methods of calculation should be used. If the approximate method is to be used, considerable excess area must be provided over that called for by the computed stresses. In very shallow, heavy girders, such as used in building work, even greater differences will be found in fibre stress given by exact and approximate methods.

145. Lengths of Cover Plates.—The required lengths of the several plates can be determined by the methods given in Art. 111. Bending moments for 5-ft. points have been calculated and are given in the table on page 193, Art. 140. A moment curve plotted from these values is shown in Fig. 43. Calculating the moments of resistance of

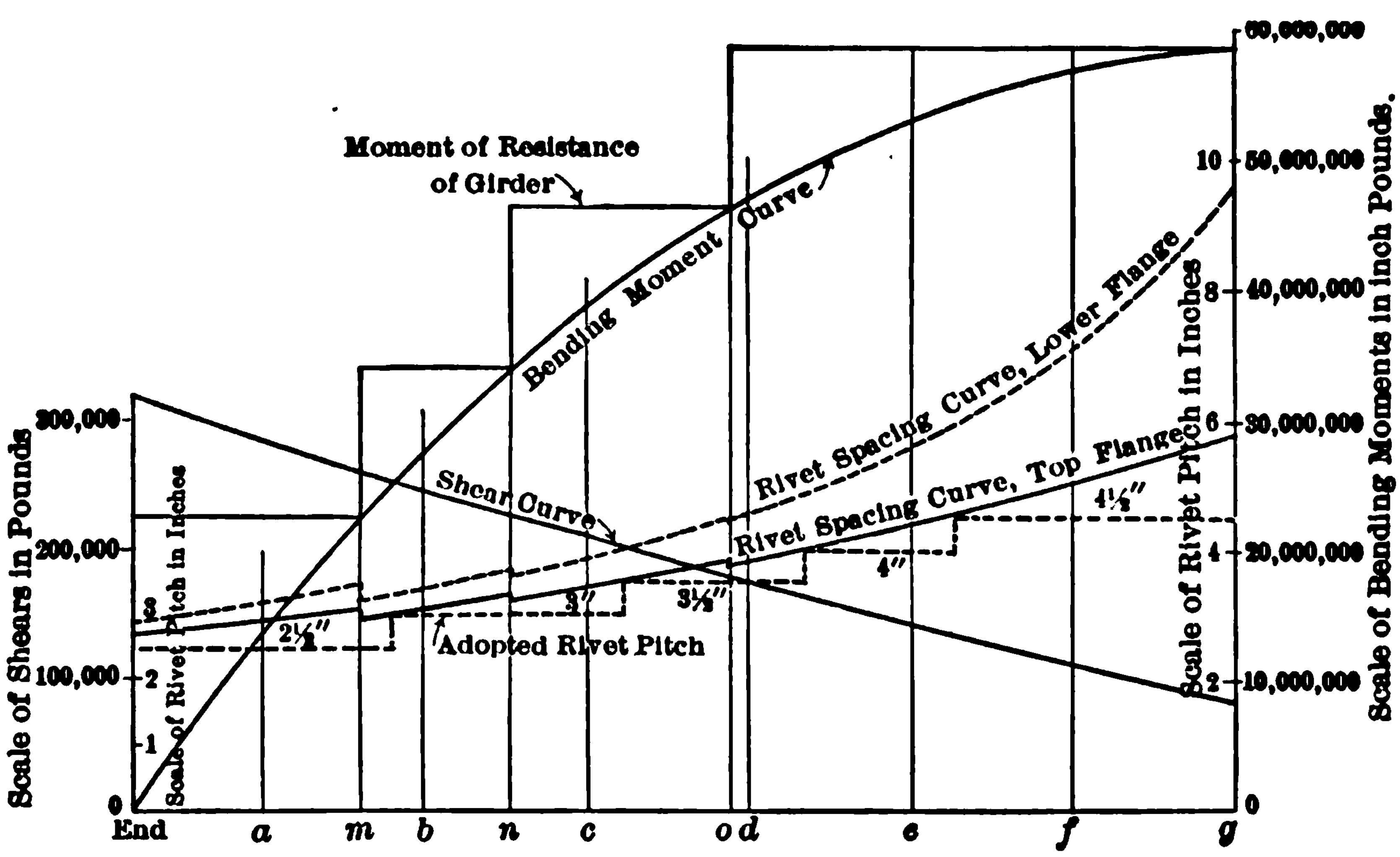


FIG. 43.

the girder for the flange angles and one, two, or three cover plates, we have the values shown by the horizontal lines in Fig. 43.

The calculations follow:

Moment of Resistance of Two Angles and Three Cover Plates.—Total net area of flange, 1/8 of web included, = 45.26 sq. ins. Effective depth = 80.72 ins. Moment of resistance = 45.26 × 80.72 × 16,000 = 58,454,200 in.-lbs.

Two Angles and Two Cover Plates.—Net area of two angles, two

cover plates, and ⅛ web area = 13.88 + 18.0 + 4.38 = 36.26 sq. ins. A new effective depth must be determined. By the same method as used in Art. 143, the centre of gravity of the flange section is found to be 0.38 in. inside the backs of the angles. · Effective depth = 80.5 − (2 × 0.38) = 79.74 ins. Resisting moment = 36.26 × 79.74 × 16,000 = 46,262,000 in.-lbs.

Two Angles and One Cover Plate.—The net area of two angles, one cover plate, and ⅛ web area is 13.88 + 9.0 + 4.38 = 27.26 sq. ins. Effective depth, found as before, is 78.60 ins. Resisting moment = 27.26 × 78.60 × 16,000 = 34,284,000 in.-lbs.

Two Angles—Net area of two angles and ⅛ web area = 13.88 + 4.38 = 18.26 sq. ins. Effective depth = 80.5 − (2 × 1.78) = 76.94 ins. Resisting moment = 18.26 × 76.94 × 16,000 = 22,479,000 in.-lbs.

The points of intersection of the lines representing the above values and the plotted moment curve of Fig. 43 determine the theoretical lengths of the several plates. Scaling the distances from Fig. 43, we find that the top plate is required for a length of 15.8 ft. from the centre of the girder; the middle plate for a distance of 22.15 ft. from the centre; and the plate next to the angles, 27.0 ft. from the centre of the girder. To provide a few extra rivets at the ends of the plates, the above distances will be increased about a foot in each case.

From Art. 78, Specifications, we have

(78) Where flange plates are used, one cover plate of top flange shall extend the whole length of the girder.

To meet this requirement, the cover plate next to the angles of the top flange will be made full length. The general details are shown on the general drawing, Plate I.

The lengths of the cover plates may also be determined approximately by assuming a parabolic variation of moment and using eq. (22) of Art. 111. Substituting the proper values for one, two and three plates, we have,

Outside plate:

$$x_1 = l\sqrt{\frac{a_1}{A}} \qquad = 68\sqrt{\frac{9}{45.26}} = 30.4 \text{ ft.}$$

Middle plate:

$$x_2 = l\sqrt{\frac{a_1 + a_2}{A}} \qquad = 68\sqrt{\frac{18}{45.26}} = 42.9 \text{ ft.}$$

Plate next to angles:

$$x_3 = l\sqrt{\frac{a_1 + a_2 + a_3}{A}} = 68\sqrt{\frac{27}{45.26}} = 52.5 \text{ ft.}$$

By the more exact method given above, the lengths of the plates were found to be 31.6, 44.3, and 54 ft. respectively.

146. Rivet Pitch in Flange Angles.—

SPECIFICATIONS.—(18) Shearing: Shop-driven rivets, 12,000 lbs. per sq. in.; field-driven rivets, 10,000 lbs. per sq. in.

(19) Bearing: Shop-driven rivets, 24,000 lbs. per sq. in.; field-driven rivets, 20,000 lbs. per sq. in.

(27) In proportioning rivets the nominal diameter of the rivet shall be used.

(31) The flanges of plate girders shall be connected to the web with a sufficient number of rivets to transfer the total shear at any point in a distance equal to the effective depth of the girder at that point, combined with any load that is applied directly on the flanges. The wheel loads, where the ties rest on the flanges, shall be assumed to be distributed over three ties.

(39) (in part) The minimum distance between centres of rivet holes shall be three diameters of the rivet; but the distance shall preferably be not less than 3 ins. for $\frac{7}{8}$-in. rivets, ... The maximum pitch in the line of stress for members composed of plates and shapes shall be 6 ins. for $\frac{7}{8}$-in. rivets, ... For angles with two-gage lines and rivets staggered the maximum shall be twice the above in each line.

Art. 31 of the Specifications outlines a method of calculation equivalent to the use of eq. (23) of Art. 114, stating that the *whole* of the shear will be assumed as transferred to the angles; but as we have already assumed one-eighth of the web section as flange, it will be more exact to use eqs. (26) and (27) of Art. 115.

The rivet pitch will be calculated at points 5 ft. apart, for which points the shears are given in Art. 140, and at the theoretical ending of each cover plate. Since there is an abrupt change of flange area, and also of effective depth at such points, there will also be a change in the rivet spacing. It is usual, however, to use a smooth curve, neglecting the changes in pitch at the ends of the cover plates.

The value of one rivet in bearing on a $\frac{7}{16}$-in. plate at 24,000 lbs. per sq. in. is given in the tables in Appendix B, as 9,190 lbs. Using the shears given in Art. 140 and the effective depths calculated

in Art. 145, we find by eq. (26), Art. 115, for the end point of the lower flange,

$$p = \frac{rh}{V}\,\frac{F + \dfrac{A_w}{8}}{F} = \frac{9.19 \times 76.94}{319.0} \times \frac{18.26}{13.88} = 2.92 \text{ ins.}$$

Values for other points are given in the table below.

For the upper flange, eq. (27) of Art. 115 must be used. In this equation w is the vertical load per inch due to the weight of one engine driver distributed over three ties (Art. 31, Specifications), and to the load per inch due to the weight of the floor and track, which is 466 lbs. per ft., as calculated in Art. 139. Since the driver load comes very suddenly upon the rivets, the impact coefficient is obtained by placing $L = 0$ in the impact formula of Art. 9, Specifications, or impact = 100 per cent.

The value of w is

$$w = \frac{60{,}000}{3 \times 16} + \frac{233}{12} = 1{,}250 + 19.4 = 1{,}270 \text{ lbs. per in.}$$

Substituting in eq. (27) of Art. 115 we have for the end point

$$p = \frac{9.19}{\sqrt{(1.27)^2 + \left(\dfrac{13.88}{18.26} \times \dfrac{319.0}{76.94}\right)^2}} = 2.71 \text{ ins.}$$

Values for other points are given in the table below.

TABLE OF RIVET SPACING

Point	Distance from End in Feet	Shear in Thousand Pounds (V)	Effective Depth in Inches (h)	Flange Area (F)	Flange Area Plus ⅛ Web Area $(F + \frac{A_w}{8})$	Rivet Pitch, Lower Flange Eq. (26) Art. (115)	Rivet Pitch, Top Flange Eq. (27) Art. (115)
End.......	0	319.0	76.94	13.88	18.26	2.92	2.71
a..........	4	285.3	76.94	13.88	18.26	3.26	2.97
m..........	7	262.5	76.94	13.88	18.26	3.53	3.19
m..........	7	262.5	78.60	22.88	27.26	3.26	2.98
b..........	9	246.6	78.60	22.88	27.26	3.48	3.14
n..........	11.85	226.0	78.60	22.88	27.26	3.80	3.36
n..........	11.85	226.0	79.74	31.88	36.26	3.68	3.28
c..........	14	211.3	79.74	31.88	36.26	3.96	3.46
o..........	18.2	182.5	79.74	31.88	36.26	4.56	3.86
o..........	18.2	182.5	80.72	40.88	45.26	4.50	3.82
d..........	19	177.8	80.72	40.88	45.26	4.64	3.89
e..........	24	145.7	80.72	40.88	45.26	5.63	4.45
f..........	29	114.3	80.72	40.88	45.26	7.16	5.10
g..........	34	85.1	80.72	40.88	45.26	9.64	5.80

In this table the two values given at points *m*, *n*, and *o*, are respectively the rivet pitches just to the left and to the right of the theoretical ending of a cover plate. The values given in the table are plotted on Fig. 43, from which the spacing at any point may be determined by scaling from the curve.

The spacing actually used will depend upon the position of the stiffener angles and splices. As a rule, the pitch is changed by quarters or halves of an inch, the changes in pitch being made at or near the stiffeners. The rivet pitch adopted is shown on the general drawing. On Fig. 43 the horizontal dotted lines show that at all points the adopted pitch is within the calculated value at the point in question.

147. Rivet Pitch in Flange Plates.—In Art. 116 the horizontal shearing stress at any point in the flange is given by eq. (28) as

$$v_n = \frac{V}{h} \cdot \frac{a_1 + a_2 + \ldots a_n}{A_f + \frac{A_w}{8}}$$

The rivet pitch is then

$$p = \frac{r}{v_n} = \frac{r\,h}{V} \cdot \frac{A_f + \frac{A_w}{8}}{a_1 + a_2 + \ldots a_n}$$

Thus at the end of the outside cover plate, where the spacing will be smallest, the three cover plates have a net area of 27.0 sq. ins., while the total net flange area, including ⅛ web, is 45.26 sq. ins. The rivets are in single shear and have a value of 7,220 lbs. By the equation given above we have, using values of *V* and *h* given in the table of Art. 146 for point *o*,

$$p = \frac{7.22 \times 80.72}{182.5} \times \frac{45.26}{27.0} = 5.36 \text{ ins.}$$

This value is for a single line of rivets. Since there are two rows of rivets in the flange plates, the pitch in each row will be twice as great, or 10.72 ins. for each row. As this spacing is in excess of the maximum spacing allowed by Art. 39, Specifications, the adopted spacing for each row will be made not to exceed 6 ins., in order to conform to the specifications.

The rivet pitch for flange plates can also be found by comparing the shearing stress along the surface in question with that which

exists between the flange and the web. Thus at all points where the three plates are used the shear existing between the inner plate and the angle is to that between angles and web as the area of the three plates is to the total flange area (1/8 web not included), or as 27 is to 40.88. The value of a rivet in single shear is 7,220 lbs. and in bearing on the web the value is 9,190 lbs. From Art. 146 the rivet pitch at point *o* (bottom flange), between angles and web is 4½ ins.

Then the pitch for flange plate rivets is

$$p = 4.5 \times \frac{7{,}220}{9{,}190} \times \frac{40.88}{27.0} = 5.36 \text{ ins., as calculated above.}$$

148. End Stiffeners.—The conditions governing the design of end stiffeners are given in Arts. 130 and 133, and in Art. 79, Specifications.

(79) (in part) The stiffeners at ends and at points of concentrated loads shall be proportioned by the formula of Art. 16, the effective length being assumed as one-half the depth of girders. End stiffeners and those under concentrated loads shall be on fillers and have their outstanding legs as wide as the flange angles will allow and shall fit tightly against them.

For the end stiffeners, the load is equal to the end shear, or 319,070 lbs. The allowable unit stress is given by the column formula $16{,}000 - 70\frac{l}{r}$, with a maximum of 14,000 lbs. per sq. in. (Art. 16, Specifications). The length of the column is taken as one-half the depth of the girders back to back of angles, or 40¼ ins. The end stiffener angles will be assumed as four angles 5 × 3½ × ¾-in., area = 23.24

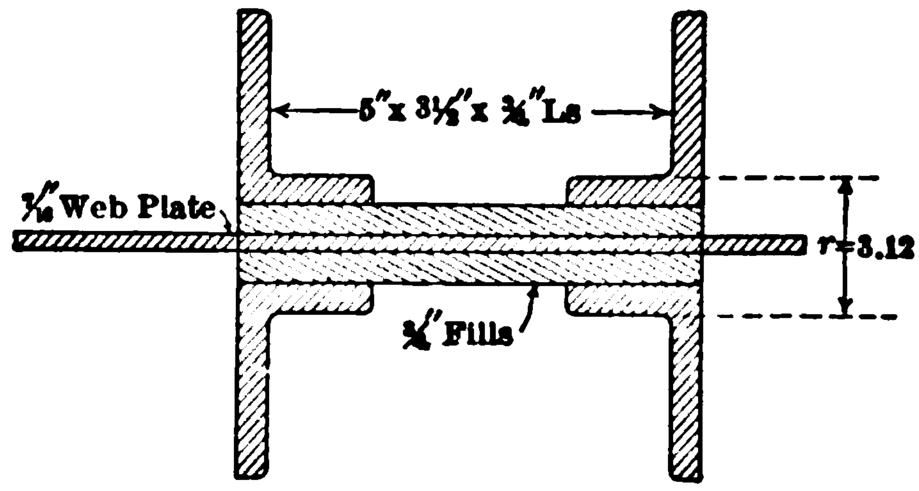


FIG. 44.

sq. ins. To avoid crimping the stiffeners, a filler plate equal in thickness to the flange angle, or ¾ in. in this case, is placed between the stiffener angles and the web plate as shown in Fig. 44. For the

section shown the value of r in a plane perpendicular to the web plate is found to be 3.12 ins. In calculating this value of r the area of the filler and web plates has been neglected. The allowable stress given by the column formula with $r = 3.12$ and $l = 40\frac{1}{4}$ is 15,100 lbs. per sq. in. As the specifications limit the working stress to 14,000 lbs. per sq. in., the required area is 319,070 / 14,000 = 22.8 sq. ins. The assumed angles furnish sufficient area.

149. Intermediate Stiffeners.—The portions of Art. 79, Specifications, which refer to intermediate stiffeners are as follows:

(79) (in part) There shall be web stiffeners generally in pairs, ... where the thickness of the web is less than $^1/_{60}$ of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a maximum limit of six feet (and not greater than the clear depth of the web); $d = \frac{t}{40}(12{,}000 - s)$, where d = clear distance between stiffeners or flange angles; t = thickness of web; s = shear per sq. in.

Intermediate stiffeners may be offset or on fillers, and their outstanding legs shall be not less than one-thirtieth of the depth of the girder plus 2 ins.

From the condition governing the width of the outstanding leg of the stiffeners we have

$$\left(\tfrac{1}{30} \text{ depth} + 2\right) = \frac{80.5}{30} + 2 = 4.68 \text{ ins.}$$

To answer this requirement, $5 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles will be used with the 5-in. leg outstanding. Filler plates $3\frac{1}{2} \times \frac{3}{4}$-in. will be placed between the stiffeners and the web to avoid crimping the stiffeners around the flange angles. The distance between the pairs of stiffeners is determined by the equation $d = \frac{t}{40}(12{,}000 - s)$, and by the condition (Art. 29, Specifications) that the unsupported length of web plate must not be greater than 160 times the thickness of the plate; in this case $160 \times 7/16 = 70$ ins.

At a point 4 ft. from the left end of the girder, where the shear is 285,300 lbs. (Art. 140) and the web area is 35.0 sq. ins. (Art. 142),

we have $s = 285{,}300 / 35 = 8{,}150$ lbs. per sq. in. Then from the above formula

$$d = \frac{7/16}{40}(12{,}000 - 8{,}150) = 42 \text{ ins.}$$

The values for other points are as follows:

STIFFENER SPACING

Distance from left end of girder in feet.....	4	9	14	19	24	29	34
Stiffener spacing in inches = d...........	42	54	65	75.6	86	95.5	104.8

From this table it can be seen that a spacing of 70 ins. governs beyond the 14-ft. point, as the calculated distances exceed 160 *t*.

The spacing of stiffeners to be used depends also upon the form of the lateral bracing of the top chord, and upon the position of the

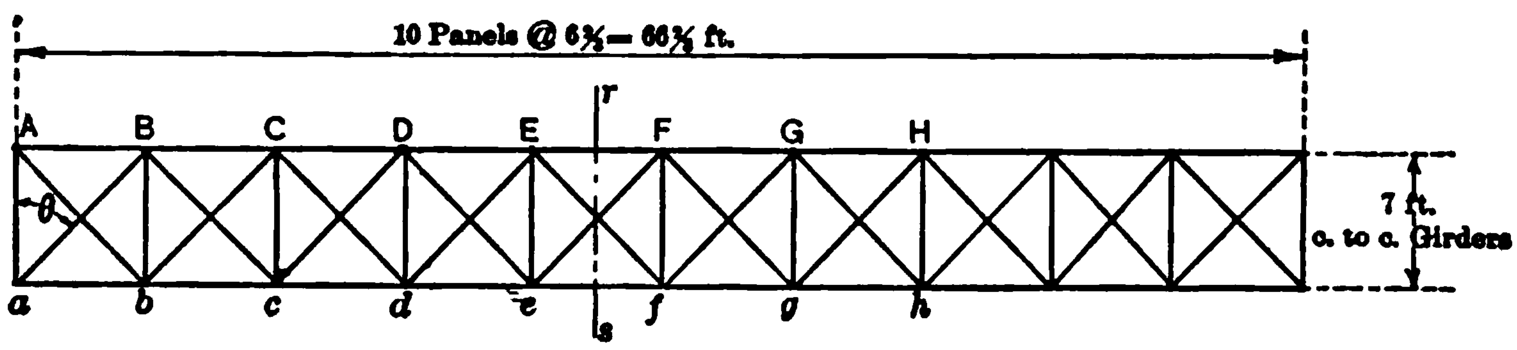


FIG. 45.

cross-frames. A study of the general drawing Plate I will bring out these points, which are also discussed in the next article.

150. The Lateral Bracing.—In order to avoid secondary stresses in the flanges, due to the effect of the lateral system as discussed in Art. 78, a double system of diagonals will be used with a transverse member at each panel point.* The top laterals will be divided into panels of a length such that the inclination of the diagonal members will be approximately 45°. For this girder it will be found that 10 equal panels of about 6 ⅔ ft. each can be used (Fig. 45). A cross-strut will be placed at each panel point and two diagonal members will be placed in each panel. The maximum distance between these cross-struts, or the maximum panel length, is subject to the conditions of Art. 30, Specifications.

*Since plate girders are comparatively free from other secondary stresses the effect of the laterals discussed in Art. 78 is not of great importance in this case. However, the single Warren system of lateral bracing is, in general, objectionable and may as well be avoided.

(30) (in part) The stress per square inch in the compression flange of a girder shall not exceed $16{,}000 - 200\frac{l}{b}$, when the flange consists of angles only or if cover consists of flat plates, where l = unsupported distance and b = width of flange.

As this unsupported distance is the panel length of the top lateral system, we must see that safe limits are not exceeded by our adopted value. From Art. 143 the flange stress on the compression flange is 58,243,200 / 80.72 = 721,500 lbs. The gross area of the compression flange is 54.21 sq. ins. (1/6 web included) and the flange stress per square inch is 721,500 / 54.21 = 13,300 lbs. Then with b, the width of flange, equal to 14 ins., the maximum allowable value of l is such that $16{,}000 - 200\frac{l}{b} = 13{,}300$, or $l = 189$ ins. or 15.8 ft. The panel length of 6 2/3 ft. is then well within required limits. Cross-frames will be located at the end of each second panel, four cross-frames being used. No bottom laterals will be used. The lower member of each cross-frame will provide the necessary connection between the bottom flanges of the girders.

The above arrangement of the laterals will determine the position of the stiffeners. A pair of stiffeners must be placed at each cross-frame in order to provide a means of fastening the frame to the girder. The position of the other stiffeners will be determined with respect to the cross-frame stiffeners. The distance from the ends of the girders to the first cross-frame will be divided into four equal parts, giving a stiffener spacing of 3 ft. 4 1/4 ins., as shown on the general drawing. Between the other cross-frames the distance will be divided into three equal parts, giving stiffener spacings of 4 ft. 6 ins. As all of these distances are well within the limiting values calculated in the preceding article, this spacing will be adopted as final.

The lateral or wind force to be provided for is specified in Art. 10, Specifications.

(10) All spans shall be designed for a lateral force on the loaded chord of 200 lbs. per linear ft. plus 10 per cent of the specified train load on one track, and 200 lbs. per linear ft. on the unloaded chord; these forces being considered as moving.

Other details of lateral bracing are governed by the following articles from the Specifications:

(70) Lateral, longitudinal and transverse bracing in all structures shall be composed of rigid members.

(74) The minimum sized angle to be used in lateral bracing shall be $3\frac{1}{2} \times 3 \times \frac{3}{8}$ in. Not less than three rivets through the end of angles shall be used at the connection.

(75) Lateral bracing shall be far enough below the flange to clear the ties.

Since a bottom lateral system is not provided in this girder, the top lateral bracing must take care of the total load specified in Art. 10, Specifications, for both the loaded and unloaded chords. It will be assumed that the cross frames take the lateral forces on the unloaded chord and distribute them to the top lateral system as a uniformly distributed load. The total lateral force under these conditions for Cooper's E–60 loading is $400 + \frac{1}{10} \times 6{,}000 = 1{,}000$ lbs. per linear foot, which is to be considered as a moving uniform load.

From the general drawing of Plate I, the panels of the top lateral system are seen to be approximately $6\frac{2}{3}$ ft. long. The wind panel load is then $1{,}000 \times 6\frac{2}{3} = 6{,}670$ lbs.

Using the conventional method of calculation for uniform loads, as outlined in Chapter IV, Part I, the shears and stresses in the various panels are as follows, all values being in thousands of pounds:

Panel	*ab*	*bc*	*cd*	*de*	*ef*
Shear	30.0	24.0	18.7	14.0	10.0
Stress	20.3	16.2	12.6	9.5	6.7

In determining the stresses given in this table, the shear in each panel is assumed to be taken equally by the two members, one in tension, the other in compression. From the general drawing the value of sec θ is found to be 1.35.

Since the laterals in each panel are rigidly fastened at the ends and at the centre points, the unsupported length can be taken as one-half the total length, or 4.25 ft. From Art. 20, Specifications:

(20) The lengths of main compression members shall not exceed 100 times their least radius of gyration, and those for wind and sway bracing 120 times their least radius of gyration.

The least permissible value of r is then $\frac{4.25 \times 12}{120} = 0.425$ in. From the handbooks we find that this value is furnished by a $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. angle. Angles with $2\frac{1}{2} \times 2\frac{1}{2}$ and 3×3-in. legs also answer the requirement for least r, but Art. 74, Specifications, given above, limits us to $3\frac{1}{2} \times 3$-in. angles as the minimum.

From the column formula of Art. 16, Specifications, the allowable stress for the assumed $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. angle is $16{,}000 - 70\frac{l}{r} = 16{,}000 - 70\frac{4.25 \times 12}{0.62} = 10{,}250$ lbs. per sq. in. The area required for the diagonals in the end panel, where the stress is greatest, is $20{,}300 / 10{,}250 = 1.98$ sq. in. The area provided by a $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. angle is 2.30 sq. in. As these diagonals must carry tension also, the net area must be investigated. Area required in tension is $20{,}300 / 16{,}000 = 1.27$ sq. in. The net area provided, deducting one rivet hole, is $2.30 - 0.38 = 1.92$ sq. ins. As a $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. angle is ample, it will be used throughout. The stresses in the other panels are smaller than those in the end panel, but the Specifications will not allow the use of a smaller angle.

The number of rivets required in each diagonal is determined by their value in single shear at 10,000 lbs. per sq. in. (Art. 18, Specifications) as the rivets in the laterals are usually field driven. The value of a $\frac{7}{8}$-in. rivet in single shear at 10,000 lbs. per sq. in. is 6,010 lbs. The number of rivets required in a diagonal in the end panel is $20{,}300 / 6{,}010 = 4$ rivets. The same number of rivets will be used in all diagonal members.

The stresses in the cross struts are each equal to one-half of a wind-panel load of 6,670 lbs. The struts will be made of the same size throughout, single $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles being used.

151. Cross Frames.—The conditions governing the design of cross-frames are the same as for the laterals, with the following addition:

SPECIFICATIONS.—(73) Deck spans shall have transverse bracing at each end proportioned to carry the lateral load to the support.

Each end cross frame carries one-half the total lateral load to the abutments. The load to be taken care of is then $1{,}000 \times 34 =$

34,000 lbs. Considering this load as carried equally by each diagonal, the stress is 17,000 × sec θ = 23,600 lbs. tension or compression. A $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$-in. angle is sufficiently large. In most cases these end frames are made somewhat larger than the calculations call for, as it is desirable that the girder be very rigidly braced at the ends. All members of the end cross frame will be made of $5 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles, as shown on the general drawing, Plate I.

The stresses in the members of the intermediate cross frames are nominal and the minimum angles, $3\frac{1}{2} \times 3 \times \frac{3}{8}$ in., will be used throughout.

152. Effect of Lateral Forces on Flange Stresses.—As the top flanges of the girders act as chord members for the lateral system, they will receive considerable stress due to lateral forces. This stress will consist of two parts; one part due to a lateral force of 1,000 lbs. per ft., as in Art. 151, the other part due to the overturning effect of the lateral load.

The maximum chord stress for the top lateral system, due to the lateral load of 1,000 lbs. per ft., will occur in members EF and ef of Fig. 45. Cutting a section r-s and taking moments, using panel loads of 6,670 lbs. at each point, as determined in Art. 150, we find by the methods of Art. 183, Chapter VI, Part I,—

Stress in

$$ef = \left[\, 30{,}000 \times \frac{9}{2} - \frac{6{,}670}{2}\left(1 + 3 + 5 + 7\right)\right]\frac{6.67}{7} = 77{,}800 \text{ lbs.}$$

The flange stress due to overturning is calculated by the methods given in Art. 190, Chapter VI, Part I. The lateral force causing overturning is 10 per cent of the live train load (Art. 10, Specifications), or 600 lbs. per ft., considered as applied 7 ft. above the rail (Art. 11, Specifications). As the rail is about 1 ft. above the plane of the laterals, and the girders are 7 ft. apart, the overturning load is $600 \times 8/7 =$ 685 lbs. per ft. uniform load. The bending moment at the centre of the girders is then $\dfrac{685 \times 68^2}{8} = 396{,}000$ ft.-lbs. and the resulting flange stress is $\dfrac{396{,}000 \times 12}{80.72} = 58{,}900$ lbs. The total flange stress for the windward girder due to lateral truss effect and overturning is

77,800 − 58,000 = 19,800 lbs. compression. From Art. 143, the flange stress due to the dead, live, and impact loads is 721,500 lbs. The stresses due to lateral loads are then $\frac{19{,}800}{721{,}500} = 2.74$ per cent of those due to vertical loading.

From Art. 25, Specifications:

(25) For stresses produced by longitudinal and lateral or wind forces combined with those from live and dead loads and centrifugal force, the unit stress may be increased 25 per cent over those given above (Arts. 15 and 16, Specifications); but the section shall not be less than required for live and dead loads and centrifugal force.

This specification allows lateral chord stresses less than 25 per cent of the stress due to vertical load stresses to be neglected. As the lateral stresses for this girder are only 2.74 per cent of those for vertical loading, the flange section designed in Art. 143 need not be increased.

153. The Web Splice.—The position of the web splice will depend upon the arrangement of stiffeners and upon the manufactured lengths of web plates of the size desired. From the handbooks (Cambria 1912 edition) we find that an 80 × 7/16-in. plate can be obtained in lengths up to 500 ins. (41 ft. 8 ins.). With plates of this length it is possible to locate the web splice at the centre of this girder. Web splices should, if possible, be located at a point where excess flange area is present. This excess area can then be depended upon to furnish all or a part of the splice necessary for that part of the web covered by the flange angles. With these conditions in view, we shall divide the web plate into three parts by locating a splice under the third stiffener each side of the centre of the girder. The three parts of the web are thus nearly equal in length, and are of a size that can readily be handled in the shop during fabrication.

The splice will be designed on the assumption that the full resisting moment of the web plate in bending is to be developed and the maximum vertical shear on the section is to be provided for. A splice of the form shown in Fig. 46 will be used.

The maximum vertical shear at the splice is found to be 154,300 lbs. The maximum fibre stress on the net flange section is to be

taken as 16,000 lbs. per sq. in., which is a necessary condition so that the full resisting moment of the web be developed. To find the corresponding fibre stress on the gross flange section the maximum fibre stress must be reduced in proportion of net to gross flange area.

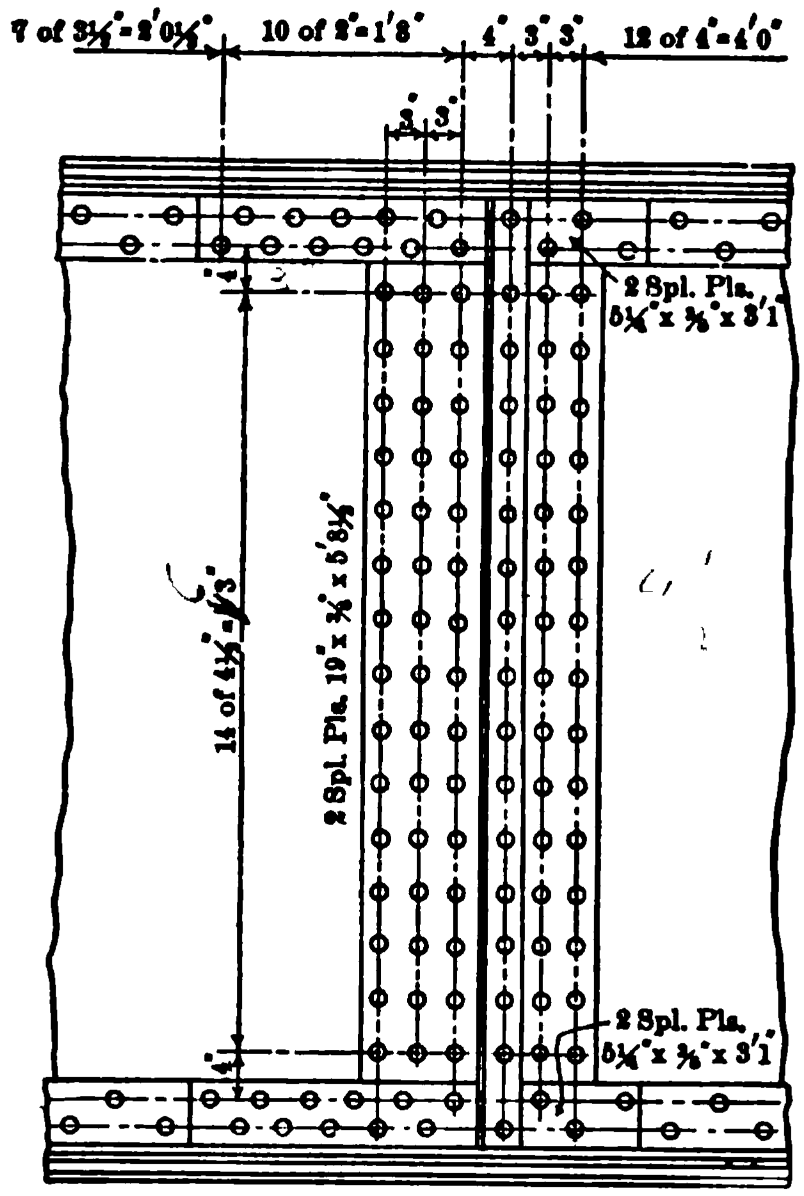


FIG. 46.

At the splice, the flange section consists of two $6 \times 6 \times \frac{3}{4}$-in. angles and three $14 \times \frac{3}{4}$-in. cover plates.

The areas are:	Gross Area, $\frac{1}{6}$ of web included	Net Area, $\frac{1}{8}$ of web included
2 Ls $6 \times 6 \times \frac{3}{4}$ in.	16.88	13.88
3 Plates $14 \times \frac{3}{4}$ in.	31.50	27.00
Web area	5.83	4.38
	54.21	45.26

Then we have $f = 16{,}000 \times \dfrac{45.26}{54.21} = 13{,}350$ lbs. per sq. in. From eq. (47), Art. 123, with $t = \frac{7}{16}$ in., $h = 80$ ins., the depth of the web plate; $r = 9{,}190$ lbs., the bearing value of a rivet on a $\frac{7}{16}$-in. plate;

$V = 154{,}300$ lbs.; and $f = 13{,}350$ lbs. per sq. in., we have

$$p = \frac{9.19}{\left[\left(13.35 \times 7/16\right)^2 + \left(\frac{154.3}{80}\right)^2\right]^{\frac{1}{2}}} = 1.5 \text{ in.}$$

Using three rows of rivets, or a pitch of $4\frac{1}{2}$ ins., the proportion of web area effective for moment is $\frac{1}{6}\,\frac{4.5 - 1}{4.5} = 0.1295$, or 13 per cent. If two rows of rivets are used, or 3-in. pitch, the above percentage becomes 11.1 per cent. Therefore in order to provide $\frac{1}{8}$ of the web effective for bending, the $4\frac{1}{2}$-in. pitch must be used. The thickness of the two splice plates must be equal to the thickness of the web plate. Two $\frac{3}{8}$-in. plates will be used, as this is the minimum thickness allowable.

The portion of the web under the flange angles will be spliced by side plates placed on the vertical legs of the angles. The area of web plate under the angles is $6 \times 7/16 = 2.62$ sq. ins. The fibre stress on the gross flange area was found to be 13,350 lbs. per sq. in. At the centre of the web plate under the flange angles the fibre stress will be $13{,}350 \times \frac{74.5}{80.72} = 12{,}300$ lbs. per sq. in. The stress on the web under the flange angles is then $12{,}300 \times 2.62 = 32{,}200$ lbs. Side plates $5\frac{1}{4} \times \frac{3}{8}$ in. placed on each angle will provide sufficient area for this stress. The rivets on the side of the splice toward the centre of the girder are in double shear, and those on the abutment side are in bearing on the web, as shown by the discussion in Art. 123. The double shearing value of a $\frac{7}{8}$-in. shop-driven rivet is 14,440 lbs. and the bearing on a $7/16$-in. web is 9,190 lbs. Since this is an indirect splice, it is subject to the conditions of Art. 57, Specifications.

(57) Where splice plates are not in direct contact with the parts they connect, rivets should be used on each side of the joint in excess of the number theoretically required to the extent of one-third of the number for each intervening plate.

Therefore, $1\frac{1}{3} \times \frac{32{,}200}{14{,}440} = 3$ rivets are required on the side of the splice toward the centre of the girder, and $1\frac{1}{3} \times \frac{32{,}200}{9{,}190} = 5$ riv-

ets are required on the abutment side. From the discussion given in Art. 123 we see that no additional rivets need be placed in position on the side of the splice toward the centre of the girder. On the side of the splice toward the abutment the rivets receive double duty, so that 5 rivets in bearing on the web plate must be provided in addition to those called for by the calculations given in Art. 146. This can be done by shortening the required rivet pitch calculated in Art. 146, so as to take in the additional rivets. If p = calculated or required pitch at the splice; q = revised or shortened pitch, n = number of additional rivets required; and y = number of spaces of length p which must be shortened to q in order to provide n additional rivets, we have

$$y = \frac{n\,q}{p - q}$$

In this case, we find from the rivet spacing curve of Fig. 43 that $p = 4$ ins. Suppose this to be reduced one-half, or $q = 2$ ins. With $n = 5$, the number of additional rivets as calculated above, we have $y = \frac{5 \times 2}{4 - 2} = 5$ spaces. That is, for 5 spaces, or 20 ins. to the left of the splice, the rivet pitch is to be shortened to 2 ins. The details are as shown in Fig. 46 and on the general drawing.

The splice designed above will develop the full bending strength of the plate. The general drawing Plate I shows that this splice is located a short distance to the right of the end of a cover plate. We then have some excess flange area at the splice, and therefore the fibre stress f of eq. (47) is less than for a fully stressed flange, as assumed in the above design.

A splice for the web will now be designed which will take into account the actual stress conditions. The maximum moment at the splice is found to be 51,930,000 in.-lbs. and the simultaneous shear is 103,700 lbs. Then

$$f = \frac{51{,}930{,}000}{80.72 \times 45.26} \times \frac{45.26}{54.21} = 11{,}860 \text{ lbs. per sq. in.}$$

and

$$p = \frac{9.19}{\left[(11.86 \times 7/16)^2 + \left(\frac{103.7}{80}\right)^2\right]^{\frac{1}{2}}} = 1.72 \text{ ins.}$$

Using two rows of rivets at $3\frac{1}{2}$-in. pitch, or three rows at 5 ins., will develop 11.9 and 13.3 per cent respectively of the web for bending.

We must also consider the case of maximum shear at the splice together with the simultaneous moment. The maximum shear is found to be 154,300 lbs. and the simultaneous moment is 49,190,000 in.-lbs. The value of f is found to be 11,230 lbs. per sq. in. Then $p = 1.73$ ins., again requiring either two rows at $3\frac{1}{2}$ ins. or three rows at 5 ins.

The area of the web under the flange angles is $6 \times \frac{7}{16} = 2.62$ sq. ins. as before, and the stress to be carried when the moment is a maximum is $2.62 \times 11,860 \times 74.5/80.72 = 28,700$ lbs. It will not be necessary to use side plates on the vertical legs of the flange angles, provided this extra stress can be carried without exceeding the allowable stress on the gross flange area, which has been calculated above as 13,350 lbs. per sq. in. The stress of 28,700 lbs. can be assumed as uniformly distributed over the angles and cover plates, whose gross area at the splice is 48.38 sq. ins., giving a unit stress of $\dfrac{28,700}{48.38} = 595$ lbs. per sq. in. The total fibre stress is $11,860 + 595 = 12,455$ lbs. per sq. in. Therefore side plates need not be used. As the stress of 28,700 lbs. is in addition to the required duty of the flanges, additional rivets must be placed in position, as in the splice previously designed. No additional rivets are required on the side of the splice toward the centre of the girder. On the abutment side the rivets are in bearing on the web plate and $\dfrac{28,700}{9,190} = 4$ additional rivets are required. These rivets can be provided by shortening the required spacing as before.

If, in any case, the above excess load cannot be entirely carried by the flanges, splice plates must be provided which will take care of the difference. Thus in the above case the flanges can take care of an excess load of $(13,350 - 11,860)\ 48.38 = 72,000$ lbs. Any load greater than this must be carried by splice plates designed as before.

154. Design of End Bearings.—As stated in Art. 137 the type of end bearing shown in Fig. 30 (*c*), Art. 133, will be adopted for this girder. This type of bearing consists of a cast base bearing on a sole plate riveted to the lower flange angles. At one end of the girder

slotted holes are provided which allow for movement due to temperature changes.

SPECIFICATIONS.—(59) Provision for expansion to the extent of ⅛ in for each 10 ft. shall be made for all bridge structures.

Since the span under consideration is 68 ft. centre to centre of bearings, the provision for expansion is about ⅞ in. The girder is connected to the cast shoe by 1¼-in. bolts. At the fixed end a 1⅜-in. hole is provided, and at the expansion end a 1⅜ × 2¼-in. slotted hole allows the required expansion to take place.

The end shoe is usually made of cast iron or cast steel. In practice the bases of shoes in girders of the size under consideration are made about 6 ins. larger all around than the size of top required for

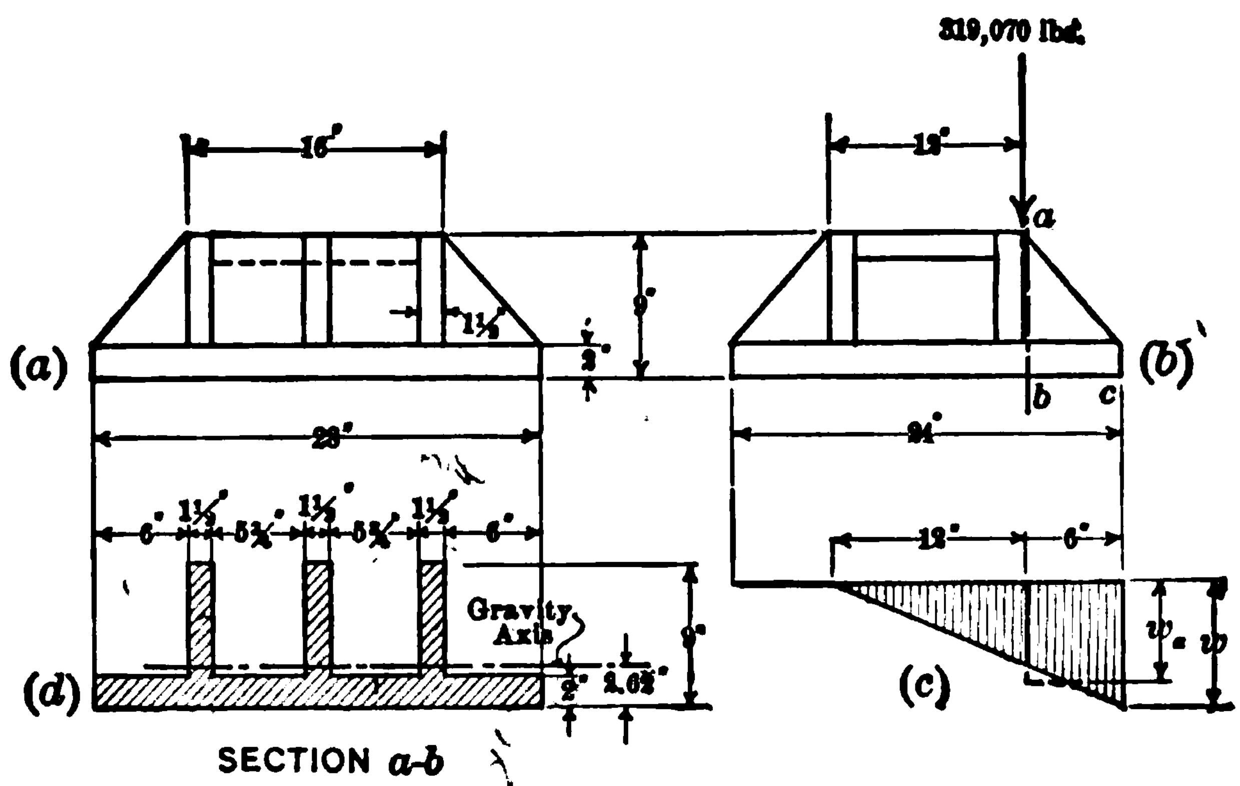


FIG. 47.

convenient connection to the girder. In this case a top 12 ins. long and 16 ins. wide is required. The base will then be made 24 ins. long and 28 ins. wide, giving a base area of 672 sq. ins. A height of 9 ins. will be used.

SPECIFICATIONS.—(19) Bearing on masonry, 600 lbs. per sq. in.

From Art. 140 the end reaction is 319,070 lbs. The base area required is 319,070 / 600 = 532 sq. ins. A base of the size given above furnishes considerable excess area. Since this type of base is rigidly attached to the girder, any deflection of the girder tends to

shift the pressure away from the centre of the shoe. This tends to increase the bearing pressure near the edges of the base. For this reason excess area is desirable, and the assumed base will be used.

Art. 133 outlines briefly the methods to be used in the design of cast bases. Fig. 47 (*a*) and (*b*) shows the general dimensions of the assumed base. In Fig. (*b*) the entire girder reaction is shown as applied at the inside edge of the top of the shoe. Assuming no tension to exist between the base and the masonry, the variation in bearing pressure is shown by Fig. (*c*). The centre of gravity of the pressure triangle is located directly under the applied load. On this assumption, the bearing at the edge of the base can be determined by placing the area of the pressure triangle equal to the pressure per inch on the width of the shoe, or $\frac{w}{2} \times 18 = 319{,}070 / 28$, from which $w = 1{,}266$ lbs. per sq. in.

The bending moment to be carried on section *a b* can be taken as the moment about point *b* of a load due to the average pressure on cantilever *b c*. This average pressure is $w_a = {}^{15}/_{18}\ w = 1{,}056$ lbs. per sq. in. The total load is $1{,}056 \times 6 \times 28 = 177{,}200$ lbs. and the moment on section *a b* is $177{,}200 \times 3 = 531{,}600$ in.-lbs. Fig. (*d*) shows the dimensions of section *a b* of Fig. (*b*). The moment of inertia of this section about the gravity axis is found to be 555.2 ins.^{4} The extreme fibre at *b*, the tension side, has a fibre stress of $531{,}600 \times 2.62 / 555.2 = 2{,}500$ lbs. per sq. in. Assuming the material to be cast iron, the fibre stress is within allowable limits, which can be taken at from 2,500 to 3,000 lbs. per sq. in.

155. Estimated Weight.—The dead-load moments and shears given in Art. 140 were calculated from a dead weight of girder estimated by the general formula of Art. 139. An examination of the moment and shear tables given in Art. 140 shows that the dead-load values are a relatively small percentage of the totals. A considerable error can thus be made in the estimated weight without any danger to the safety of the structure. Therefore, it is usually not considered necessary to make any further estimate of weight and the preliminary value is taken as final. For very long girders the dead weight may become much larger than in the above design and it may

be necessary to check up the assumed weight. The error in the assumed weight allowed by some designers is one per cent of the total dead plus live load. In the girder considered in this chapter the allowable error by this rule will be 0.01 (1,500 + 6,000) = 75 lbs. per ft. of girder.

In estimating the cost of a structure, a more precise determination of weight is required than can be obtained by the use of a general formula. After the shop drawings have been made, the weight of the span can be determined by means of information given in the rolling-mill handbooks.

The following tabulation gives the weight of the girder designed

ESTIMATED WEIGHT
70-Foot Deck Plate Girder Span

Item	No. of Pieces				Total Weight, Pounds
Flange angles....	8	6 × 6 × ¾ × 70′ 0″	28.7	560′ 0″	16,072.0
Web plates	4	80 × 7/16 × 23′ 6¾″	119.0	94′ 3″	16,639.8
" "	2	80 × 7/16 × 22′ 9½″	119.0	45′ 7″	
Cover plates......	2	14 × ¾ × 70′ 0″	35.7	140′ 0″	
" "	2	14 × ¾ × 56′ 1″	35.7	112′ 2″	20,569.3
" "	4	14 × ¾ × 48′ 0″	35.7	192′ 0″	
" "	4	14 × ¾ × 33′ 0″	35.7	132′ 0″	
End stiffener angles	16	5 × 3½ × ¾ × 6′ 7″	19.8	105′ 4″	2,085.8
End stiffener fills.	8	10 × ¾ × 5′ 8″	25.5	45′ 4″	1,156.2
Int. stiffener angles	64	5 × 3½ × ⅜ × 6′ 7″	10.4	421′ 4″	4,382.0
Int. stiffener fills...	64	3½ × ¾ × 5′ 8″	8.93	362′ 8″	3,238.6
Splice plates	8	19 × ⅜ × 5′ 8½″	24.22	45′ 8″	1,105.8
Splice plates	16	5¼ × ⅜ × 3′ 1″	6.69	49′ 4″	330.0
Sole plates........	4	14 × ¾ × 1′ 6″	35.7	6′ 0″	214.2
Lateral angles.....	10	3½ × 3 × ⅜ × 8′ 9″	7.9	87′ 6″	691.3
" "	10	3½ × 3 × ⅜ × 4′ 2″	7.9	41′ 8″	329.2
" "	10	3½ × 3 × ⅜ × 4′ 2½″	7 9	42′ 1″	332.4
" "	5	3½ × 3½ × ⅜ × 6′ 6½″	8.5	32′ 8½″	278.0
Cross frames......	8	3½ × 3½ × ⅜ × 6′ 6½″	8.5	52′ 4″	444.8
" "	8	3½ × 3 × ⅜ × 7′ 10¾″	7.9	63′ 2″	499.0
" "	4	5 × 3½ × ⅜ × 6′ 6½″	10.4	26′ 2″	272.2
" "	4	5 × 3½ × ⅜ × 7′ 6⅝″	10.4	30′ 3½″	315.0
Gusset plates......	16	13 × ⅜ × 1′ 1¾″	....	....	303.9
" "	4	8 × ⅜ × 0′ 8″	....	....	27.2
" "	8	14 × ⅜ × 1′ 3½″	....	....	184.6
" "	2	8½ × ⅜ × 0′ 9″	....	....	16.3
Lateral plates.....	4	13½ × ⅜ × 1′ 6¼″	....	....	97.9
" "	18	12 × ⅜ × 3′ 0″	....	....	768.8
" "	10	9 × ⅜ × 2′ 4½″	....	....	199.4
					70,553.7
7,856 Rivet heads @ 0.185 lbs. per head...........................					1,453.4
Total..					72,007.1

in this chapter. The weights of the various sections are as given in the Cambria Handbook, 1912 Edition.

From the estimate of weight, we find that the weight of the span is 72,007.1 / 70 = 1,030 lbs. per ft. The weight estimated by the formulas of Art. 139 was 1,040 lbs. per ft. The error in the assumed weight is then only 10 lbs. per ft. of girder, which is well within the limits specified above, and no revision of dead-load stresses is necessary.

Design of a Through Plate Girder

156. Through Plate Girders.—General Data.—Plate II. is a general drawing of a through plate girder of the same over-all dimensions as the deck girder designed in the preceding articles. The span is 70 ft. over all, and the live load is E-60. The clearance diagram will be taken as shown in Fig. 1, Art. 174. This requires a girder spacing of 16 ft. in order to clear the top flanges. A stringer spacing of 7 ft. will be used. As this spacing is the same as for the girders of the deck bridge, the ties will be the same as those designed in Art. 138.

In a structure with floor-beams and stringers an odd number of panels is preferable to an even number. When an odd number of panels is used in a girder bridge, the maximum moment occurs at a point half a panel length from the span centre, and when an even number of panels is used the maximum moment occurs at the girder centre. The maximum moment for the first arrangement is therefore somewhat less than for the second arrangement.

In the girder in question, five 14-ft. panels provide a convenient arrangement of stringers and floor-beams. The end bearings will be set back 6 ins. from the end of the girder, giving a distance centre to centre of bearings of 69 ft.

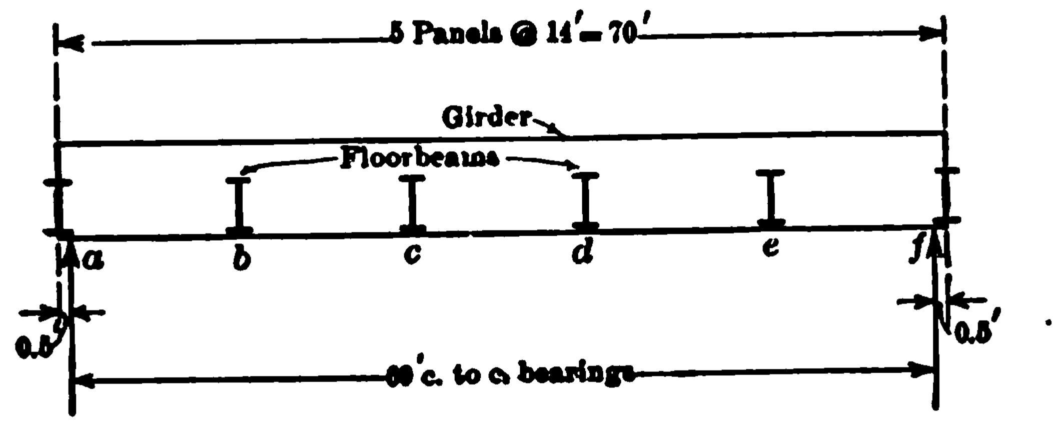
157. Moments and Shears.—From Art. 139, the dead weight of the floor and track is 466 lbs. per ft. of bridge. The weight of steel in the span is given by the formulas of Art. 66, Part I, as $w = 1.1 (14l + 450)$. With $l = 69$ ft., $w = 1554$ lbs. This gives a total dead load of 2020 lbs. per ft. of bridge. A load of 2000 lbs. per ft. was used in the design. Table A gives the resulting dead-load moments and shears.

The live-load moments and shears are calculated as for a truss of the same span by the methods given in Chap. V, Part I. Table A gives the calculated values.

TABLE A

DEAD- AND LIVE-LOAD MOMENTS

Point	*a*	*b*	*c*
Dead-Load Moment	0	378,000	574,000
Live-Load Moment	0	1,600,000	2,390,000
Live-Load Impact	0	1,300,000	1,945,000
Total Foot-Pounds	0	3,278,000	4,909,000
Total Inch-Pounds	0	39,350,000	59,000,000



DEAD- AND LIVE-LOAD SHEARS

Panel	End	*ab*	*bc*	*cd*
Dead-Load Shear	35,000	28,000	14,000	0
Live-Load Shear	165,800	117,000	71,800	34,900
Impact Shear	135,000	96,600	61,600	31,200
Total Shear	335,800	241,600	147,400	66,100

158. Design of Girders.—The methods of design are exactly the same as for the deck girder. From eq. 61 of Art. 136, a 68 × 3/8-in. web plate is large enough to take care of the shear in panel *ab*. Economical depth conditions, as given by eq. 60, Art. 136, requires a 95 × 9/16-in. web. As a compromise an 82 × 7/16-in. web will be adopted.

A trial shows that the flange section can be made the same as for the deck girder. This flange section is given in Art. 143. It will be found that the effective depth is 82.72 ins., flange stress = 714,000 lbs., and area required = 44.60 sq. ins. The assumed flange, plus one-eighth of the web area, provides an effective net flange area of 45.36 sq. ins.

An investigation of the allowable unsupported length of the top flange, subject to the conditions of Art. 30, specifications, must now be made. The gross area of the flanges, plus one-sixth of the web area, is 54.35 sq. ins. This gives a unit flange stress of 714,000/54.35 = 13,120 lbs. per sq. in. Since the cover plates are 14 ins. wide, the allowable unsupported length of flange, as determined from

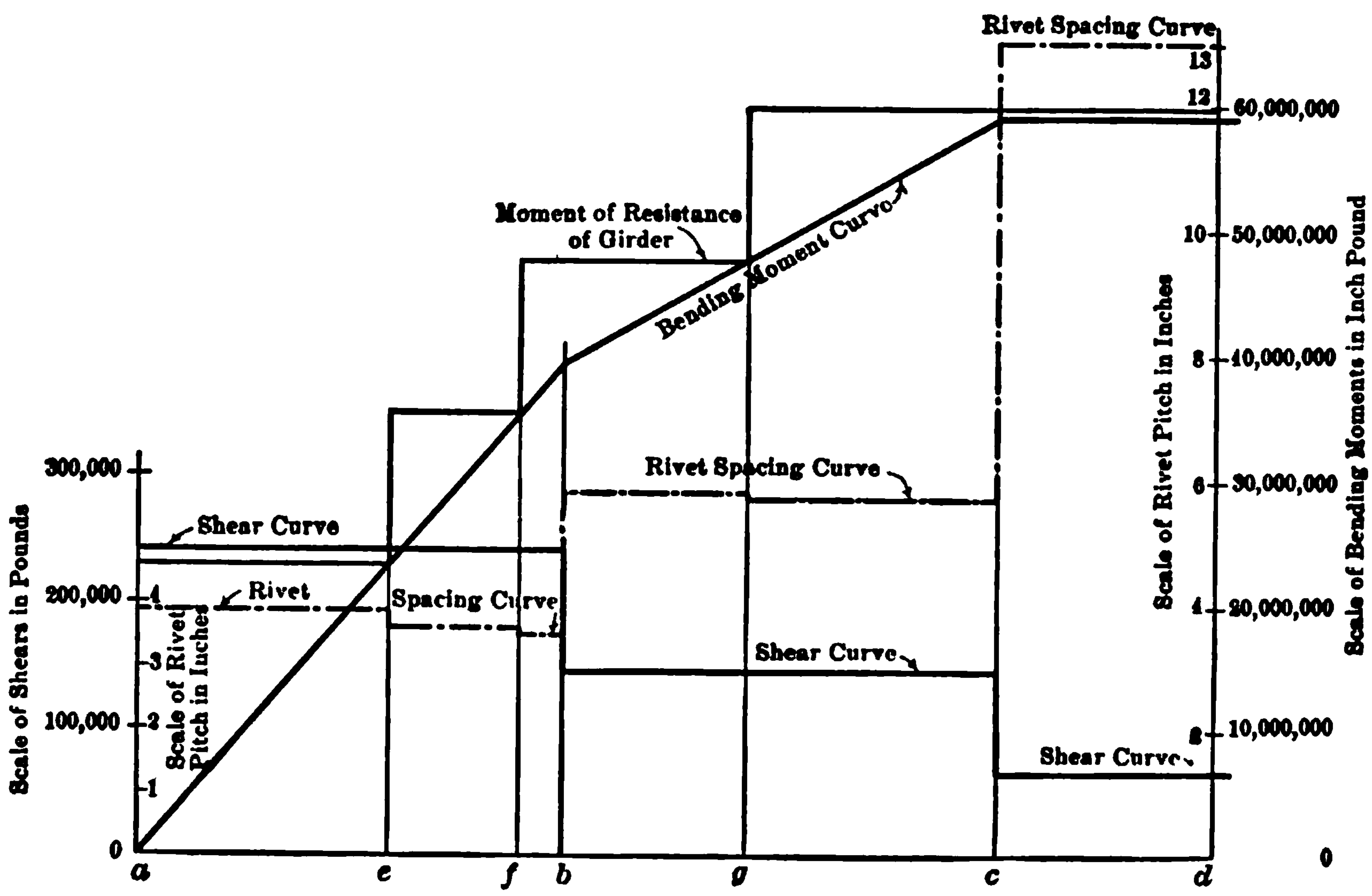


FIG. 48.

the formula $p = 16{,}000 - 200\ 1/b$ is about 16.8 ft. As the panels are 14 ft. long, the proper support is given by the floor beams.

Fig. 48 gives the points of cut-off of cover plates. The calculations are similar to those given in Art. 145. On the same figure are also given the shear and rivet spacing curves. Since no vertical load is carried by the flanges, the rivet spacing is calculated by eq. (26) of Art. 115.

The web splice, stiffener spacing, and lateral bracing are calculated by the same methods as used in the deck girder. All details are as shown on the general drawing.

The end stiffeners consist of a pair of $6 \times 4 \times \frac{3}{4}$ in. angles placed at the end of the girder, and a pair of $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in. angles

placed with their backs a foot from the end of the girder, as shown on the general drawing. Since the outer pair of angles carry the end floor beam, they are made heavier than the inner pair. In designing the end stiffeners two conditions of loading are to be considered. Considered as a column, subject to the conditions of Art. 79, specifications, the stiffeners must take the shear in panel *ab*. Also, the area of the angles must be sufficient to take the maximum end reaction in bearing. The adopted arrangement will be found to provide some excess area.

In order to give a neat appearance to the structure, the top flange is curved at the ends of the girder, and the cover plate next to the angles is brought down over the backs of the outer end stiffeners.

159. Design of the Floor System.—The stringers are simple beams equal in length to a panel, or 14 ft. They carry the weight of the track and floor, the live load, and their own weight. From Art. 139, the floor weighs 466 lbs. per ft. of bridge. The formula $w = 9/8 (12l + 100)$ gives the weight per ft. of the stringers. With $l =$ 14 ft. this formula gives $w = 302$ lbs. The total dead weight is then 768 lbs. per ft. of bridge, or 385 lbs. per ft. per stringer. For this load, the centre moment is 9,400 ft. lbs. and the end shear is 2,700 lbs.

From the table on page 245, Part I, the live-load moment for E-60 loading is found to be 165,000 ft.-lbs. and the end shear is 57,-900 lbs. The impact allowance is to be determined for a loaded length of 14 ft. From these values, the total centre moment is found to be 3,980,000 in. lbs. and the end shear is 115,900 lbs.

A stringer depth equal to about one-seventh of the panel length is usually adopted for short panels. In this case the stringers will be made 24.25 ins. back to back of angles. The web will be placed flush with the backs of the top angles in order to comply with Art. 128, specifications. Using the same working stress as for the deck plate girder, the web area required is 11.6 sq. ins. A $24 \times \frac{1}{2}$-in. web plate furnishes 12 sq. ins. A flange consisting of two $6 \times 6 \times \frac{1}{2}$-in. angles will be assumed. For these angles, the effective depth is 20.89 ins., flange stress 190,500 lbs., net flange area required 11.9 sq. ins. Assuming one-eighth of the web as flange area, the angles must provide

10.4 sq. ins. The assumed angles furnish a net area of 10.5 sq. ins. after deducting one rivet hole from each angle.

The rivet spacing is calculated from eq. 27, Art. 115. As the wheel and floor loads are the same as for the deck girder, the vertical load on the top flange is 1,270 lbs. per in. as given in Art. 146. The rivets are in bearing on a ½-in. web plate and have a value of 10,500 lbs. At the end of the girder the rivet pitch is

$$p = \frac{10.5}{\sqrt{\left(\frac{115.9}{20.89} \times \frac{10.5}{12}\right)^2 + \left(1.27\right)^2}} = 2.10 \text{ in.}$$

In calculating the rivet pitch at the quarter and centre points, the shear can be determined by the approximate rule of Art. 170, Part I.

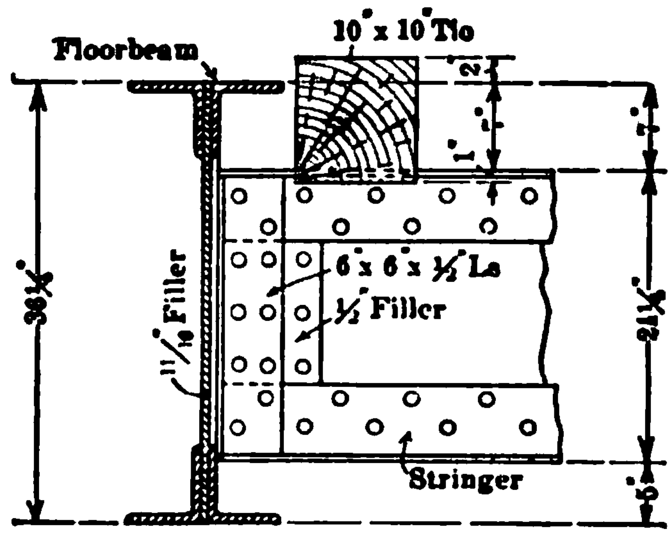


FIG. 49.

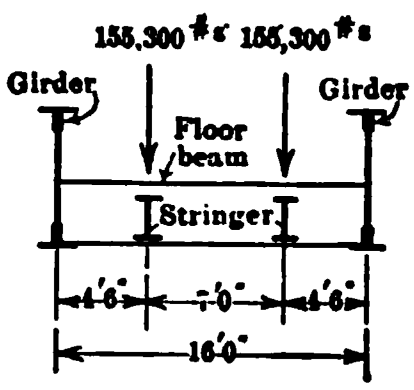


FIG. 50.

With values thus obtained, the rivet pitch at these points is found to be 3.21 and 5.68 ins. respectively.

According to Art. 79, Specifications, the thickness of the web plate is such that no web stiffeners are required.

The end connection between stringer and floor beam will be by means of two 6 × 6 × ½-in. angles. The details are shown in Fig. 49. As the end shear is 115,900 lbs., 9 shop rivets are required in double shear, and 11 in bearing on the web. Fig. 49 shows 10 rivets in double shear in the angles, and 13 rivets in bearing in the angles and filler. Since the stringers are short and well sheltered by the main girders, no stringer lateral bracing will be provided.

The intermediate floor beams will be made 12 ins. deeper than the stringers, or $36\frac{1}{4}$ ins. back to back of angles. Fig. 49 shows the details of the framing between stringer and floor beam. From the table on page 245, Part I, the maximum floor beam concentration is 78,300 lbs., and from the calculations given above, the dead-load reaction for two stringers is found to be 5,400 lbs. Including impact, loaded length two panels, the total load concentrated at the stringer connection is 155,300 lbs. As shown in Fig. 50, the distance from the main girder to the stringer is 54 ins. The bending moment at the stringer is then 8,400,000 in. lbs. Assuming the floor beam to weigh 2,500 lbs. the dead-load end shear and centre moment are 1,250 lbs. and 60,000 in. lbs. respectively. The total end shear is then 156,550 lbs. and the total bending moment is 8,460,000 in. lbs.

A floor-beam section consisting of four $6 \times 6 \times \frac{11}{16}$ in. angles and a $36 \times \frac{1}{2}$ in. web plate will be found to provide the required flange and web areas. As the methods of calculation are exactly the same as for the stringer, the detail work is left for the student.

The rivet pitch in the flange angles is given by eq. 26, Art. 115. As the shear is practically uniform from the girder to the stringer, varying slightly due to the dead load, the rivet pitch can be made constant between these points. The rivet spacing is $16.43/14.18 \times 32.75 \times 10{,}500/156{,}550 = 2.55$ ins. Between the stringers, where the shear is practically zero, the rivet pitch can be made the maximum allowable. Plate II. shows the adopted arrangement.

The number of rivets required in the flange angles between the girder and the stringer may also be determined by calculating the number required to transmit from the angles to the web plate the total stress in the flange angles at the stringer. This stress may be assumed to be the same as that at the centre of the floor beam, where the stress in the angles is $14.18/16.43 \times 8{,}460{,}000/32.75 = 223{,}000$ lbs. The rivets are in bearing on the $\frac{1}{2}$-in. web plate and $223{,}000/10{,}500 = 22$ are required.

The connection between the floor beam and the main girders is made by means of $6 \times 6 \times \frac{1}{2}$ in. angles. To connect these angles to the floor beams, 11 rivets are required in double shear, and 15 in bearing on the web. The details of this connection are similar to those shown in Fig. 49 for the stringer. To connect the floor beam to the

web of the main girder, 156,550/6010 = 26 field rivets in single shear are required. Plate II shows the adopted floor-beam details.

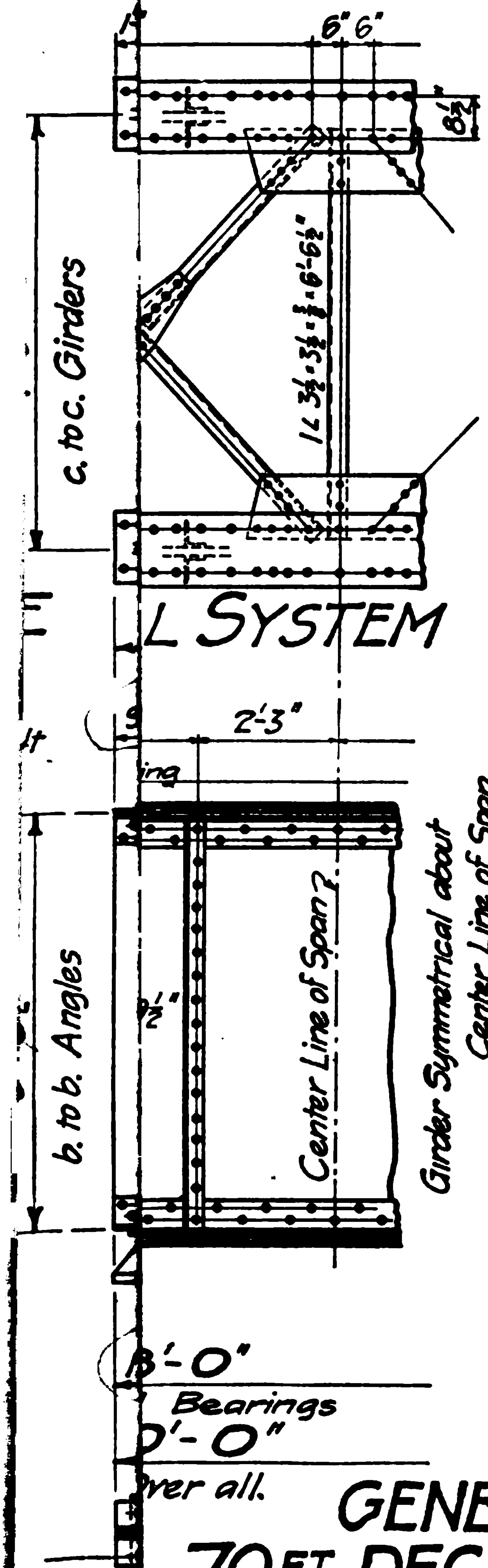
The connection between the stringer and floor-beam must contain sufficient field rivets to take the floor-beam concentration in bearing on the web plate, or the end shear for a stringer in single shear. For the floor-beam concentration, 155,300/8750 = 18 rivets are required, and for the stringer end shear, 115,900/6010 = 20 rivets are required. The adopted arrangement is shown on the general drawing, Plate II.

In order that the floor beam may be made available as a top flange support, a bracket will be riveted to the top of the floor beam and to the web stiffener as shown on Plate II. This bracket is to fit up tight under the top flange, and is to extend along the floor-beam as far as the clearance diagram will allow. An efficient bracket can be made of a $\frac{3}{8}$-in. web plate and $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles.

The end floor beam is designed to carry the load brought to the beam by the stringers in the end panel. A beam section composed of a $36 \times \frac{3}{8}$-in. web plate and $6 \times 4 \times \frac{5}{8}$-in. angles will be found to be sufficient. The 6-in. legs will be placed against the web plate, thus providing a narrow flange in the direction of the secondary bending stresses discussed in Art. 80.

As shown in the general drawing, the end floor beam is attached to the outer pair of end stiffener angles. By this arrangement part of the end reaction is carried to the outer edge of the girder, thus tending to prevent tipping of the shoe due to the deflection of the main girders.

PLATE I



GENERAL NOTES

Specifications; A.R.E.A. 1910
Material; Medium O.H. Steel.
Rivets; ⅞-in. Diam.

GENERAL DRAWING
70 FT DECK PLATE GIRDER
COOPER'S E 60 LOADING

CHAPTER VII

DESIGN OF TRUSS BRIDGES

160. General Principles.—In previous chapters the design of individual members and of riveted joints has been considered, and certain principles and requirements pertaining thereto have been discussed. In the present chapter some of the general requirements will be considered with reference to the arrangement of members and proportioning of parts so as to produce a satisfactory truss.

A railroad bridge truss is subject to severe usage. For ordinary spans the larger part of its load consists, generally, of a rapidly moving train load which is applied under conditions that result in a large dynamic effect, or "impact," as shown in Chapter II. Such effect is not only vertical but horizontal, and results in vertical and horizontal vibrations of the structure and of its members which produce increased stresses and, frequently, wear of parts. The effects of wind pressure and temperature changes must also be provided for; and, finally, deterioration from corrosion is often a very important and troublesome element of the problem.

A truss should be looked upon as an elastic structure, not only with respect to its several members but also with respect to its details in all their parts. Deflection and distortion cannot be prevented, and, in fact, if such action did not exist it would be impossible for a structure of any kind to support a load. It is due to the elasticity of material that it is possible to distribute stress from a joint into a member or from a member into a joint; and a clear understanding of the necessary relation between stress and distortion, and the fact that the distribution of stress is often determined by the relative amounts of possible distortion in the various parts of a member or joint, will be of much assistance in the proportioning of effective details.

While it is necessary that a truss should be elastic and deform under a load, it is important to arrange members and details so as

to insure that the deformations will be a minimum consistent with economy, and also that these deformations will be of an elastic nature, that is to say, such that the material will recover its original form and dimensions after each application of the live load. Elastic deformation is necessary and has no injurious effect; inelastic deformation means over-stressed members or details and permanent injury to the structure.

To secure the desired rigidity of structure and to prevent inelastic deformation or overstrain, certain general principles of design should be followed. The general proportions of the truss, for a simple span bridge, should be about as indicated in Art. 8. Very shallow trusses show large deflections, mainly due to the chord stresses, while very deep or high trusses show large deflections from web stresses and, at the same time, possess a low degree of stability against lateral forces. Fortunately those proportions giving maximum rigidity correspond closely with those for maximum economy.

The spacing of trusses centre to centre must be sufficient to give a lateral truss of reasonable rigidity. For through, single-track spans exceeding about 300 ft. in length, this condition requires a spacing greater than necessary for clearance. (See Art. 8.) Long and slender members should be avoided, even though of sufficient calculated strength, as the action of the live load causes undesirable vibration of such members, resulting in excessive local stresses and, sometimes, wear of parts.

Members should be so connected that their gravity axes meet at a common point. Violation of this principle results in secondary bending moments in the members and increased flexibility of the truss. Where such an arrangement must be deviated from, provision should be made for the resulting additional stresses. Riveted connections should be concentric with the gravity axes of the member connected. (See Art. 91.) Members should be made symmetrical about an axis through the central plane of the truss and also, where possible, about a transverse axis. Symmetry facilitates simplicity of details and concentricity of connections.

Excessive secondary stresses should be avoided by the use of proper proportions and arrangement of members as shown in Chapter IV, Art. 82. Large secondary stresses are likely to result in over-

strained details, loose rivets, and some permanent set. The steel superstructure being flexible and of variable length, due to temperature and stress effects, free movement of one end should be provided for by suitable roller bearings. Where such bearings do not operate freely, or are entirely inoperative, as frequently happens, both the truss and the masonry are subject to large stresses for which they are not designed.

The function of the floor system is quite distinct from that of the truss, and the method of connecting the one to the other, and the effect of the action of each upon the other, require careful consideration. Attention must also be given to proper protection against corrosion. Unless subject to special conditions the chief requirement is to arrange the members and details so that all parts will be accessible for cleaning and painting. For special cases, such as structures crossing other railroads, special methods of protection may be required as, for example, the use of a protective covering of cement or concrete.

Clauses of the Specifications, Appendix A, touching upon general features of design are as follows:

33. Structures shall be so designed that all parts will be accessible for inspection, cleaning, and painting.

34. Pockets or depressions which would hold water shall have drain holes, or be filled with waterproof material.

35. Main members shall be so designed that the neutral axis will be as nearly as practicable in the centre of the section, and the neutral axes of intersecting main members shall meet at a common point.

36. Rigid counters are preferred; and where subject to reversal of stress shall preferably have riveted connections to the chords. Adjustable counters shall have open turnbuckles.

37. The strength of connections shall be sufficient to develop the full strength of the member, even though the computed strength is less, the kind of stress to which the member is subjected being considered.

82. Hip verticals and similar members, and the two end panels of the bottom chords of single-track pin-connected trusses, shall be rigid.

161. Lower Chord Members.—In pin-connected spans eye-bars are generally used for all lower chord members except the end two panels of single-track spans (member *a b c*, of Fig. 1, (*a*) and (*b*)). (See above requirement of the Specifications.) For these two panels stiff members are used to provide for any possible reversal of stress

due to lateral forces and tractive effect, and to give greater rigidity to the structure than the relatively light eye-bars which would otherwise be required. Stiff members in these panels are desirable, also,

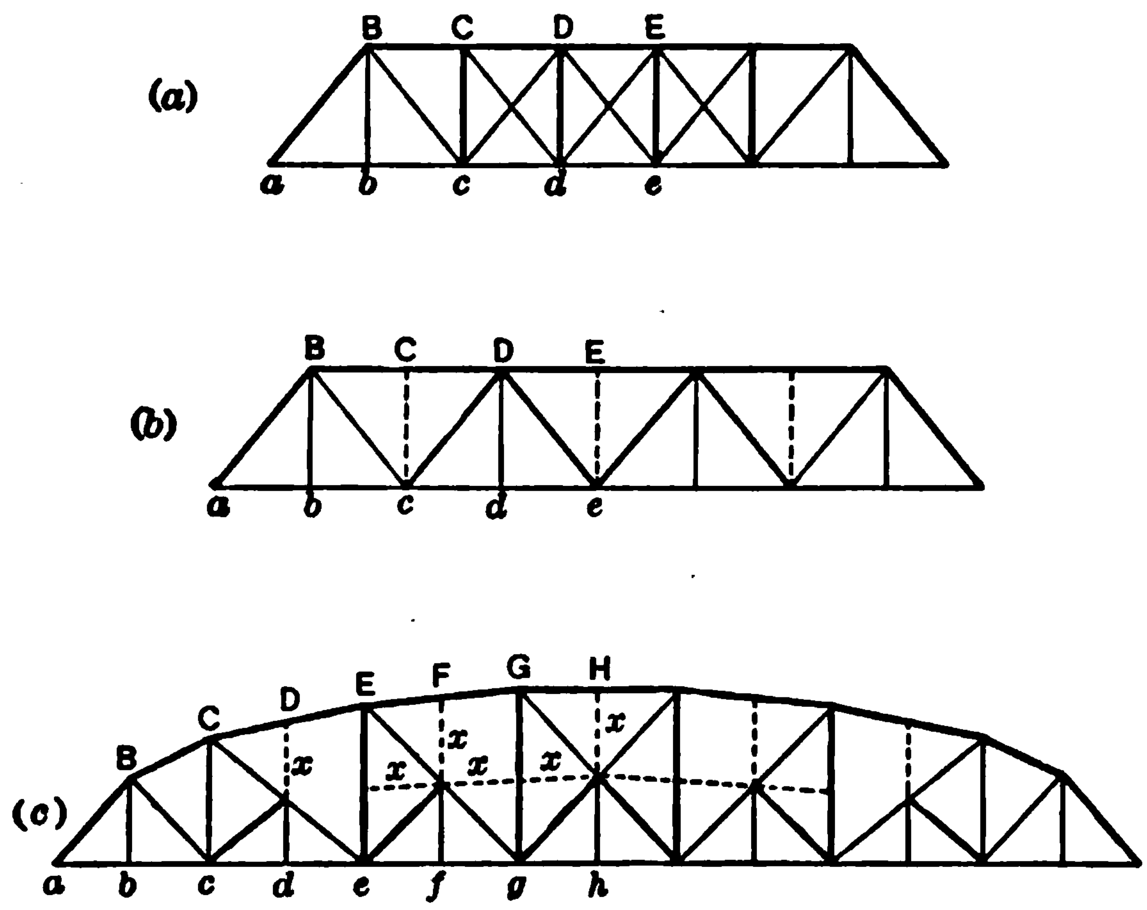


FIG. I.

as offering greater resistance to the action of a derailed car striking the end post, and thus, in some cases, preventing a collapse of the structure.

Stiff end members are conveniently made of rolled or built channels with the flanges turned out, so as to permit the vertical post at the third panel point to pass inside. (See Plate III.) Occasionally, in the case of trusses of only five or six panels the form used in the end panels is used for the entire bottom chord, the necessary additional section for the central panels being furnished by additional side plates. This gives, perhaps, a better appearance than the use of eye-bars for one or two panels only, but if pin joints are used it would seem desirable to secure the advantage of the economy of eye-bars for the members of these panels. From the standpoint of rigidity, also, eye-bars for these panels will be sufficiently large to form satisfactory members. In double-track bridges the effect of lateral stresses upon the lower chord is much less than in single-track structures, so that eye-bars can be used throughout.

The best relative dimensions of eye-bars are determined from

considerations of convenience of packing bars on pins, size of pins required, and ease of manufacture. Thick bars occupy much space and cause large bending moments in the pins, thus requiring large pins. Very thin bars are difficult to manufacture and hard to handle in shipping, and also require large pins in order to furnish sufficient bearing area. These considerations lead to the use of bars ranging in thickness from 1/6 to 1/4 of the width for bars from 5 to 10 ins. wide. The maximum thickness employed is about two inches. A considerable disadvantage of pin-connections and of eye-bars for the lower chord is the difficulty of securing good connections between the lower laterals, floor-beams, and truss members. A riveted lower chord gives much simpler details in this respect. (Compare Figs. 7 and 8.)

In riveted trusses the lower chord is made of a variety of sections. For small spans the sections shown in Fig. 2 (*a*) and (*b*) are suitable and convenient. For longer spans, rolled and built-up channel sections are used, as shown in Fig. 2 (*c*) to (*g*). Turning flanges inward

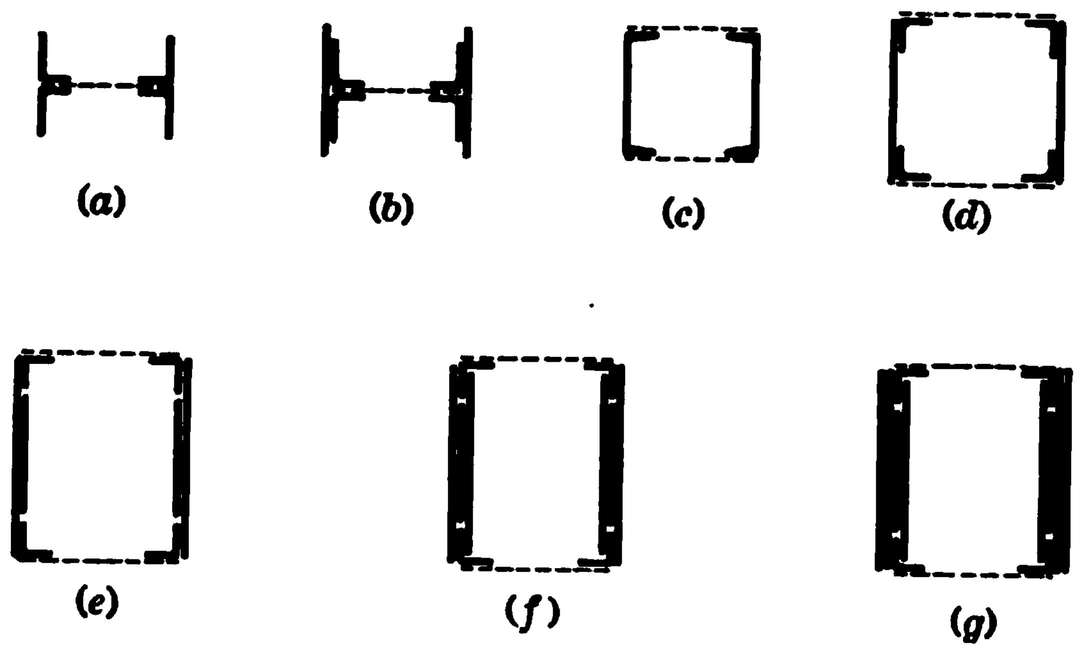


FIG. 2.

gives a more convenient connection for floor-beams and lower laterals than to turn them out, and also requires less material for lacing and tie-plates. In the former case the joints or gusset plates are placed outside and in the latter case inside of the web plates. Increase of section from member to member is provided by means of additional side plates. The distance back to back of angles must be made constant.

Riveted tension members should have the material concentrated mainly in the two web plates and plates connected thereto rather

than in large angles. There is no necessity of spreading out the material so as to secure a large moment of inertia, as is the case in compression members. Heavy lacing connecting the two parts is also unnecessary. For small members, simple tie-plates are sufficient, and for large members lacing of moderate size, the purpose being to keep the member straight during transportation and erection. At the ends of the member tie-plates should be used of a length about equal to the width of the member.

162. Upper Chord Members.—The upper chord members are the largest compression members of the truss and require much care in proportioning and detailing to secure the desired strength with due attention to economy and the relation of other connecting members. This is especially true in the case of pin-connected trusses. The forms of sections commonly used are shown in Fig. 3. Fig. (*a*) may be used for light, highway riveted bridges. The angles have long vertical legs in order to provide convenient connections for the web members, and give to the section a reasonable moment of inertia. Figs. (*b*) to (*f*) are used for railway bridges, the form of section depending largely upon the area required.

Cover plates are almost invariably used on top chord sections. They serve not only to connect the two or more vertical webs much more thoroughly than lacing, but also to prevent corrosion, to a considerable extent, by keeping out water from the joints and other interior parts of the members. In the case of pin joints, cover plates are somewhat objectionable from the fact that their stress must be transmitted to the pin in an indirect manner through other members. This condition leads to the practice of making the cover plates only sufficiently thick to be able to resist local buckling, and then to vary the cross-section of the different segments of the chord by adding area in the form of web or side plates. Area added to the cover plate tends also to raise the centre of gravity of the section, which is undesirable, especially for pin joints.

In pin-connected trusses of moderate span-length the vertical dimensions of the end segment of the top chord are usually fixed by the size of the eye-bar heads which must be placed inside, and the horizontal dimensions by the size of the vertical post at the second top joint. The width must also be such as to make the radius of

gyration about a vertical axis at least as great as about a horizontal axis. Pins should preferably be placed at the gravity axis of the top chord, but in some cases the requirement of space for eye-bar heads makes it impossible to do this without the use of considerable excess material in the member. A small eccentricity of pin is, therefore,

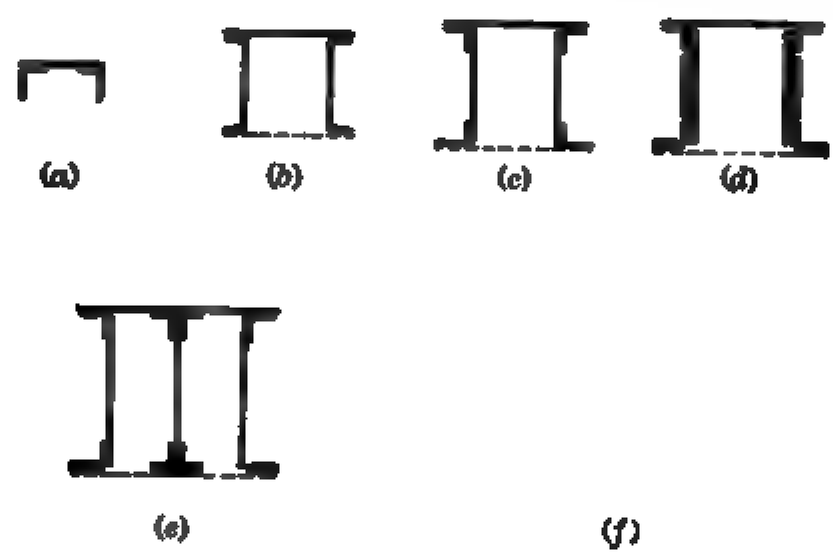


Fig. 3.

often allowed, in which case the resulting bending moments should be investigated as explained in Art. 66. In riveted trusses the gravity lines can readily be made to intersect, except for slight variations in position of the gravity lines in adjoining members of the top chord due to change of cross-section.

The end segment of the chord having been proportioned, the increase in area required in segments toward the centre is made by increasing the thickness of angles and of web plates, and by adding outside side plates. In riveted trusses it is convenient to use the same web thickness throughout in order to avoid fillers at the joints. For large sections, three or four webs are used, as in Figs. 3 (*e*) and (*f*). Where a number of plates are riveted together to give increased section the thickness of the individual plates should be as great as practicable, a few thick plates giving a better distribution of stress than several thin plates. The limit of thickness in ordinary structures is usually taken at ¾ in., as this is about the limiting thickness for which punching of holes is permissible. (See Art. 118, Specifications.) It may be questioned if it is not better practice to use thick plates,

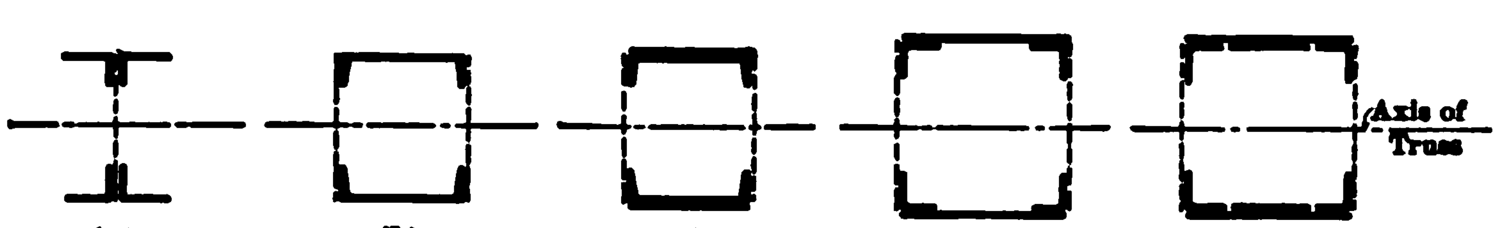
with drilled holes, and so avoid, in many cases, the use of multiple plates and stitch rivets.

Upper chord members which are used also as beams to support ties, or closely spaced floor-beams, must be made relatively deep to secure an economical section. Excessively deep chords, however, should be avoided on account of the resulting increased secondary stresses due to deflection and other secondary effects. In subdivided trusses attention should be given to the support of the top chord so as to avoid excessive secondary stresses. (See Art. 75.)

End posts are generally made of the same form and general dimensions as the top chord section. The stress is about the same as in the end chord segment, and, in the case of a through bridge, the use of a cover plate is even more important on account of the bending moments developed in the portal bracing.

163. Tension Web Members.—In pin-connected trusses all diagonal tension web members are generally made of eye-bars, counters being in the form of adjustable bars. For diagonals carrying heavy stress, such as those near the end of the truss, eye-bars form very satisfactory members, but near the centre where the stresses are small, and especially where the height is great, long eye-bars are likely to be so slender as to be very vibratory. Sometimes built-up members are used in those panels where counters would otherwise be needed, but if such are used they should be riveted to the chords and not pin-ended, as pin-ended members subject to reversing stresses will cause wear on the pins.

In riveted trusses tension diagonals (members *Bc* and *De*, Fig. 1 (*b*)) are conveniently made of the forms shown in Fig. 4, depending on the section required and the connecting details. Hip verticals



(*a*) (*b*) (*c*) (*d*) (*e*)

FIG. 4.

of pin-connected trusses were formerly made of eye-bars connected to a plate to which the floor-beam was riveted. This design has been displaced by rigid verticals, generally of the four-angle type. Fig.

2 (*a*), as giving increased rigidity against vibration, and greater resistance against any upward force from ice or drift-wood. This form is also commonly used for all of the verticals of a truss of the form shown in Fig. 1 (*b*). A vertical plate or diaphragm must be used between the angles for a height at least the full depth of the beam in order to distribute the load. Where the lower chord consists of a deep built-up member the elongation of vertical suspenders is likely to cause heavy bending or secondary stresses in the chord member. These can be reduced by using a liberal cross-section in the suspender, or by making it shorter than the theoretical length to allow for live-load distortion.

164. Compression Web Members.—In the Pratt truss the compression web members are the "posts" *Cc*, *Dd*, etc., Fig. 1 (*a*). These are made of convenient form for attachment of floor-beams and generally of a two-channel section, rolled channels for small spans and built channels for larger spans. (Fig. 5.) In these members the flanges are generally turned inward in order to enable them to enter

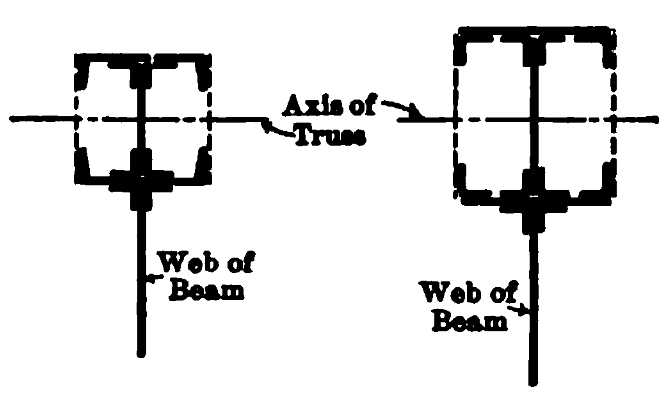


FIG. 5.

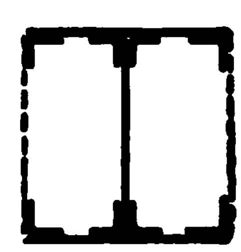


FIG. 6.

the top chord without cutting the flanges. This arrangement also requires less weight of lacing than with flanges turned outward. Where it is necessary to turn flanges outward and then to trim them off at the ends, careful attention must be given to maintain the full strength of the member by suitable reinforcement.

Tie-plates should extend as near to the end of the compression member as possible. Where necessarily placed some distance from the end connection, as often occurs in pin joints, the separate halves of the member, or "forked ends," are subject to some unequal loading and must be carefully reinforced. (See Specifications, Art. 53.) The lateral spacing of the channels must be sufficient to give a radius

of gyration about an axis parallel to the channels at least as great as in the other direction. The spacing should, however, not be excessive, as this will increase the bending stresses due to beam deflection. (See Art. 79.) Web compression members should be made symmetrical in section in both directions.

In the case of large members, lacing should not be entirely depended upon to resist the column shear in order that the two segments may act as a single unit. To insure this assumed action, central webs should be used throughout the entire length of the member, as in Fig. 6. This arrangement is very effective and serves the same purpose as the cover plate of the top chord section. At the ends of the member the stress carried by this web can be transferred to additional side plates, or to the joint gusset plates, in the case of riveted joints.

In riveted trusses, diagonal compression members are generally made of the same form as the vertical posts above described. In small trusses the four-angle form, Fig. 4 (*a*), is sometimes used, but it is not as satisfactory for compression members as the channel section. (See Art. 55.) Small secondary struts, such as those supporting the top chord or vertical post (members, x, x, of Fig. 1 (*c*)), are made of two light channels or four angles laced. Sub-struts of Fig. 1 (*c*) are generally made of two channels.

165. Lateral Bracing.—The lateral bracing of a bridge truss is designed not only to resist certain definite lateral forces due to wind pressure, but to secure reasonable lateral rigidity of the structure against shocks and impact from rapidly moving trains. Rigidity is promoted by wide truss spacing, by the use of members of relatively large cross-section so that the unit stresses will be small, by stiff forms of members, and by well-designed joint details. Riveted or built-up types of members are preferable to rods and bars, partly on account of the lesser flexibility of the member itself, and partly because of the reduced stress on gross section. Stiff forms are now generally used for all laterals.

For lower laterals of through spans and upper laterals of deck spans the floor-beams form the struts and the diagonals are generally made of two angles placed back to back. The two diagonals are connected at their intersection and supported by the stringers. End

connections should be as centrally arranged as possible; this is very important at the end joints. In pin-connected spans the joint plates

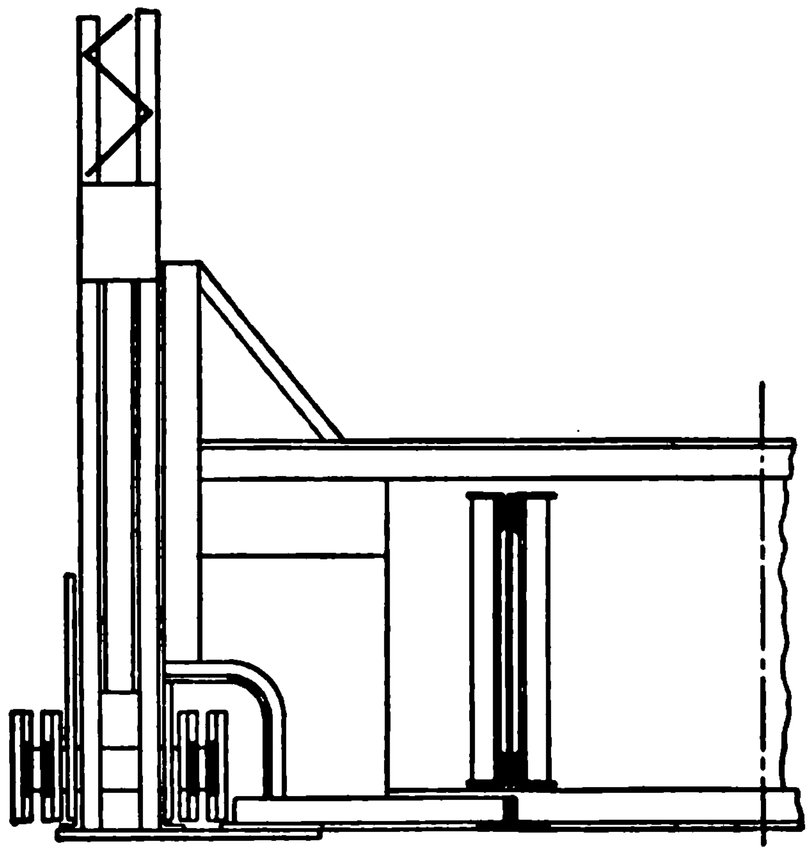


FIG. 7.

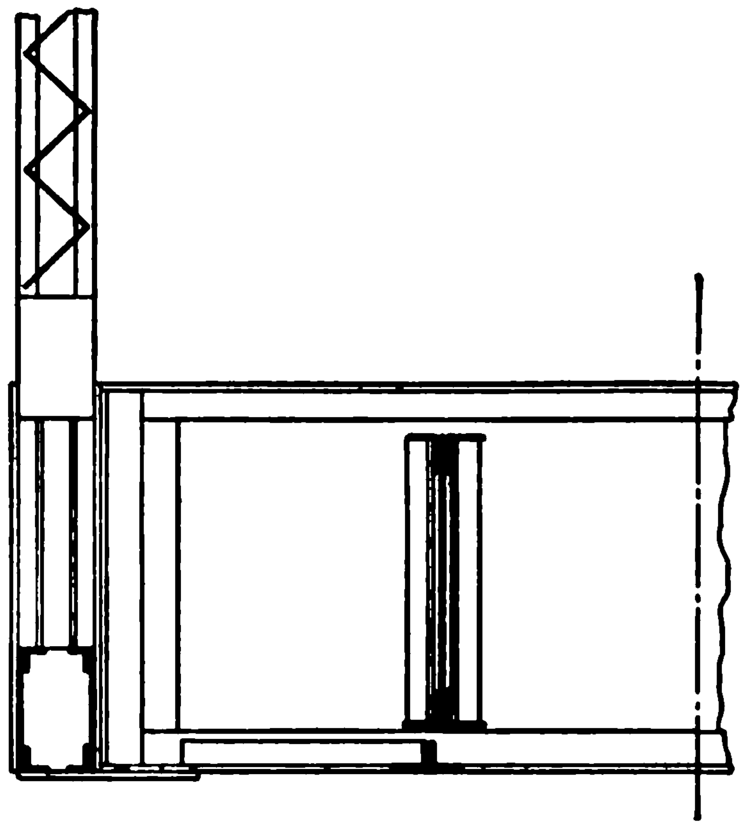


FIG. 8.

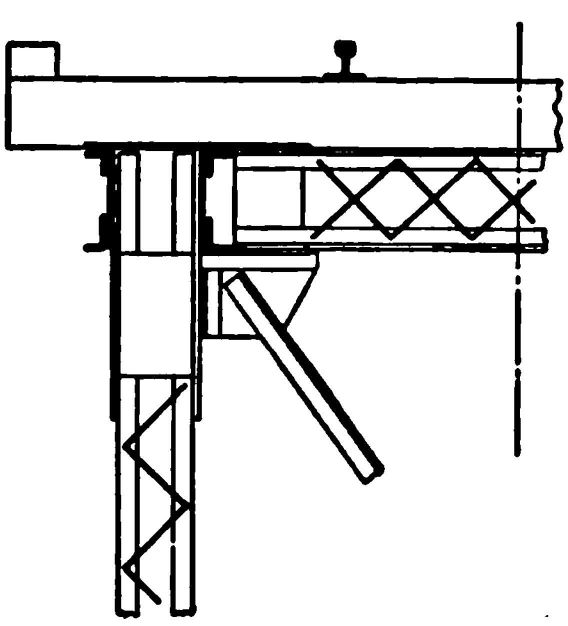


FIG. 9.

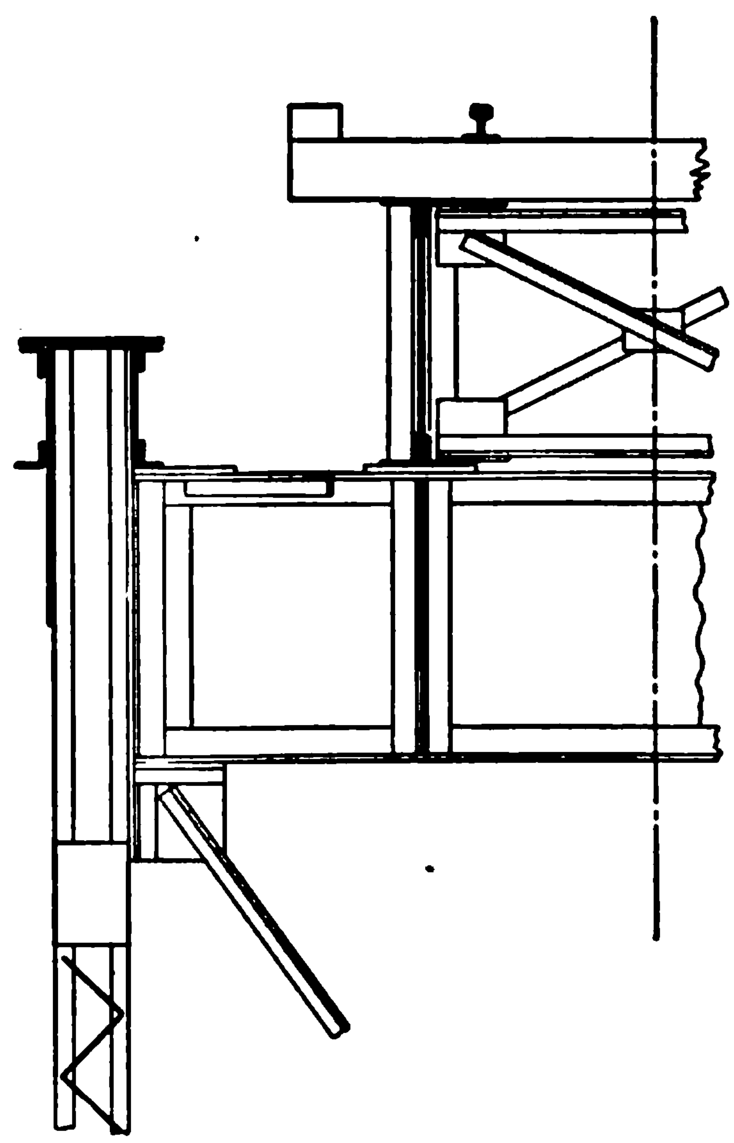


FIG. 10.

are riveted to post and beam and in riveted spans to the chord also. (See Figs. 7 and 8.) This detail is much more satisfactory in the

riveted span. Lateral members are made slightly shorter than the theoretical length, so that when stretched into place by drift pins they will have some initial tension.

The top lateral diagonals of through spans are generally made of four angles, spaced vertically the full depth of the chord, and connected to both top and bottom flanges. Such a member made of minimum size angles will generally have excess cross-section, but rigidity in this bracing is very important, as the two chords constitute together a long compression member from end to end of the bridge, and should be well braced so as to act as a unit. At each joint, lateral struts are used consisting of the same form of section; or, where deep sway-bracing is used, the two top angles of this bracing may constitute the strut. (See Fig. 11.)

The lower laterals of deck bridges are not as important as the

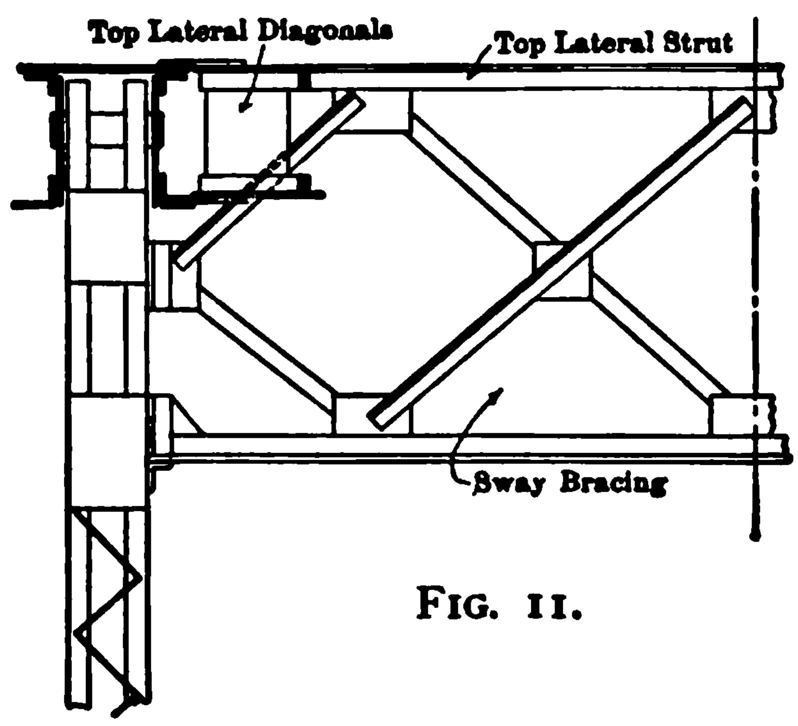


FIG. 11.

top laterals of through bridges. In the former case the chords are tension members and need less bracing than compression members. Also, in a deck bridge, transverse bracing is usually inserted in each panel of the full depth of the truss, which, together with the upper laterals, furnishes a complete system of bracing without the lower laterals. The lower laterals are therefore made small, the diagonals being made of two angles and the cross-struts of two or four angles. The connection must be made on the post in the case of pin-connected spans, but can be made on the chord in riveted spans. (See Figs. 12 and 13.)

166. Portal and Sway-Bracing.—In through bridges the portal

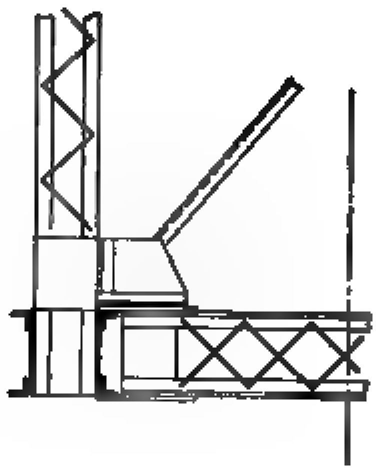


FIG. 12.

FIG. 13.

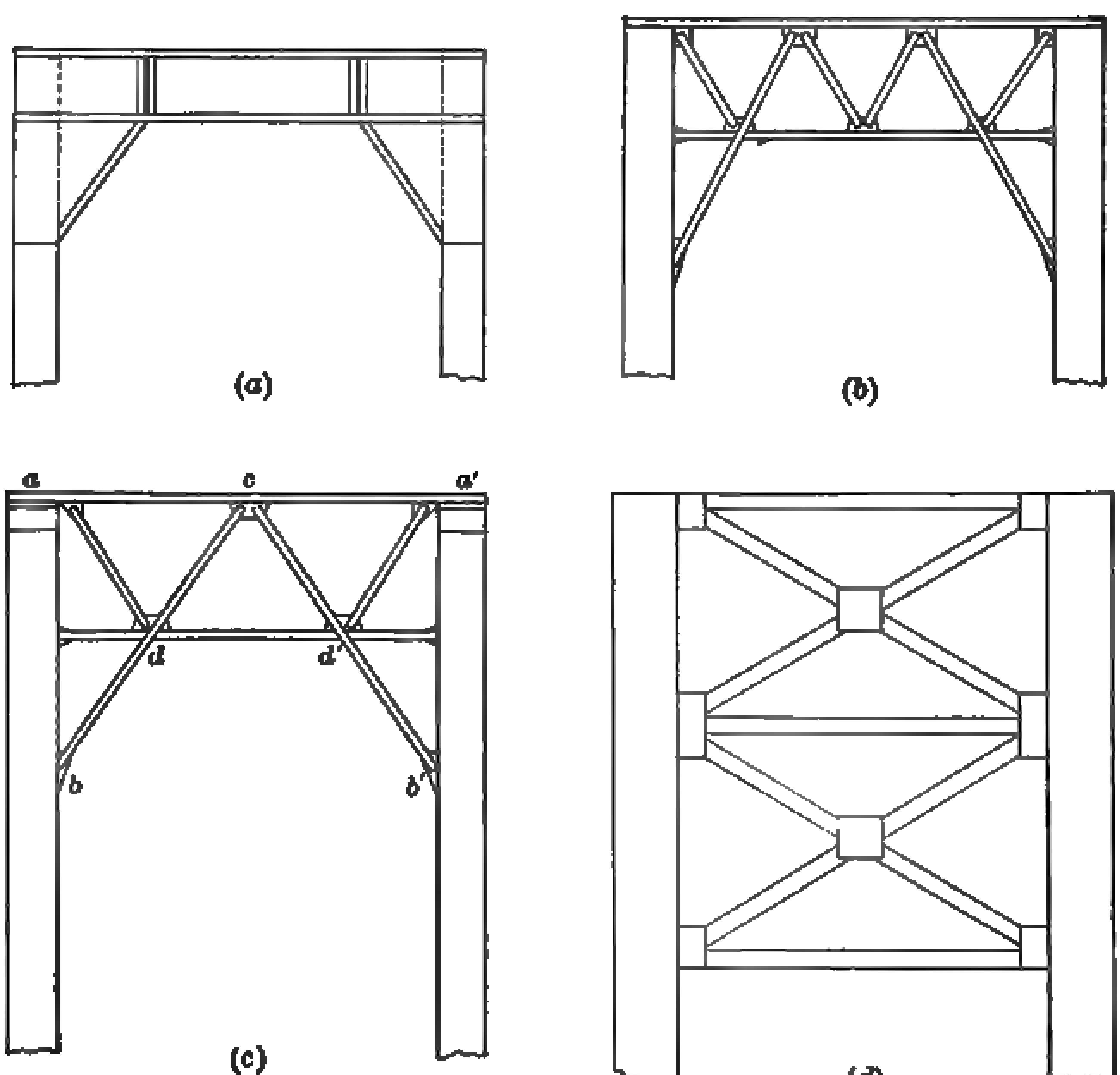


FIG. 14. Forms of portal bracing.

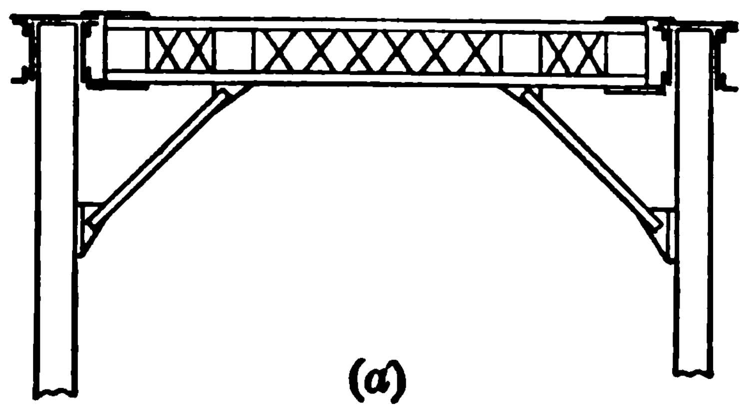
bracing carries down to the abutment the accumulated lateral force brought to the ends of the truss by the upper laterals. As the portal transmits its load partly by the bending resistance of the end posts, it is a more flexible form of bracing than a full cross-frame and should, therefore, be employed only when the cross-frame cannot be used, as in the through bridge.

To make the portal as rigid as possible, it should be made as deep as the head room will allow, and the end posts should be braced as low as possible by the use of brackets or inclined bracing, as in Fig. 14 (*a*), (*b*), and (*c*). The bending moment in the portal as a whole is a minimum at the centre (theoretically zero) and a maximum at the ends. Hence its connections to the end posts need to be made strong and effective.

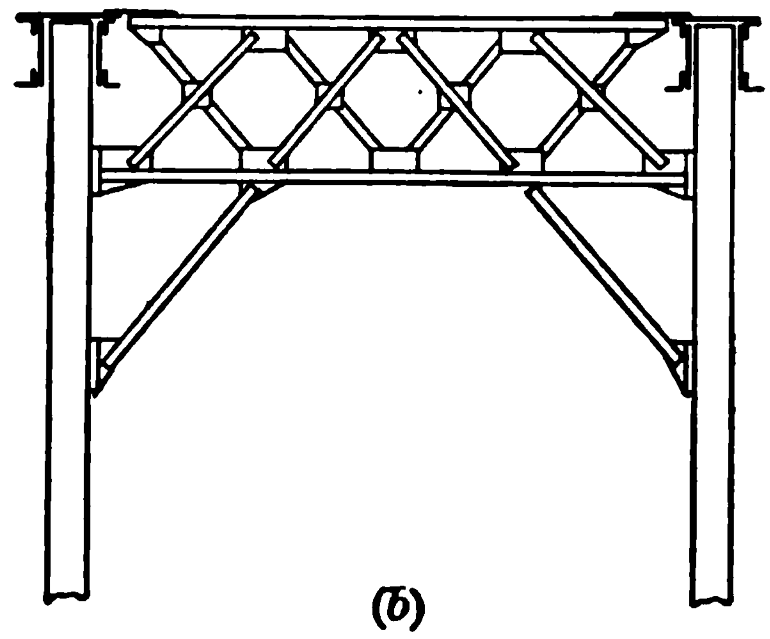
The form of portal used depends largely upon the head room available. For a minimum of room a solid plate-girder type is the most effective, Fig. 14 (*a*). For an increased depth the plate girder is replaced by a lattice girder, Fig. (*b*), and for still greater depth the "A" frame type, Fig. (*c*), is used, or a deep lattice girder or double cross-frame, Fig. (*d*). For trusses of moderate span the "A" frame is very simple and effective, as it secures maximum depth of bracing for the end posts and minimum number of parts. Theoretically the entire stress is taken by the strut $a\ a'$ and braces bc and $b'c$, the members ad and $a'd'$ serving as stiffeners. Except for small spans the individual members of the portal should be made of either two or four angles and of the full depth of the end post so as to connect to both top and bottom flanges.

Sway-bracing is generally placed in each panel and made as deep as the head room will allow. Its purpose is to stiffen the structure laterally somewhat, and to maintain a rectangular cross-section. It is made of minimum size angles in the form of a lattice girder. (Fig. 15.) Where head room is limited, brackets are generally used, connected to top strut and post, Fig. (*a*). For very deep trusses a double cross-frame is a convenient form. Sometimes in this case the intermediate strut a is omitted. This is objectionable, as the absence of this strut will result in some lateral bending of the posts at this level. (See Art. 79.) Besides stiffening the truss as a whole the sway bracing serves to stiffen the vertical posts and so shorten their column

length in a transverse direction. At the same time this arrangement also acts to increase somewhat the effect of floor-beam deflection. In deck bridges the main end bracing is placed in the plane of the



(a)



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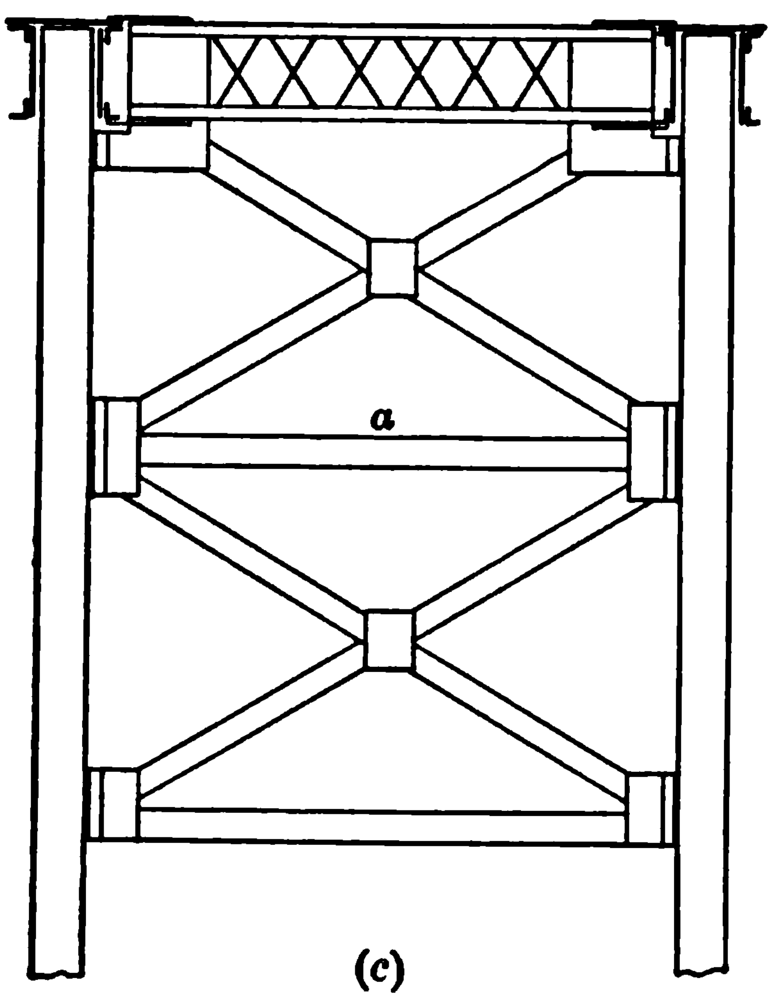


(c)

FIG. 15. Forms of sway-bracing.

heavy inclined end posts and made the full depth of the truss. The sway-bracing now also becomes cross-bracing of full depth.

In the case of double-track bridges the action of sway-bracing equalizes, to some extent, the load on the trusses when one track only is loaded. This effect is, however, not great, probably not more than about 10 per cent (see Part II, Art. 271), as the result depends also upon the lateral rigidity of the upper and lower laterals. However, by maintaining the trusses parallel, or nearly so, under eccentric loads, the bending of floor-beams and vertical posts due to unequal deflections of the trusses is largely avoided. Instead of the beams and posts being bent by such action, the entire cross-section is turned through a small angle. In some recent designs of large structures with two or more tracks, the floor-beams are supported by details provided with pin joints having the axes of the pins parallel to the truss axis.

167. End Bearings.—Truss bridges are supported on shoes through pin bearings or their equivalent. At one end the shoe rests on rollers to permit of free longitudinal movement. In pin-connected spans the end truss pin is used also in the shoe; in riveted trusses the shoe pins may be placed at any convenient point vertically below the centre of the end joint. (See Fig. 16.)

The objects to be kept in mind in designing a shoe and pedestal are: (1) to distribute the load from the pin uniformly over the several rollers, both longitudinally and laterally; (2) to provide adequate size and number of rollers to give proper stress and free movement; (3) to so arrange the rollers that they can readily be cleaned of accumulated dirt and dust; (4) to arrange the details so that constant movement will not cause the rollers to become displaced, and (5) to distribute the load from the rollers, or the fixed shoe, evenly to the masonry below. To secure uniformity of load on the rollers requires the shoe to be rigid in a longitudinal direction, and to secure this it should be made fairly high, measured from pin centre to top of rollers. This height should not be less than about one-half the length of bearing upon the rollers. Laterally, the pressure is well distributed by the pin bearings if the bed plate is set true. If not, then the pressure will be greater at one end of the rollers than at the other. Adjustment in a lateral direction is not usually provided for, the desired evenness of bearing being secured by careful setting of the bed plate. A form of bearing in which lateral adjustment is provided for is shown in Fig. 17, a design by Mr. Geo. H. Morison. This consists of a cast steel rocker bearing placed between the shoe and the rollers as shown. Between the main castings is inserted a "rocker plate," whose upper and lower surfaces are finished in cylindrical form, the upper one with axis transverse to the bridge and the lower one with axis parallel thereto. By this arrangement the bearing will adjust itself in setting to any small angle between shoe and bed plate or masonry. This bearing is well adapted to large riveted trusses where it can be used without pins. Fig. 18 shows the use of an I-beam grillage to secure lateral stiffness and to widen out the bearing.

In modern practice rollers are made of much larger diameter than formerly, 6 ins. being about the minimum now used. (See

Specifications, Art. 62.) Experience with small rollers has shown that it is very difficult to prevent them from becoming clogged with dirt and rusted so that they fail to act. Rollers should be kept clean and should act freely, and both of these requirements are secured by using large sizes. To avoid too wide spacing the rollers

FIG. 16. FIG. 17.

are usually cut away on the sides, forming "segmental" rollers. (See Fig. 18.) This makes it necessary to insure that the rollers are properly set and so arranged that they are always upright. To prevent displacement, one of the rollers should be geared at top and bottom to notches in the plates. (Fig. 18.) Such an arrangement also prevents the roller nest from becoming skewed, as it tends to do under unequal pressures.

The allowable pressure on rollers is given by the formula

$$p = 600\,D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p = pressure per lineal inch of roller and D = diameter of roller. This formula makes the allowable pressure proportional to the diameter, and for a 6-in. roller it is 3,600 lbs. per inch. Formerly, it was estimated by the application of a very rough theory, that the safe pressure was proportional to $\sqrt{D}$, which obviously was less

favorable to large rollers than to small ones. Experiments to determine the elastic-limit pressure have shown,* however, that it is closely proportional to the diameter of the roller.

Beneath the rollers a fairly rigid bed plate is desirable. A form frequently used is one made from rails planed so as to fit together, as shown in Fig. 18. This arrangement tends to keep dust and dirt

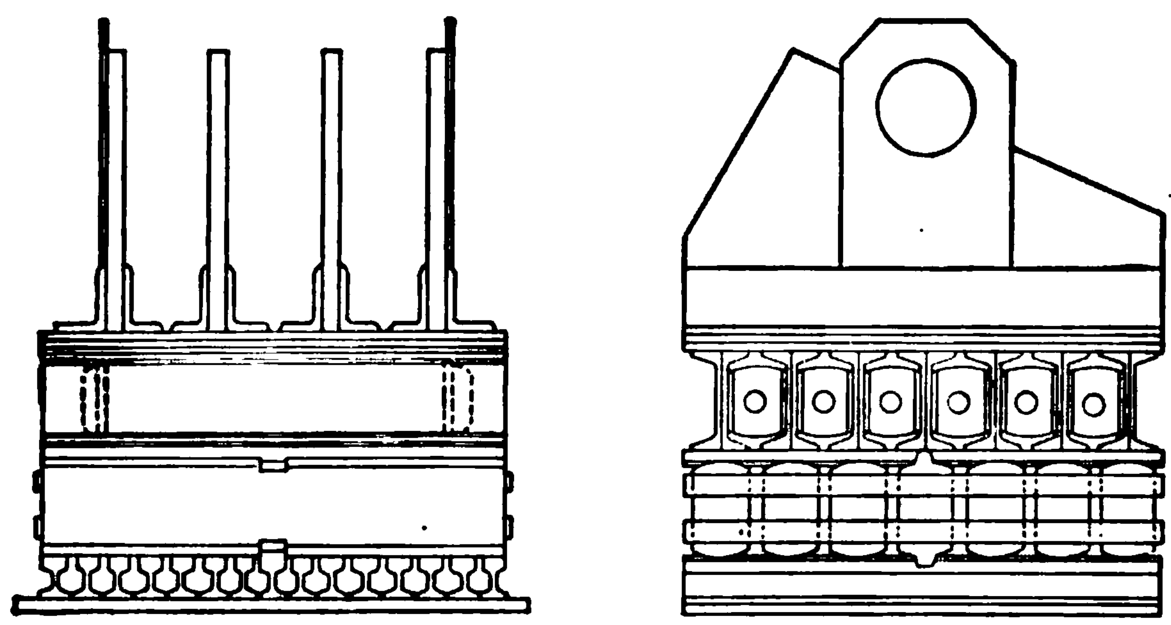


FIG. 18.

from the roller bearings. A planed casting is also a satisfactory form, and frequently used (Fig. 19). At the fixed end the bed plate or casting is usually made deeper than at the roller end, by an amount equal to the height of rollers. This enables the masonry elevations to be made the same at both ends.

168. Design of Pins and Packing of Members.—While pin connections are, in special cases, used for built-up members, they are not in general well adapted to such forms. They necessarily require the connecting members to overlap, and, therefore, to be made of various widths, and where many members connect this is not convenient. Pin connections are primarily suited to designs where most of the tension members are eye-bars, as such members can be made up of any convenient number of parts, so that they may be readily arranged on the pin. Eye-bars can also be given a slight angle with respect to the plane of the truss so that the bars of the same member are not quite parallel. This also facilitates packing of members at the joints.

* See paper by Professors Crandall and Marston, Trans. Am. Soc. C. E., Vol. 32, p. 99.

The design of pin joints is, in some respects, more difficult than riveted joints. It involves several distinct features, namely: the arrangement of members, or "packing"; the calculation of the stresses in the pin and the determination of its size; the calculation of the necessary bearing areas, or thicknesses of the pin plates for the built-

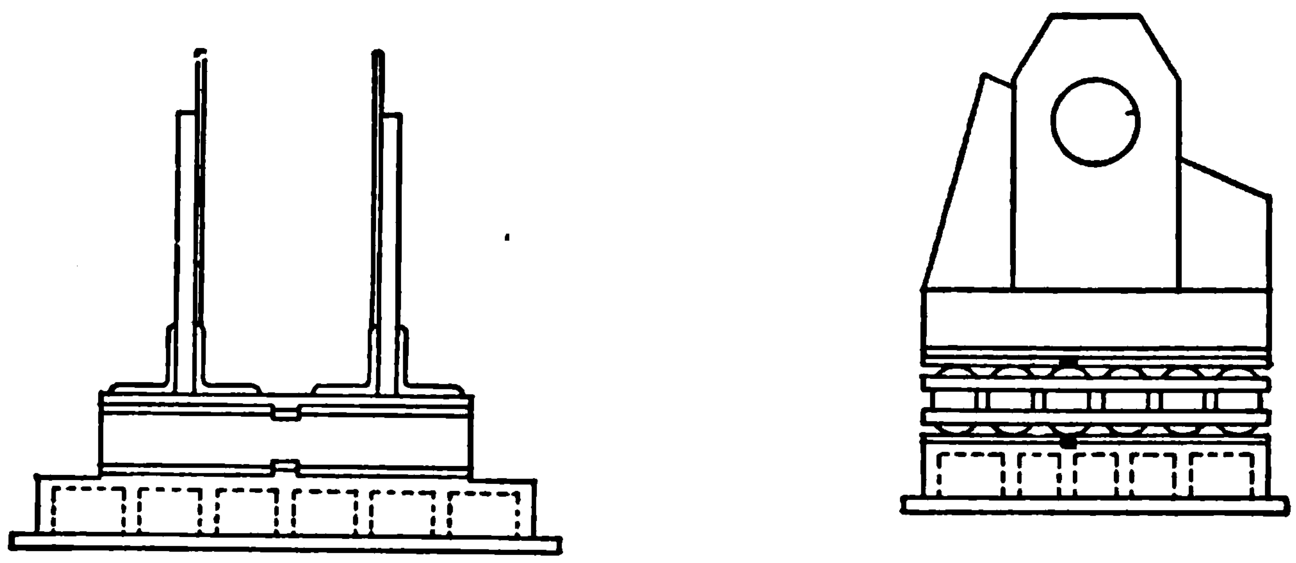


FIG. 19.

up members; and the determination of the lengths and riveting of the pin plates so as to preserve the full assumed strength of the members. The sizes of eye-bar heads, and sometimes of the eye-bars, are also dependent upon the size of pin. The various steps of the calculation are interdependent, and must first be made tentatively, and then revised until accordant results are reached.

It is generally convenient to assume, first, a size of pin, based on other similar designs, then the thicknesses of bearings and sizes of eye-bar heads can be determined. The packing of members is then arranged, and, finally, the stresses in the pin calculated and the exact size determined. The other calculations are then revised if necessary. Small variations in size in the different pins of the same bridge are undesirable, only two or three sizes generally being used.

A pin must be analyzed as a short beam subjected to heavy shearing and bending stresses. The bearing pressures of the members on the pin are also high. A pin must then be designed with reference to (1) shear, (2) bending, and (3) bearing or crushing.

(1) *Shearing Stresses.*—The shearing stresses on a pin are found, as in any beam, by summing up the forces acting on one side of any given section. In dealing with pins it is convenient to resolve all forces into horizontal and vertical components and then to sum up

the components separately and get the resultant where desired. The shearing stress is calculated as in rivets, as the average stress on the entire cross-section, and the allowed unit (12,000 lbs. per sq. in. in the Specifications, Art. 18) is applied in the same way. It is seldom that the shearing stresses will determine the pin size.

(2) *Bending Moments.*—The bending moments are generally calculated by assuming the loads as concentrated loads, applied at the centres of bearing of the several members. This gives slightly larger values for moments than if the loads are assumed as distributed over the areas of contact, and sometimes it is specified that the loads may be assumed as distributed over one-half the widths of the bearings. The difference in result is small, however, and the former method is sufficiently accurate.

In the calculation of moments the arrangement of the members on the pin is first tentatively fixed upon, such arrangement being made with particular reference to maintaining small bending moments. The moments are then calculated for the critical sections, as may be determined from inspection or by trial. The horizontal and vertical components of the forces are separately considered and the resultant moment at any section found by the formula $M = \sqrt{M^2_h + M_v^2}$, where M = total moment and M_h and M_v are, respectively, the moments from the horizontal and the vertical components.

The forces to be assumed in the calculation of shear and moments are not, in general, the maximum stresses in all the members attached to the pin. These maximum stresses do not generally act simultaneously. Obviously, the pin is in equilibrium at all times under balanced forces, and the stresses assumed in the members must, therefore, be in equilibrium. For pins in the lower chord it is generally necessary to consider two conditions, first, when one of the chord stresses is a maximum, and second, when the stress in the diagonal is a maximum. Thus for the pin at joint *c*, Fig. 20, the stress in *cd* is first taken at its maximum value. The simultaneous stresses in all the other connecting members are then determined. Knowing the position of load for maximum stress in *cd* the corresponding stress in one of the other members, as *bc*, is calculated. Then from the equilibrium of the joint the corresponding stresses in *Bc* and *Cc* are quickly found. A small force diagram is convenient for this purpose.

With the system of forces thus determined, the maximum moment in the pin is found. Then the maximum stress in member *Bc* is assumed, the corresponding stress in *bc* or *cd* calculated from the loading, and the stresses in the remaining two members found from the joint equilibrium. The maximum moment for this case is found

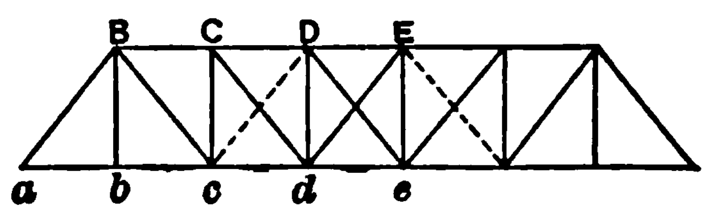


FIG. 20.

and, finally, the greater of the two moments used in determining the pin diameter.

Considering the pins of the upper chord, the maximum moments at *B* will generally occur when the stress in the hanger *Bb* is a maximum. The corresponding stress in *Bc* will then be nearly at its maximum. At the other top joints the maximum moments occur for maximum stress in the diagonal. For horizontal chords the upper chord members are built continuous at the pinholes so that only the difference between chord stresses is transmitted to the pin. In the case of inclined chords the joint is made at the pin and generally all the stress is transmitted through the pin. In this case, however, the centres of bearing of the chords are nearly opposite each other, so that the resulting pin moment is still mainly dependent upon the difference in chord stress, or maximum diagonal stress, as in the other case. (See Chapter VIII for examples of pin calculation.)

The maximum bending moment being known, the pin diameter is determined from the usual formula for fibre stress in beams, $f = Mc/I$. In this case c = radius of pin $= r$, and $I = \pi r^4/4$. Expressing the result in terms of the diameter D we have $f = 10.2\, M/D^3$ and $D = \sqrt[3]{10.2\, M/f} = 2.17\sqrt[3]{M/f}$. Tables of resisting moments for various unit stresses and pin sizes are given in the handbooks.

The allowable bending stress in pins may be taken fairly high, 24,000 lbs. per sq. in. in the Specifications, Appendix A. This is for the reason that the assumption of concentrated loads made in the calculations gives higher values than the true moments, and for the further reason that any slight bending of the pin results in some

readjustment of the forces acting thereon, which tends to reduce the moments. The pin is also not subject to large additional stresses of a secondary nature, as is the case with the members themselves. The unit stresses employed are the result of many years' experience of bridge engineers in the behavior of bridges long in service.

(3) *Bearing Stresses.*—The bearing stress on pins is taken, as in the case of rivets, at so many pounds per square inch on the area found by multiplying the thickness of the bearing by the diameter of the pin. For riveted members the necessary bearing area is provided by adding pin-plates to the member until the requisite thickness is secured. For eye-bars the bearing stresses may be limited to the desired value by a general provision limiting the ratio of pin diameter to the width of the widest connected bar. Thus, if the allowed unit tensile stress in the eye-bar is 16,000 lbs. per sq. in. and the bearing stress is 24,000 lbs. per sq. in., then this bearing pressure will not be exceeded if the pin diameter is not less than $16,000/24,000 = 2/3$ of the width of the widest connected bar.

Packing of Members on Pins.—To avoid large bending moments on pins, the packing, or arrangement of bars and members on the pin, must be made with reference to the moments produced thereby. In general, to keep the moments small, a bar or member acting in one direction should be placed closely adjoining a bar or member acting in the opposite direction, the two forces thus acting approximately as a couple with lever arm equal to the distance between centres of bearing. Then where there is a large number of bars the next pair should be arranged in the opposite way so that the next couple will be of opposite sign. In this way the pin is subjected to moments due to couples of opposite sign, and the bending does not accumulate towards the centre of the pin.

For the same reason diagonal bars should be placed close to the vertical posts, to keep the vertical moments small, and also close to a chord bar pulling towards the centre from the pin in question to keep the horizontal moments small. Fig. 21 shows the packing of the lower chord of the truss designed in Chapter VIII (see Plate III). The dotted lines represent the diagonal members. At joint *c* the diagonal is placed next to the post and next comes a chord bar *cd*, since the horizontal component of the diagonal should be balanced

by a chord bar pulling towards the right. Next in order is the member *bc* and then the last bar *cd*. At joint *d*, Fig. 22 shows two arrangements. Fig. (*a*) is the same as in Fig. 21. If the stresses in 6 and 3 were equal to the stresses in 5 and 4 the arrangement in Fig. (*a*) would give a much smaller moment on the pin at bar 1 than arrange-

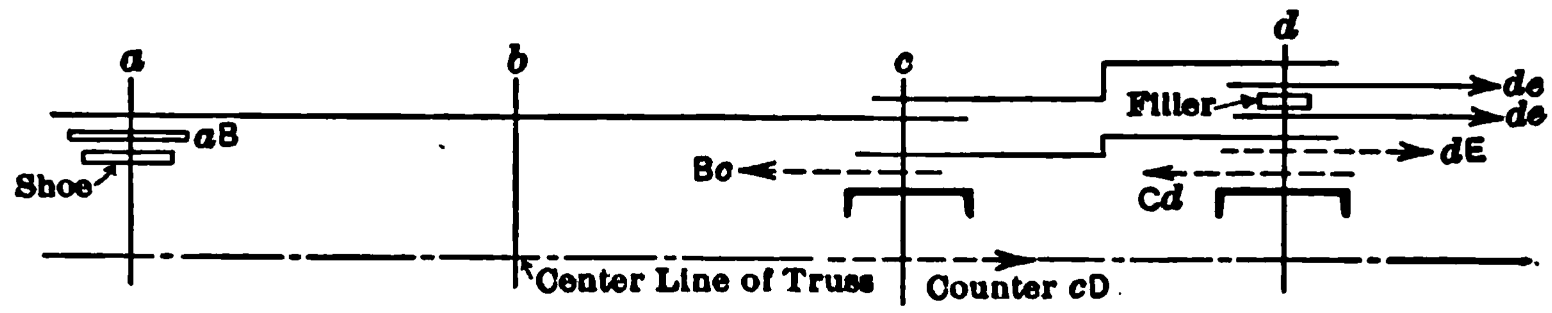


FIG. 21.

ment (*b*). But the stresses in 5 and 4 are considerably larger than those in 6 and 3, so that the actual moment at bar 1 is less in (*b*) than in (*a*). Arrangement (*a*) was finally adopted because of the excessive slope required for bar 3 in (*b*). The true variation for the arrangement of Fig. (*a*) is shown in Fig. 23. For maximum stresses in the chords the stress in member *dE* (bar 2) is small.

Where four bars are needed for the diagonal, two are placed inside and two outside of the post channels and adjacent thereto.

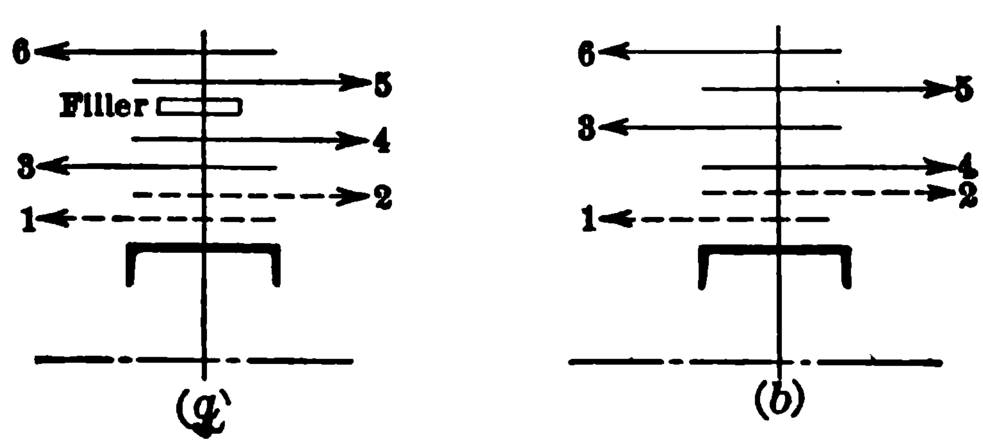
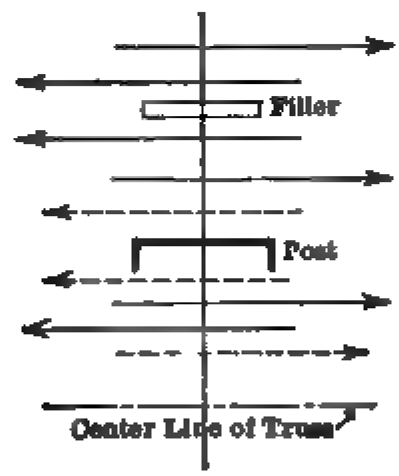


FIG. 22.

Counters are placed within the post, single counters being placed at the centre of the pin. When six or more chord bars are needed, two or more are placed inside of the post channels and just inside of the main diagonals, as shown in Fig. 24. Where bars are placed inside the post the channel flanges are cut away to allow the bars to be placed close to the pin plates on the post. Two bars of the same member placed adjacent, as at joint *d*, Fig. 21, must be separated at least an inch by a filler to permit of painting.

Where a large number of chord bars are packed consecutively on a pin the advantage of the arrangement shown in Fig. 22*b* can be secured by making the outside bar thinner than the others. Then



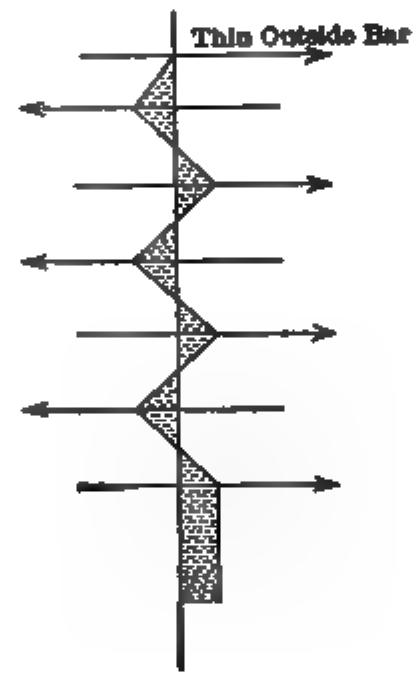


FIG. 23. FIG. 24. FIG. 25.

an alternating arrangement of bars does not tend to produce large moments. The general form of the resulting moment curve is shown in Fig. 25.

At the top chord the packing of the truss of Chapter VIII is shown in Fig. 26. At the hip joint *B*, the diagonals and hanger act in the same direction vertically, giving a large vertical bending moment. This cannot be avoided except by placing the diagonals outside of the posts, which is objectionable as requiring the flanges to be cut away and also requiring an excessive inclination to the diagonals to bring them to the proper point at the lower joint *c*. (See Specifications, Art. 83.) Where four bars are required, two must either be placed outside, or a centre web or diaphragm placed in the top chord and end post to support the pin, as is common in large trusses. See Fig. 3.

In arranging members on a pin and in calculating centre to centre distances it is necessary to allow certain clearances for irregularities of surfaces, rivet heads, etc. The following are the usual allowances: Between two eye-bars, $1/16$ inch; between two parts of riveted members without rivets, or with rivets countersunk and chipped, $1/4$ inch;

between eye-bars and riveted members about 1/8 inch. For projecting rivets, flattened or otherwise, add the height of rivet to the above clearances.

169. Pin Plates.—Riveted members attached to pins generally require the pin hole to be reinforced with additional plates in order

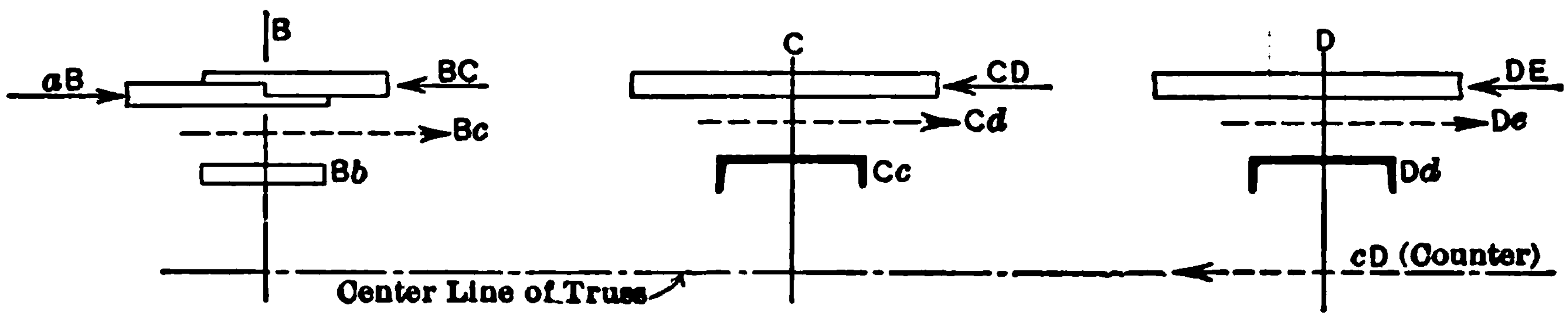


FIG. 26.

to furnish the required bearing area on the pin. These pin plates must be so attached to the members as to transmit the stress they receive from the pin to the body of the member without overstressing any of its parts. At the pin hole it is assumed that the pressure transferred from pin to plate is proportional to the thickness of plate, and in the body of the member the stress is assumed to be uniformly distributed over the cross-section. The pin plates must be designed to secure this transfer of stress from pin to body of member without overstressing any part. In the compression member, Fig. 27, for example, the pin pressure P is given over to the web d, and the pin plates a, b, c, and e in proportion to their thicknesses. The stresses carried by these plates must be transferred to the main parts of the member, the angles and top plate, so that at section q beyond the last plate, the stress will be uniformly distributed. The rivets connecting the pin plates must, in the first place, be sufficient to carry the stress from the plates to some part of the main member; secondly, the stress to be carried by the angles and top plate, which do not bear on the pin, must be transferred into these parts by a sufficient number of rivets to the left of the line q, at which section it is assumed that all main parts of the member are uniformly stressed; and, finally, the transfer of stress from pin plates to top and bottom angles must be made without overstressing the web d.

In Fig. 28 the pin pressure P acts towards the end of the member. To insure safety against rupture on section r, through the pin hole,

the net area on this section should be considerably greater than the net section through the body of the member, 25 per cent by Art. 28, Appendix A; also the net section back of the pin hole, section *s*, is usually required to be as large as the net section in the body of

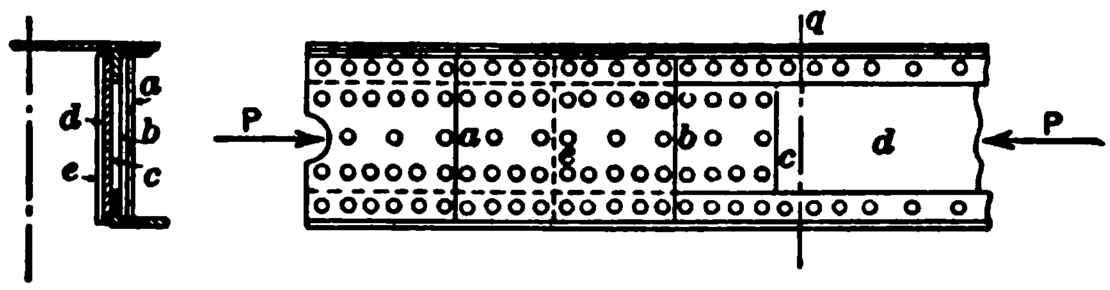


FIG. 27.

the member. In designing the pin plates it is assumed, (1) that the load taken by the several plates is proportional to their thicknesses, (2) that the stress on section *r* is uniform over the cross-section, and (3) that the stress on section *q* is also uniform over the cross-section. To secure these conditions there must be sufficient rivets on the *left* of section *r* to distribute the stress received by the pin plates to the angles, to the extent necessary to secure uniformity of stress at *r*; and there must be sufficient rivets in the plates to the right of *r* to distribute the remaining stresses in the plates to the angles, so that the stress at *q* will be uniform over the section at that point.

In securing the general results described above, the principles of rivet grouping discussed in Chapter V should be followed so far as

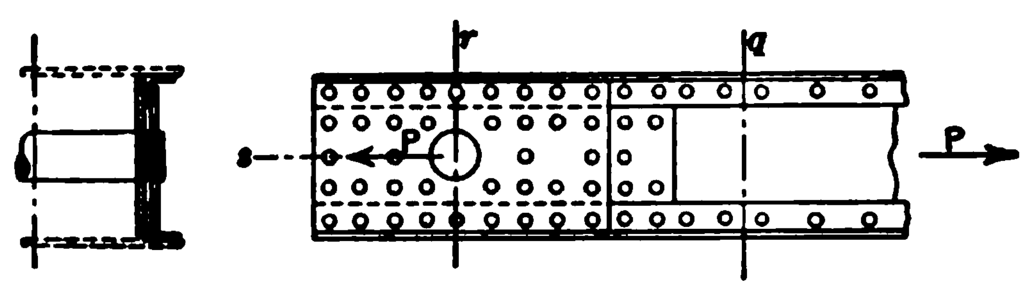


FIG. 28.

practicable. Detailed examples of pin plate design will be found in Chapter VIII.

170. Riveted Joints.—Joints in riveted trusses are usually simpler in design than pin joints. Both tension and compression diagonals are riveted to gusset-plates which are attached to the webs of the chord members. Generally, the web members are placed inside the gusset-plates and the latter inside the top chord, and inside or outside

of the bottom chord according as the flanges of the chord are turned out or in.

Fig. 29 illustrates a top chord joint. The size of gusset-plate will generally be determined by the space required for the necessary number of rivets. The thickness of gusset plate should therefore be sufficient to give a bearing value equal to the shearing value of a

FIG. 29.

rivet in single or double shear as the case may be. For $\frac{7}{8}$-inch rivets in double shear this would require $^{11}/_{16}$-inch plates.

To secure uniformity of stress on the rivets, so far as possible, the group of rivets in each member should be symmetrical about the gravity axis. In the case of heavy stresses it may be necessary to investigate the tensile strength of the gusset-plate along a line *ab* or *cd*, approximately at right angles to the stress, the amount of stress at any section being equal to the total value of the rivets outside of the section. If all gravity lines meet at a point, then the resultant of forces B, C and D will be equal to E minus A, and will be applied along the line $A-E$. The rivets in $A-E$ will, therefore, if arranged symmetrical about the centre line of member $A-E$, be also central with respect to the stress transferred to this member.

Short, connecting angles, *k*, *k*, are often used. These enable the joint to be shortened somewhat, but are not necessary for the purpose

of giving a central connection to the channels or angles composing the member. These segments cannot bend in a plane parallel to the webs of the channels, and the tie plates near the ends of the diagonal members prevent bending in the other plane. Therefore, as shown in Art. 92, the stress in the member will be uniformly distributed, and hitch angles are unnecessary for this purpose.

171. Splices in Chord Members.—Riveted tension chords are spliced at any convenient point, generally near a joint, and on the side towards the end of the truss, or on the side of the lesser stress. Splice-plates and rivets must of course be sufficient to carry the entire stress on net section, and so arranged as to transfer the stress in all elements of the member across the joint to corresponding parts on the other side without overstressing any part, following the same general principles as discussed in the preceding articles. In short spans two panel lengths of the chord are often riveted up and shipped as a single piece.

Compression chords are not generally fully spliced. In trusses with horizontal chords the members are faced to a true bearing and the splice made only with reference to holding the two members securely in place. Such a splice is made on the side of the joint towards the end of the truss. In pin-connected trusses with polygonal chords the splice must be made at the joint, and in that case each member is commonly made full pin bearing instead of a butt bearing. This requires much more riveting in the attachment of pin plates, but it is difficult to make a satisfactory butt joint at such a place. In riveted trusses a butt joint is easier to make, but in that case the splice is generally made strong enough to carry a large percentage, 50 per cent to 75 per cent, of the stress.

In large structures butt joints are very difficult to make so as to be effective and secure an even distribution of stress. Great accuracy is required in the shop to secure the exact angle of cut required, and the deflections of members during erection and the camber of the truss tend to disturb the correct adjustment of the joint. Joints intended to act as butt joints should not be fully riveted up until the bridge is free of the false work.

172. Floors of Truss Bridges.—The usual steel floor system of a railway truss bridge consists of floor-beams at each panel point, and

stringers riveted between or resting upon the floor-beams and supporting the ties directly. Two stringers are generally used for each track, spaced from $6\frac{1}{2}$ to $7\frac{1}{2}$ feet apart. A narrow spacing results in large bending moments in the beam, and a wide spacing gives large bending moments in the ties. A too narrow spacing also requires an extra pair of safety stringers outside of the main stringers to provide for derailed cars. Such an arrangement is sometimes used, but a single set of stringers is more economical, gives some desired elasticity to the track, and is the common practice.

In through bridges, stringers are riveted between the beams and the beams riveted to the vertical members of the truss. The attachment of the beam to the post and the lower laterals to the beam and post, or chord, so as to make a satisfactory detail, is a difficult matter. In a riveted truss Fig. 8 shows the common arrangement. This gives a good connection for the lower laterals and the distance from rail to lowest line of the truss, or clearance line, is a minimum. In a pin-connected truss the beam and laterals cannot be connected to the chord, which condition requires the use of a design like those shown in Figs. 7, 30, and 31. In Figs. 7 and 30 the lower flange of the beam is brought down a little below the eye-bar so that the lateral plate can be attached to the end of the post. This gives fair connections for the laterals and a minimum distance from rail to clearance line. The notching out of the beam is expensive and weakens the end section greatly. To provide the necessary strength of end detail, the end connection is commonly extended up the post above the upper flange by splicing the web near the end and extending the web plate upwards. See Fig. 20, Chapter VIII, for details. Notching of the beam can be avoided by using the design of Fig. 31, but this does not give as satisfactory lateral connection and increases the distance from rail to clearance line.

At the end panels stringers may rest directly on the abutment (Fig. 32), or end floor-beams may be used (Fig. 33). The latter is the more common practice. The use of a floor-beam gives more substantial construction, better connections for laterals, and insures definite and uniform action of the floor members at the expansion end of the bridge. End beams may be supported on the pins (Fig. 33), or on the bed plate inside of the end posts (Fig. 34).

As a general rule the stringers can be made of the most economical depth and the beams enough deeper to allow the stringers to be framed into the beams between the flanges. The depth of stringer will thus be from $\frac{1}{6}$ to $\frac{1}{8}$ of the panel length and the beam 10 to 14 inches deeper so that the stringer may be riveted to the beam between the

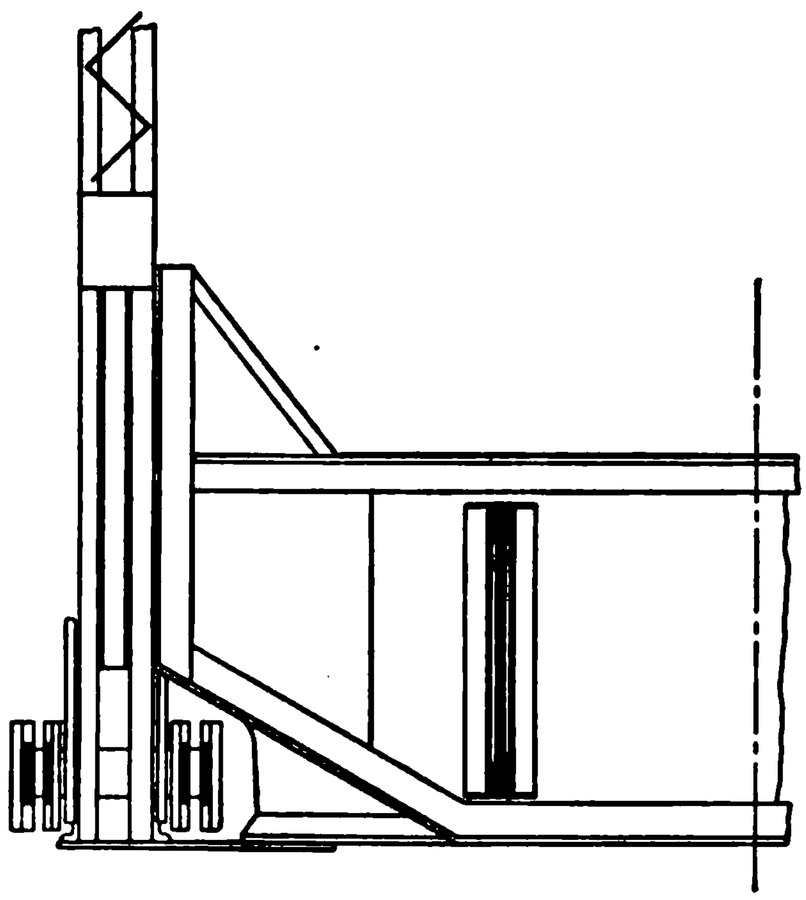


FIG. 30.

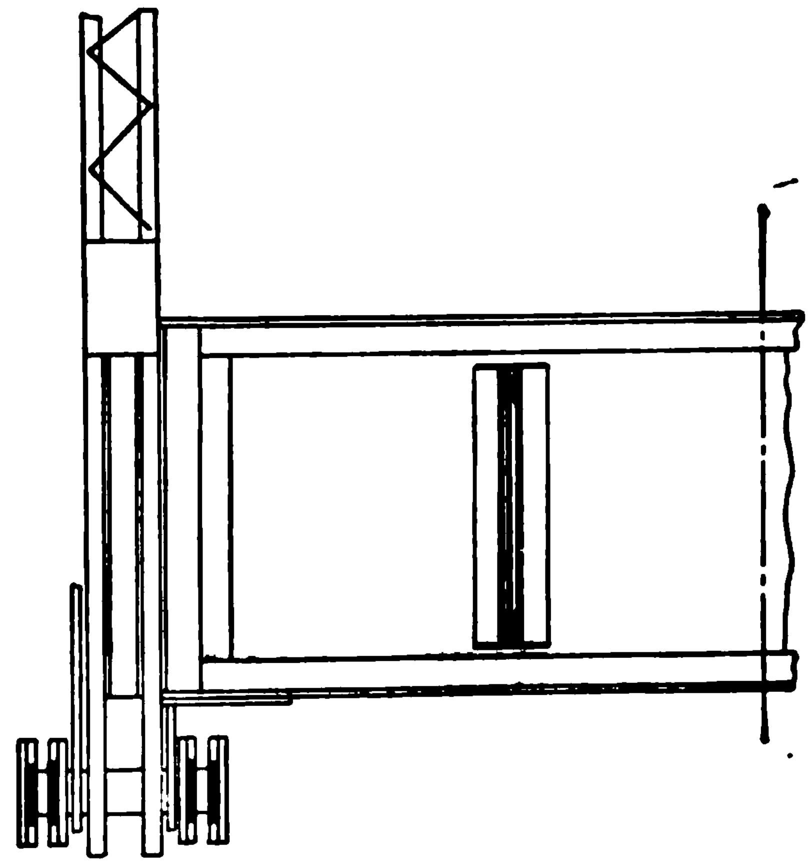


FIG. 31.

vertical legs of the flange angles. Where headroom is limited, much shallower stringers may be used and the stringer connection made lower on the beam. While the depth of beam resulting from this arrangement may be too great for economy from the standpoint of moments, the shearing stresses in the beam are high and a relatively deep beam is desirable in order to give convenient rivet spacing. Deep beams are also advantageous on account of the effect of deflection on post stresses. (See Art. 79.)

Where headroom is limited beams and stringers may need to be made much shallower than usual. This is accomplished in various ways. The panel length may be made very short, by the use of a subdivided form of truss, thereby allowing the use of much shallower floor members. The lower chord may also be made in the form of a deep riveted section and the floor member in the form of small shallow transverse beams closely spaced and riveted to the lower chord. Or, instead of using the lower chord in this way, separate longitudinal girders may be riveted to the posts below the lower chord. This

permits eye-bars to be used. The objection to this form arises from the stresses produced in the members and connections from the longitudinal extension of the lower chord under stress.

Solid floors to support ballast may be arranged between longi-

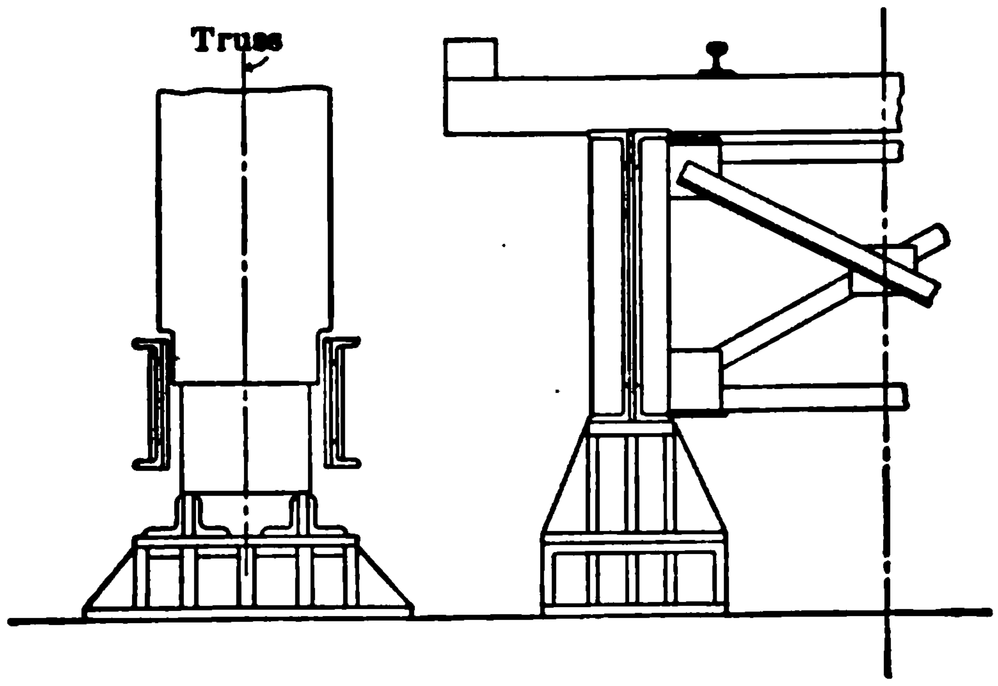


FIG. 32.

tudinal stringers, as in plate girder spans, the stringers being spaced any desired distance apart. For shallow floors the arrangement described above lends itself readily to the use of a ballasted track.

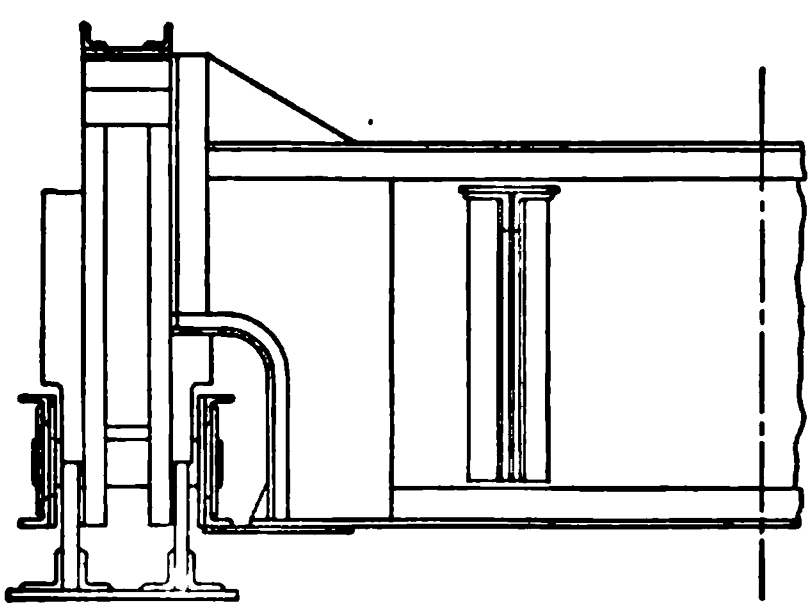


FIG. 33.

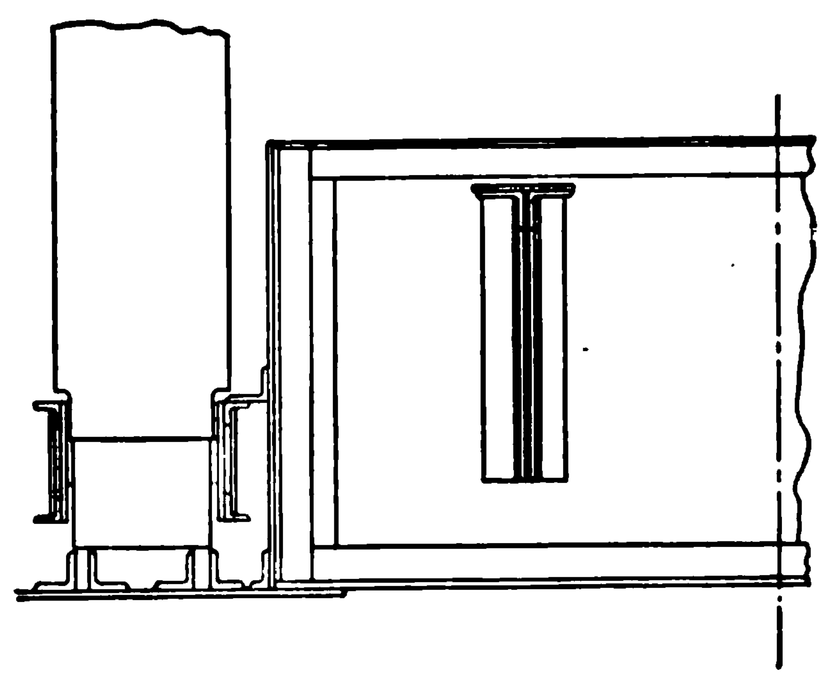


FIG. 34.

Expansion joints should be used in the stringer system for long spans in order to avoid the stresses due to chord elongation. Fig. 35 shows a convenient form for such a joint. While a desirable detail for long spans they should not be used too frequently, as they tend to decrease the rigidity of the floor system.

173. Camber.—Truss bridges are generally constructed with a slight curvature or camber, upwards, so that when fully loaded they

will deflect into a straight line. This is usually accomplished by making the top chord slightly longer than the bottom chord, the common rule being to give it an excess of ⅛ in. for every 10 feet in length of span. (See Art. 81, Specifications.) The actual amount of

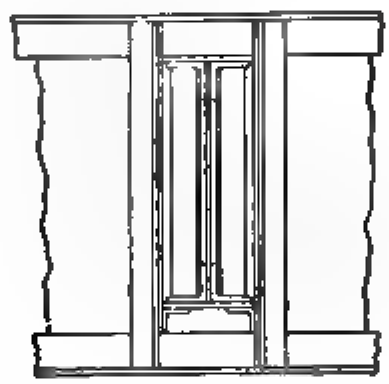


FIG. 35.

camber produced by any given difference in chord length is given by the formula

$$c = \frac{il}{8h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

in which

c = camber in inches;
i = increase of length of top chord over bottom chord in inches;
l = span length in feet;
h = height of truss in feet.

For $h/l = {}^1/_7$, and $i = ⅛$ in. per 10 ft., we have $c = 7\,i/8 = \dfrac{1}{1{,}200}$ of span length, approximately.

Sometimes the camber is produced by shortening or lengthening all members in their manufacture by an amount equal to their elongation or compression under full load. Thus each member will have its correct theoretical length when the truss is loaded. This method is desirable for large structures.

CHAPTER VIII

THE DESIGN OF A PIN-CONNECTED SINGLE-TRACK THROUGH RAILWAY BRIDGE

174. General Data.—A pin-connected railway bridge of the Pratt type will be designed to meet the requirements of the 1910 specifications of the American Railway Engineering Association. These specifications are given in Appendix A.

The span will be 175 ft., consisting of 7 panels of 25 ft. each, and the height will be 30 ft. The track will be assumed to be straight and the span will be made square ended. A live load corresponding to Cooper's E-60 will be used. The distance between trusses will be governed by the clearance diagram of Fig. 1; they will be spaced 17 ft. 9 ins. centre to centre. On comparing Fig. 1 with the clearance diagram of Art. 2, Specifications, it will be found that the adopted diagram calls for a clear width which is one foot greater than that called for by the specifications. This change has been made in order to conform to the present day tendency to use larger and wider rolling stock.

175. Design of the Wooden Floor System.—The floor system will consist of wooden ties placed on stringers, as shown in Fig. 2. The general considerations and articles from the specifications which govern the design of the ties are the same as for the ties of the plate-girder floor system designed in Art. 138. As in that case the special axle loads of Art. 7, Specifications, are to be used. Each axle carries a load of 75,000 lbs., which is assumed to be distributed over three ties. The load per tie at each rail is then 12,500 lbs.

The stringers will be spaced 7 ft. c. to c., as shown in Fig. 2. In general a comparatively wide spacing of stringers is desirable. This allows the tie to act as a cushion and thus relieves the stringer to some extent from the heavy impact stresses due to roughness of track. Wide spacing also tends to reduce the bending stresses in the floor-beams. A spacing of more than 7 ft. is, however, undesirable as it makes the size of ties, and therefore the weight of the wooden

floor, unnecessarily large. When the trusses are spaced closer together, say from 16 ft. to 16½ ft., the stringer spacing should be taken at about 6½ ft. in order to allow ample room between the stringers and vertical posts of the truss for the connections of the floor-beams to the trusses.

Proceeding as in Art. 138 we find that a 10 × 10-in. tie is required. The length of the ties will be taken as 12 ft., and they will be spaced

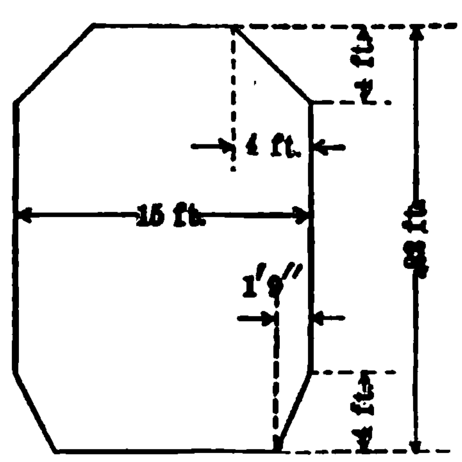


FIG. 1.

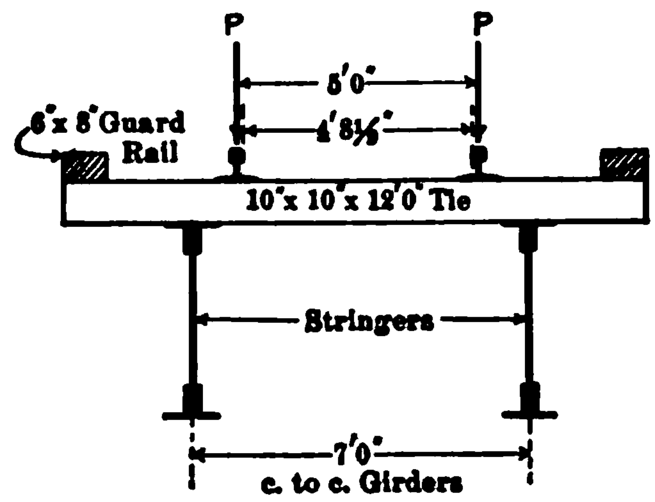


FIG. 2.

16 ins. c. to c., in order to meet the requirements of Art. 5, Specifications. To prevent bunching, a 6 × 8-in. guard rail will be provided at the ends of the ties, the guard rail to be notched over the ties.

176. The Dead Load.—From Art. 6, Specifications:

> The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weigh 4½ lbs. per ft. B. M.; and rails and fastenings, 150 lbs. per ft. of track.

Calculating the weight of track and wooden floor as in Art. 139 we find this to be 523.5 lbs. per ft. of bridge.

The exact weight of steel in the bridge cannot be determined until the design has been completed and an estimate of weight made. A preliminary estimate can be made by the use of the formula given in Art. 9, Chap. I. For the type of structure and the loading under consideration, the dead weight is closely given by the formula $w = 1\frac{1}{8}\,(8\,l + 700)$,* where w is the weight of steel per ft. of bridge, and l is the span in feet. With $l = 175$, we have $w = 2362.5$ lbs. per ft. of bridge.

The weight of steel and wooden floor is then $2362.5 + 523.5 =$

* The formula is $w = k\,(8l + 700)$.
For E-40, $k = \frac{7}{8}$; E-50, $k = 1.0$; E-60, $k = \frac{9}{8}$.

2,886 lbs. per ft. of bridge, which gives a dead panel load of $\frac{1}{2} \times 2{,}886 \times 25 = 36{,}000$ lbs. per truss. This panel load will be assumed to be distributed as stated in Art. 69, Chap. IV, Part I, namely, two-thirds of the dead panel load, or 24,000 lbs., at each lower chord joint, and one-third, or 12,000 lbs., at each top chord joint.

An exact estimate of the weight of the structure has been made, from which it was found that the preliminary estimate of weight was sufficiently close to the true weight to make a revision unnecessary.

177. Dead-load Stresses.—The dead-load stresses resulting from the panel loads determined in Art. 176 are calculated by the methods given in Art. 86, Part I. The resulting stresses are given in Table A of Art. 180.

178. Live-load Stresses.—The assumed live load is Cooper's E-60 loading. The methods of stress calculation are outlined in Arts. 141 to 145, Part I. The resulting stresses are given in Table A of Art. 180.

Attention is called to the calculation of the live-load stress in member *de*, the centre panel bottom-chord member. The method of calculation is given in Art. 147, Part I. When the shear in the centre panel is zero, as required by the conditions of the problem, we find that wheel 10 is placed at the lower chord joint *d*. The resulting moment at *d* is 13,566,800 ft.-lbs., and the stress in *de* is 452,200 lbs. tension. This stress is a little smaller than that obtained by using the maximum moment at joint *d*, which occurs when wheel 12 is at *d*, giving a moment of 13,726,300 ft.-lbs. and a stress in *de* of 457,500 lbs. tension. The difference in these stresses is found to be about 1.15 per cent of the true stress.

In practice it is usual to consider the stress in *de* as due to the maximum moment as *d*, instead of as calculated above. This assumption is reasonable and saves considerable time. It is only in the case of very large structures that any considerable saving in material can be made by using the exact stress. The problem is worked out exactly here in order to show the student the difference between the true and approximate stresses.

179. Impact Stresses.—The impact stresses are estimated by the formula given in Art. 9, Specifications:

(9) The dynamic increment of the live load shall be added to the maxi-

mum computed live-load stresses and shall be determined by the formula

$$I = L\frac{300}{l + 300}$$

where I = impact or dynamic increment to be added to live-load stress; L = computed maximum live-load stress; l = loaded length of track in feet producing the maximum stress in the member.

The loaded length to be used in the formula for all chord members may be taken as the total distance c. to c. bearings, or 175 ft. in this case. For web members, l is to be taken as the distance from wheel 1 to the right end of the bridge, assuming the loads to move toward the left. The loaded length for member *Bb*, the hip vertical is to be taken as two panels, or 50 ft., since the stress in the member is caused by the floor-beam reaction (Arts. 133 to 135, Part I). The resulting impact stresses are given in Table A, Art. 180.

180. Maximum Direct Stresses.—The maximum stress in any member is the sum of the dead, live, and impact stress for that member as calculated in the previous Articles. These stresses are tabulated in Table A. The member notation conforms to that shown in Fig. 3.

TABLE A

MAXIMUM STRESSES

Member	Dead-Load Stress	Live-Load Stress	Impact Stress	Total Stress
abc	+ 90.0	+237.4	+150.0	+477.4
cd	+150.0	+381.0	+241.0	+772.0
de	+180.0	+452.2	+286.0	+918.2
BC	−150.0	−381.0	−241.0	−772.0
CDE	−180.0	−457.5	−289.5	−927.0
aB	−140.5	−370.4	−237.5	−748.4
Bc	+ 93.6	+265.5	+181.8	+540.9
Cd	+ 46.8	+177.1	+128.5	+352.4
De	0	+102.6	+ 79.3	+181.9
Dc	− 31.2	+ 48.8	+ 40.8	+ 58.4
Bb	+ 24.0	+113.5	+ 97.4	+234.9
Cc	− 48.0	−136.2	− 98.9	−283.1
Dd	− 12.0	− 78.9	− 61.0	−151.9

+ denotes tension; − denotes compression.
All values are given in thousands of pounds.

The total stress for member *Dc*, a counter, is made up for only two-thirds of the dead-load stress, as required by Art. 23, Specifications:

(23) Wherever the live-load and dead-load stresses are of opposite character, only ⅔ of the dead-load stress shall be considered as effective in counteracting the live-load stress.

The total dead-load stress for *Dc* is 46,800 lbs. compression, and $\frac{2}{3} \times 46{,}800 = 31{,}200$ lbs., as recorded in Table A. The object of this article of the specifications is to place excess material

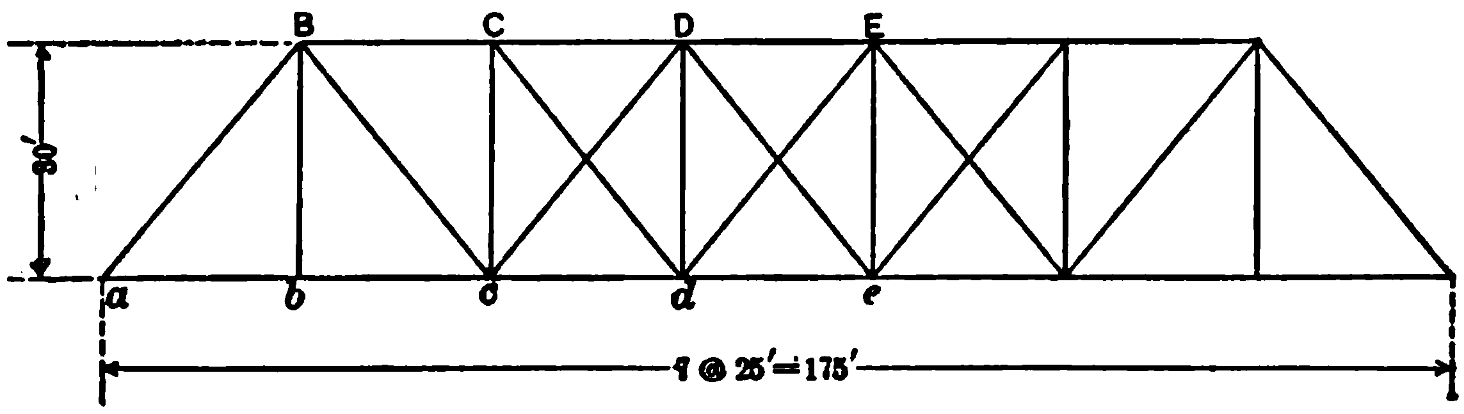


FIG. 3.

in the counters in order to provide for a future increase in live load over that for which the structure is designed.

181. Lateral Forces and Wind Loads.—The lateral forces, in the form of train vibration and wind load, which are to be provided for, are specified in Art. 10, Specifications:

(10) All spans shall be designed for a lateral force on the loaded chord of 200 lb. per linear foot plus 10 per cent of the specified train load on one track, and 200 lb. per linear foot on the unloaded chord; these forces being considered as moving.

This provides for a wind load of 200 lbs. per ft. on the bridge, and for a lateral force, which, for E-60 loading, is $0.1 \times 6{,}000 = 600$ lbs. per ft. This load of 600 lbs. per ft. is supposed to represent the combined effect of wind pressure on the train and the vibration and lateral impact due to unbalanced lateral forces in the locomotive, effect of rough track, etc.

The position and number of lateral systems necessary to provide for the above forces have been discussed in Art. 181, Part I. In this case there will be horizontal trusses in the planes of the top and bottom chords, bracing between the end posts, and transverse bracing between all vertical posts. The stresses in the top and bottom lateral trusses can be determined at once, but those in the transverse bracing depend upon considerations which will be taken up later in Arts. 210 and 211.

182. Top Lateral Stresses.—The stresses in the top lateral system are due to a uniform moving load of 200 lbs. per linear foot. (Art. 10, Specifications.) Fig. 4 shows the general arrangement of the top lateral system. Two diagonals are shown in each panel. These members have small stresses, but are long and slender and, if de-

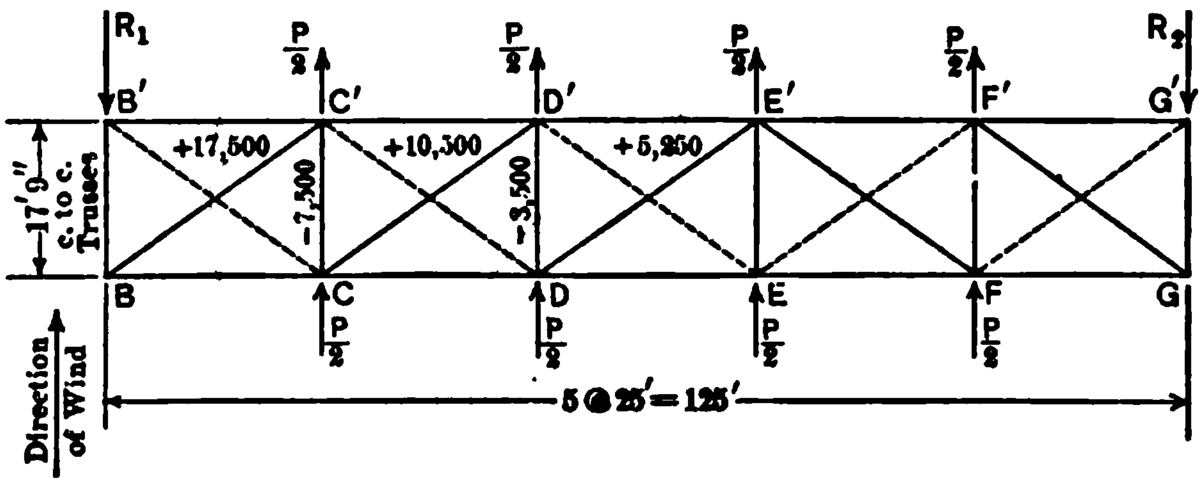


FIG. 4.

signed to take compression, a large area would have to be provided for in order to meet the requirements of the specifications regarding columns. It is therefore usual to assume that these diagonals take tension only. For the direction of wind and position of loads shown in Fig. 4, the full line members are in action. The top chords of the main trusses form also the chords for the lateral system. The chord stresses due to lateral forces are so small compared to those due to vertical loading that lateral stresses are not usually calculated for these members. According to the specifications (Art. 25) wind stresses need not be considered unless they exceed 25 per cent of the stresses due to vertical loading.

The panel load due to lateral forces is $P = 200 \times 25 = 5{,}000$ lbs. It is usually assumed that this panel load is equally divided between the two trusses, as shown in Fig. 4. The stresses in the diagonals and struts are determined by the methods given in Chap. IV, Part I. Considering the wind as a moving uniform load, and using the conventional method of calculation, the stresses are as shown on Fig. 4. The members BB' and GG' are considered as part of the portal bracing. The stresses in these members are calculated in Art. 211.

183. Bottom Lateral Stresses.—The stresses in the bottom laterals are to be determined for a lateral load of 200 lbs. per linear ft. on

the surface of the bridge, and 600 lbs. per linear ft. on the train. These loads are to be considered as moving loads (Art. 10, Specifications).

The effect of these lateral loads on the lateral system can be divided into two parts; lateral truss effect, and overturning effect. To these two effects must be added a third, the portal effect, which is due to the loads brought down at the portal from the top lateral system. The methods of calculation are given in Chap. VI, Part I. Fig. 5 shows the arrangement of members in the bottom lateral system of the truss under consideration.

(A) *Lateral Truss Effect.*—The stresses in the members of the bottom lateral system are due to a total lateral force of 800 lbs. per

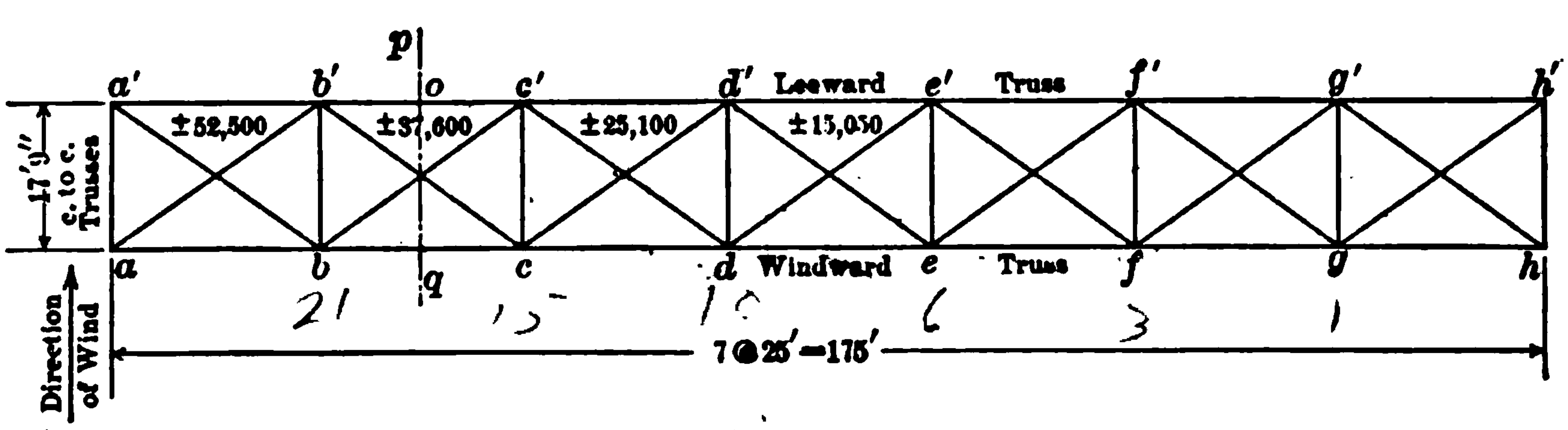


FIG. 5.

lin. ft., giving panel loads of $800 \times 25 = 20{,}000$ lbs. Since the lower part of the leeward truss is completely sheltered by the floor system, it will be assumed that the entire joint load is applied at the joint of the windward truss. The cross struts are formed by the floorbeams which are large, heavy members. As the stresses in these members are small they may be neglected.

The stresses in the diagonals will be calculated on the assumption that both members in any panel act at the same time. Since the diagonals are rigidly fastened at their intersection point, the unsupported length of the members is such that they may be designed as compression members without using excessive areas. Considering the panel loads as a system of moving loads, each diagonal taking one-half of the shear in the panel in question, we have the stresses as given in Fig. 5.

The chord stresses are calculated for panel loads of 20,000 lbs. at each panel point. As both diagonals in any panel are assumed to be in action, moments must be taken about a section taken through

the intersection of two diagonals, as explained in Art. 183, Part I. Thus for stresses in member *bc* of Fig. 5, point *O* is the required moment centre. The resulting chord stresses are given in Table B.

(B) *Overturning Effect.*—The lateral force of 600 lbs. per ft. is usually considered as applied on the side of the train at a distance of 7 ft. above the top of rail. As this horizontal force is resisted by the bottom laterals, which are located at a distance h, Fig. 6, below the point of application of forces, there results a tendency to produce rotation of the truss about a horizontal axis. This overturning effect is resisted by the vertical trusses. Loads P, shown in Fig. 6, acting downward on the leeward truss and upward on the windward truss, represent the action of the overturning force. Taking moments about point A, Fig. 6, we have $P = 600 \times 25 \times h/b =$

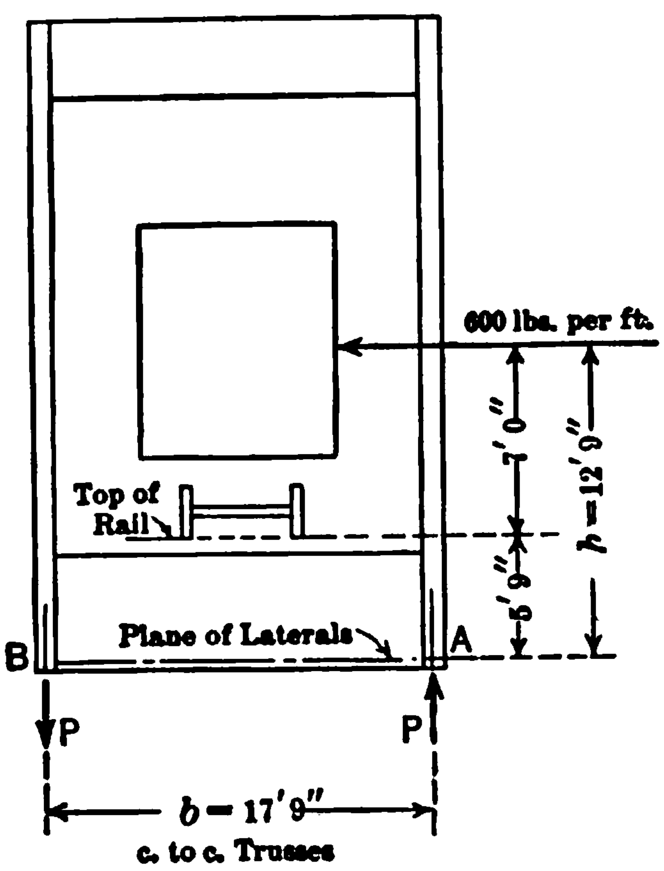


FIG. 6.

15,000 h/b, which is the amount of the vertical panel loads at points A and B. The distance h depends upon the depth from top of rail to plane of laterals, a distance which cannot be definitely determined until the floor-beams have been designed. For this truss we find from Fig. 20 that the distance from top of rail to plane of laterals is about 5 ft. 9 in. Then $h = 7 + 5.75 = 12.75$ ft., and $P = 15{,}000 \times 12.75/17.75 = 10{,}800$ lbs. per panel. In trusses of the general dimensions of the one here considered, the distance h can be taken as 12.5 ft. for preliminary calculation. After the floor-beams have

been designed the correct distance can be determined and the calculations revised if necessary.

Applying panel loads of 10,800 lbs. to the main trusses, the chord stresses are found to be as given in Table B. These stresses can also be determined by proportion from the dead-load stresses given in Table A. The dead panel load is 36,000 lbs. Therefore, overturning stress = dead-load stress × 10,800/36,000 = 0.3 dead-load stress.

(C) *Portal Effect.*—The portal effect is due to the direct stresses set up in the end post by the loads on the top lateral system. As shown in Art. 188, Part I, the portal effect is tension in the bottom chord of the leeward truss and compression in the windward truss. The amount of the portal effect depends upon the form and shape of the portal bracing, which has not yet been determined. The "A" frame type of portal will be used, as shown in Fig. 7. For pre-

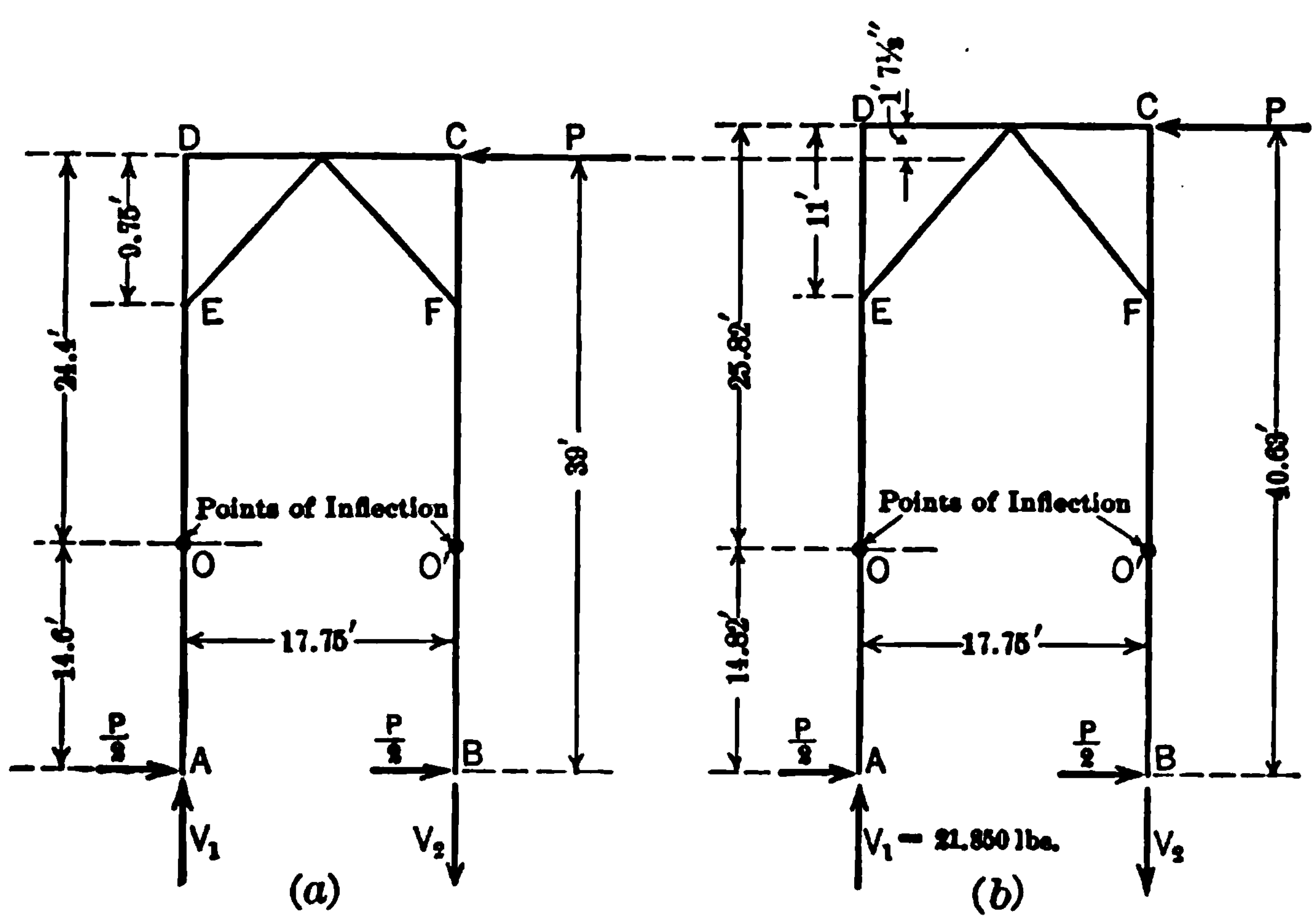


FIG. 7.

liminary calculations it will be sufficiently accurate in this case to consider, in Fig. 7 (*a*), $ED = \frac{1}{4}\,DA$, or 9.75 ft. It will be found that the posts can be considered as fixed at the base (see Art. 211), which locates the point of inflection at point O, half way between points E and A. Then the portal effect $= P \times 24.4/17.75 \sin\theta$, where θ is the angle which the end post makes with the vertical, and P is the load brought to the portal by the top bracing. Including full

joint loads at points B and G, in addition to the loads shown in Fig. 4, we have $P = 3 \times 5{,}000 = 15{,}000$ lbs. Then portal effect $= 15{,}000 \times 24.4/17.75 \times 25/39 = 13{,}200$ lbs.

After the layout of Fig. 29, Art. 211, was completed, it was found that the true dimensions of the portal are as shown in Fig. 7 (*b*). Using these dimensions, the corrected value of the portal effect is found to be 14,000 lbs., the value given in Table B.

(D) *Total Lateral Chord Stresses.*—The total stresses in the bottom chord members due to (A), (B), and (C) are given in Table B.

TABLE B

TOTAL LATERAL TRUSS CHORD STRESSES

Member	Windward Truss				Leeward Truss			
	ab	*bc*	*cd*	*de*	*a'b'*	*b'c'*	*c'd'*	*d'e'*
Lat. Truss Effect.....	−42.2	−112.8	−155.0	−169.0	+42.2	+112.8	+155.0	+169.0
Overturning Effect.....	−27.0	− 27.0	− 45.0	− 54.0	+27.0	+ 27.0	+ 45.0	+ 54.0
Portal Effect..	−14.0	− 14.0	− 14.0	− 14.0	+14.0	+ 14.0	+ 14.0	+ 14.0
Total Stress...	−83.2	−153.8	−214.0	−237.0	+83.2	+153.8	+214.0	+237.0

+ denotes tension; − denotes compression.
All stresses given in thousand-pound units.

Comparing the values given in Table B with those given in Table A for the same members, we find that in members *bc*, *cd*, and *de* the lateral truss stresses exceed 25 per cent of those due to vertical loading. The effect of lateral stresses must, therefore, be considered in determining the sectional areas of those members, according to Art. 25, Specifications. If the preliminary calculations show that the lateral chord stresses are less than 25 per cent of those due to vertical loading, no further attention need be paid to them. But if, as in this case, they must be taken account of in the design of the members, the calculations must be carried further, and true distances determined where preliminary assumed values have been used.

184. Working Stresses.—Before proceeding with the design of the truss members, an abstract will be made of those articles of the specifications relating to working stresses to which frequent reference will need to be made. The numbers refer to articles of the specifications.

Tensile Stresses:

(15) Axial tension on net section 16,000 lbs. per sq. in.

(21) The lengths of riveted tension members in horizontal or inclined positions shall not exceed 200 times their radius of gyration about the horizontal axis. The horizontal projection of the unsupported portion of the member is to be considered as the effective length.

(26) In proportioning tension members the diameter of the rivet holes shall be taken as ⅛ in. larger than the nominal diameter of the rivet.

Compressive Stresses:

(16) Axial compression on gross section of columns—$16{,}000 - 70\,l/r$, with a maximum of 14,000 lbs. per sq. in., where l is the length of the member in inches, and r is the least radius of gyration in inches.

(20) (in part) The lengths of main compression members shall not exceed 100 times their least radius of gyration.

Combined Stresses:

(24) Members subject to both axial and binding stresses shall be proportioned so that the combined fiber stresses will not exceed the allowed axial stress.

(25) For stresses produced by longitudinal and lateral or wind forces combined with those from live and dead loads and centrifugal force, the unit stress may be increased 25 per cent over those given above; but the section shall not be less than required for live and dead loads and centrifugal force.

General Requirements:

(38) The minimum thickness of metal shall be ⅜-in., except for fillers.

(41) The diameter of the rivets in any angle carrying calculated stress shall not exceed one-quarter the width of the leg in which they are driven.

(82) Hip verticals and similar members, and the end two panels of the bottom chords of single track pin-connected trusses shall be rigid.

Rivets.—The rivets will be taken as ⅞ inches in diameter. This is the size usually adopted for railway bridge work. The specifications give the following:

Working Values for Rivets:

(18) Shearing; shop driven rivets, 12,000 lbs. per sq. in.; field driven rivets 10,000 lbs. per sq. in..

(19) Bearing; shop driven rivets, 24,000 lbs. per sq. in.; field driven rivets, 20,000 lbs. per sq. in.

The following table of values for a ⅞ inch rivet will be found useful in the work to follow:

RIVET VALUES

Bearing Values in Pounds per Rivet

Thickness of Plate	3/8″	7/16″	1/2″	9/16″	5/8″	11/16″	3/4″	13/16″
Shop driven...	7,880	9,190	10,500	11,810	13,130	14,440	15,750	17,060
Field driven..	6,560	7,660	8,750	9,840	10,940	12,030	13,130	14,220

RIVET VALUES (*Continued*)

Shearing Value in Pounds per Rivet

	Single Shear	Double Shear
Shop driven	7,220	14,440
Field driven	6,010	12,020

185. Forms of Members.—The best form for the members of a truss is determined by considerations which have been discussed in Chap. VII. In the present article these considerations will be applied to the case of the truss here designed.

(*A*) *Tension Members.*—All bottom chord members, except those of the end two panels (Art. 82, Spec.), and all tension diagonals, will be made up of eye-bars. For a truss of the general dimensions of the one under consideration, four eye-bars are used for the bottom chord members *cd* and *de*, and two bars as used for each tension diagonal, except for the counter *Dc*, where one bar is used. The best relative dimensions of these bars are determined from considerations discussed in Art. 162.

The end two panels of the bottom chord, members *ab* and *be*, will be made in the form of built-up members to comply with Art. 82, Specifications, and for the reasons discussed in Art. 161. Where the stresses are small, these members are usually made up of two rolled channels; where rolled channels do not provide sufficient area, built-up channels are used, consisting of a web plate and two angles, as shown in Fig. 8. The width of the web plates depends upon the size of eye-bar heads of the several members entering joint *c*. Fig. 8 shows that the eye-bar head on *cd* must come inside the angles of the built-up channel. Also, the eye-bar head on *Bc* must not extend below the back of the lower angles on *abc*, in order not to interfere with the lateral plate which is riveted to the lower angles. The size of eye-bar heads at this joint must then be known before the width of web plates of member *abc* can be determined. But the size of eye-bar head cannot be determined until after the pins have been designed. A preliminary width of plates for *abc* must then be assumed, which is to be checked up as soon as the pins are designed. The designer must, therefore, depend upon his experience or judg-

ment in making the preliminary assumptions. Having assumed suitable sizes and proportions, the sizes of eye-bar heads are obtained from the handbooks. Usually at least three sizes of head are given for each bar. The designer is usually safe in assuming that the middle size of head is required for the members at joint *c*. The width of side plates can then be chosen accordingly. An example of this is given in the next article.

According to Art. 82, Specifications, member *Bb*, the hip vertical, must also be a built-up member. It is commonly made up of two

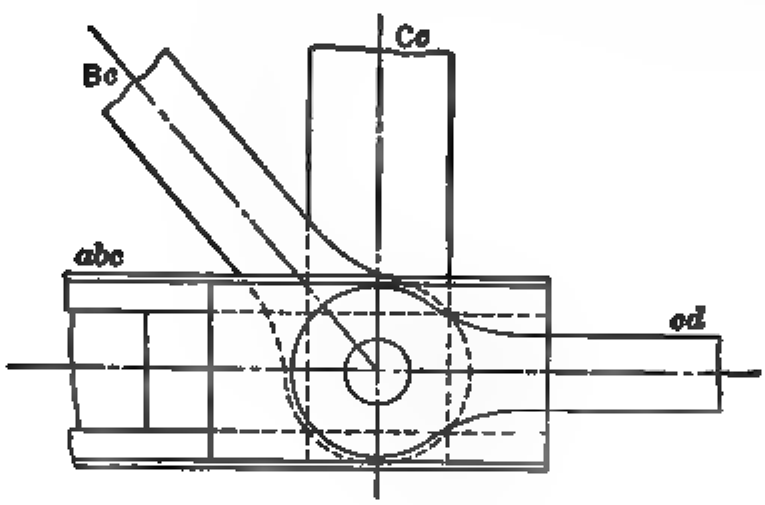


FIG. 8.

rolled or built-up channels where the packing inside the top chord will permit, or of four angles and a web plate made into an I-shaped section where space for packing is limited.

(*B*) *Compression Members.*—The vertical compression members for trusses of the size considered in this chapter can usually be made up of rolled channels laced together to form a rigid member. Some specifications limit the least width of such channels to 10 inches in order to provide a width of member such that a rigid floor-beam connection can be made. Also, the flanges of channels less than 10 inches wide are too narrow to form a satisfactory connection between the two parts of the member.

The top chord and end post sections for small trusses can also be made up of rolled channels. But for trusses of the span under consideration, it is usually necessary to use built-up sections. The chord section must be made wide enough to allow the posts and diagonals to be packed inside, and must be made relatively deep

so that the pin-centre may be located far enough from the top cover plate to allow room for the eye-bar head and still keep the pin-centre reasonably close to the neutral axis of the section. On the other hand, the chords must not be made so deep that secondary stresses become an important factor. It will usually be found that in trusses of this size, if the chords are made just deep enough to take in the eye-bar on member *Bc* at joint *B* the other requirements will be met and sections of about the minimum size will provide sufficient area for the end member *BC*. The conditions at joint *B* are shown in Fig. 9. As the size of eye-bar head governs the depth of the chord member, we must proceed as for member *abc* described in Art. 185(A).

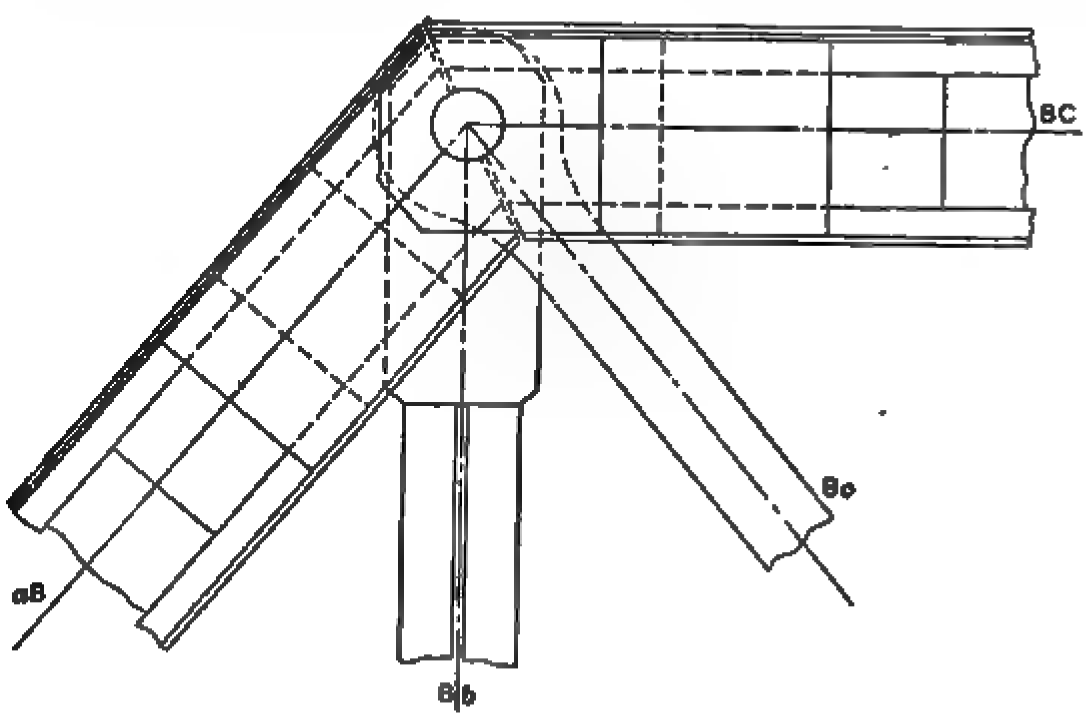


FIG. 9.

The pin at joint *B* is usually the largest in the truss. In assuming the size of eye-bar head, it is usually best to make provision for the largest size head given in the handbooks for bar *Bc*. The depth of web plate should be made a little greater than required in order to clear the eye-bar head. As such plates are usually made with rolled edges, it is best to use plates of full inches in width, avoiding fractions of an inch.

Compression chord members for trusses of the size discussed in this chapter are usually made up of a cover plate, four angles, and two web plates, with lacing across the open side of the member. Two

forms in common use are shown in Fig. 10. The conditions governing the depth of web plates have been discussed above. The distance D of Fig. 10 is usually made about ½ in. greater than the depth of

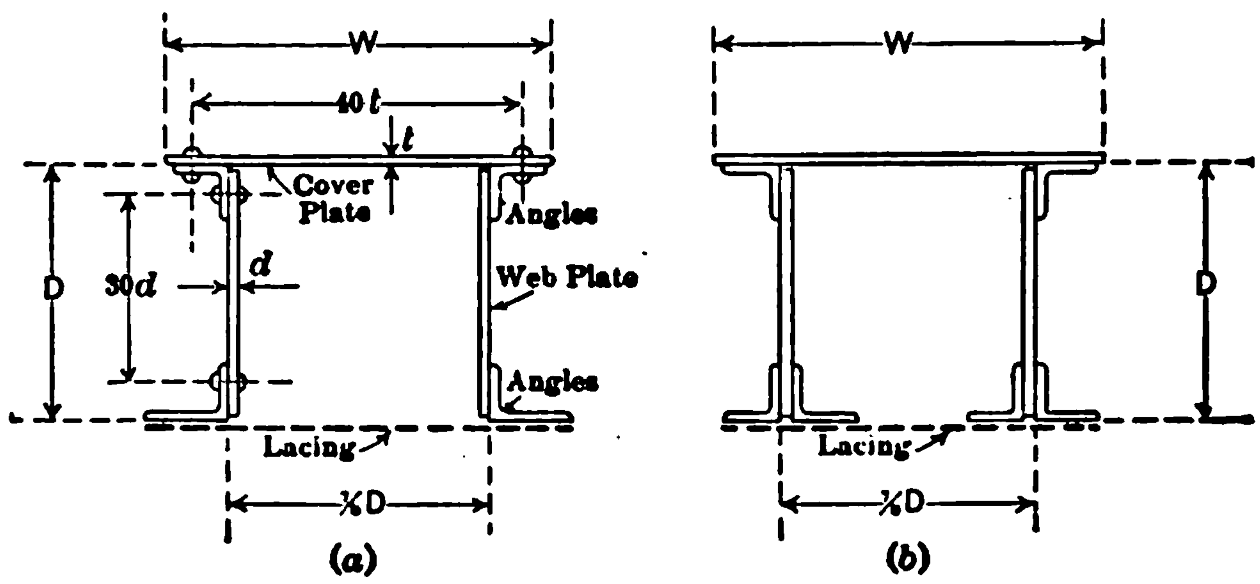


FIG. 10.

web plates. The least thickness of these plates is governed by Art. 44, Specifications:

(44) In compression members the metal shall be concentrated as much as possible in webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between its connections to the flanges. Cover plates shall have a thickness not less than one-fortieth of the distance between rivet lines.

The distance between the outside faces of the web plates is usually made about ⅞ D, as shown on Fig. 10. The width of cover plate is made such that it will project about ¼ in. beyond the top angles. As in the case of webs, plates of full inches in width should be used if possible. As specified in Art. 44, cover plates must have a thickness at least 1/40 of the distance between rivet lines. In determining the distance between rivet lines, the gage lines of the angles should be placed as close to the edges of the angles as possible in order to allow room for the rivet heads on the pin plates.

The size of angles to be used depends upon the size of the other parts. For the smaller members, with web plates from 18 to 20 ins. deep, angles with 3½-in. legs can be used for the top angles. For webs deeper than these, angles with 4-in. legs should be used. The thickness of the angles should be not less than about ¾ of the thickness of the cover plate. Such angles will be found large enough to provide an adequate connection between the cover and web plates of the section.

The lower angles in the form of member shown in Fig. 10 (*a*) are usually made larger and thicker than the top angles. This is done to provide area at the bottom of the member which will serve to partly balance the area of the cover plate and thus bring the centre of gravity of the section closer to the centre line of the web plate, as called for by Art. 35, Specifications. When 3½ in. angles are used as top angles, use 3½ × 5-in. angles at the bottom, with the 3½-in. leg against the web. Angles 6 × 4 in. are to be used with 4-in. top angles. The thickness of these angles can be made about the same as that of the cover plate. In Fig. 10 (*b*) four equal-leg angles are shown at the bottom of the web plates. At joints the inside angles are sometimes wholly cut off to allow for packing the web members inside and close to the web plates.

The general requirements given above are for the minimum sizes of material. If in any case more area is required than given by this minimum section, it can be provided by increasing the thickness of the parts, and by the use of additional web plates as described in Art. 162. For appearance sake and for convenience of detail, the width of the cover plate and depth of web plates should be kept constant for all members.

186. Design of Tension Members.—The design of the members of the truss in question, subject to the general considerations given in the preceding article, will be taken up in detail. Table C contains, in convenient form, the stresses in the various members and other data required for the determination of the sectional areas of the members.

The areas of the bottom chord tension members must be determined for two conditions of loading, the larger area to govern the design of the members. These cases are: Case (*a*), dead, live, and impact stresses, as given in Table A, at 16,000 lbs. per sq. in.; and Case (*b*), dead, live, and impact stresses from Table A, together with the total lateral chord stress for the member given in Table B, the unit stress to be 20,000 lbs. per sq. in. (Art. 25, Specifications.) It is to be noted that, for this truss, case (*b*) governs the design of all lower chord members, except *ab*. Since, at this stage in the design, the distances given on Figs. 6 and 7 are usually preliminary values, the areas determined in Table C are also preliminary areas, and must be revised later.

The eye-bars for the bottom chord members *cd* and *de* will be made 8 inches wide. This will allow a uniform width of bar for

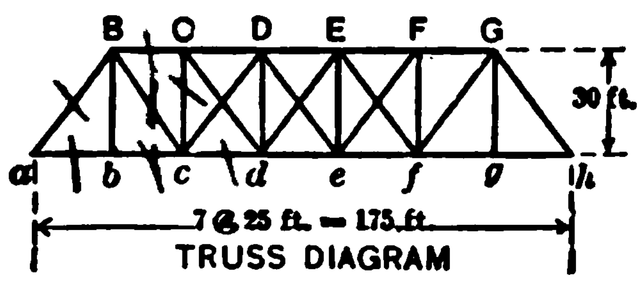


TABLE C

DESIGN OF MEMBERS

DESIGN OF TENSION MEMBERS

Member	Case A Dead, Live, and Impact Stress	Case B Dead, Live, Impact, and Lateral Stress	Area Required (Sq. ins.) Case A at 16,000 lbs. in.²	Area Required (Sq. ins.) Case B at 20,000 lbs. in.²	Section	Area Provided (sq. ins.)
ab.......	+477.4	+ 560.6	29.85	28.00	4 Ls 4×4×7/16 in.	38.24 gr.
bc.......	+477.4	+ 631.2	29.85	31.56	2 Pls 20× 5/8 in.	31.49 net
cd.......	+772.0	+ 986.0	48.30	49.30	4 Bars 8×1 9/16 in.	50.0
de.......	+918.2	+1155.2	57.40	57.76	4 Bars 8×1 13/16 in.	58.0
Bc.......	+540.9	....	33.80	.....	2 Bars 9×1 7/8 in.	33.80
Cd.......	+352.4	....	22.10	.....	2 Bars 7×1 5/8 in.	22.75
De.......	+181.9	....	11.35	.....	2 Bars 6×1 in.	12.00
Dc.......	+ 58.4	....	3.65	.....	1 Bar 6×1 in.	6.00
Bb.......	+234.9	....	14.70	.....	4 Ls 6×4×7/16 in.	21.09 gr.
					1 Pl 10×7/16 in.	16.71 net

DESIGN OF COMPRESSION MEMBERS

Member	Dead, Live, and Impact Stress	l (in.)	r (in.)	$\frac{l}{r}$	Unit Stress (lbs. in.²)	Area Req'red (sq. ins.)	Section	Area Prov. (sq. ins.)
Cc.......	−283.1	360	5.32	67.7	11,260	25.10	2[s 15" at 45 lbs.	26.48
Dd.......	−151.9	360	5.62	64.0	11,520	13.15	2[15" at 33 lbs.	19.80
aB.......	−748.4	469	8.99	52.2	12,350	60.6	1 Cov.Pl.28×5/8 in. 2Ls 4×4×7/16 in. 2WebPls22×9/16in. 2Ls 6×4×5/8 in.	60.60
BC.......	−772.0	300	8.99	33.4	13,660	56.6	1Cov.Pl28×5/8 in. 2Ls 4×4×7/16 in. 2WebPls22×9/16in. 2Ls 6×4×5/8 in.	60.60
CDE.......	−927.0	300	8.74	34.35	13,600	68.2	1Cov.Pl28×5/8 in. 2Ls 4×4×7/16 in. 2 WebPls22×3/4 in. 2 Ls 6×4×5/8 in.	68.84

+ denotes tension; − denotes compression.
Stresses given in thousands of pounds.

these members, and material less than two inches thick can be used. For member *Bc*, where two bars are used, it will be necessary to

use bars 9 inches wide in order to keep the thickness less than 2 inches. Some designers use four bars at this place, two of the bars being placed outside the top chord section. This arrangement will usually be found necessary only where the use of two bars for *Bc* requires eye-bar heads so large that excessively large top chord sections must be used in order to allow for packing of bars at joint *B*. Seven-inch bars for *Cd* and 6-inch bars for *De* and *Dc* will be found to answer all requirements.

The members *ab* and *bc* will be made up as one continuous member consisting of two built-up channels. The area required is determined for case (*b*) and is given in Table C as 31.56 sq. ins. net. As stated in the preceding article, the width of side plates for this member is determined by the eye-bar heads entering joint *c*. From the table of eye-bar heads in Appendix B we find three different heads given for the 8-in. bars of member *cd* and two heads for the 9-in. bars of member *Bc*. As a trial section we will provide for the middle size head of an 8-in. bar, which is 19 ins. in diameter, and for the smaller head for a 9-in. bar, which is 20 ins. in diameter. To accommodate these heads we will use 20-in. web plates. Referring to Fig. 11, which shows the details of this joint, it can be seen that the weakest section in the member is on the line *p–q*. The required net area must then be provided at this section. Using two plates, $20 \times \frac{5}{8}$ in., and four angles, $4 \times 4 \times \frac{7}{16}$ in., deducting four rivet holes from each plate, and one hole from each angle, subject to the conditions of Art. 26, Specifications, we have a net area of 31.49 sq. ins.

Member *Bb*, the hip vertical, will be made up of four angles and a plate, forming the section shown in Fig. 12. The area required is 14.7 sq. ins. For this member the weakest section is near the upper end, where the pin plate connections are made. Two rivet holes are to be deducted from the plate, and two from each angle as shown in Fig. 54. The width of the plate cannot be determined at once, as it depends upon the packing of members at joint *B*. A width which subsequent calculation (Art. 215) shows to be sufficient is 10 inches. Assuming four angles $6 \times 4 \times \frac{7}{16}$ in., and one plate $10 \times \frac{7}{16}$ in., with the rivet allowance stated above, the net area is 16.71 sq. ins. This area is a little larger than required, but this excess is

desirable, since it reduces the elongation of member Bb under stress. As shown in Chap. VII, Part II, the elongation of member Bb causes point b of member abc to move out of line, thereby causing large secondary stresses in the member. Therefore a liberal use of material in member Bb will reduce the secondary stresses in abc.

187. Design of Compression Members.—The allowable working stress for members in compression is given by the formula of Art. 16,

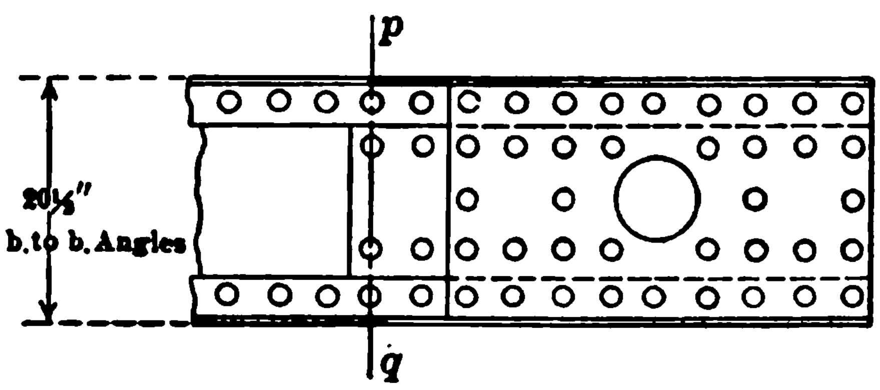


FIG. 11.

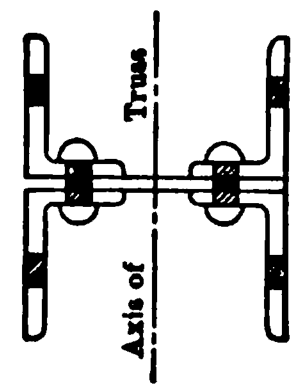


FIG. 12.

Specifications. Since this formula involves the radius of gyration of the section, we must use "cut and try" methods in the design of such members. From Art. 54 it is seen that the radius of gyration of top chord sections, such as used in this design, is approximately $4/10$ of the depth, and of sections made up of channels is approximately $3/8$ of the depth. Using these values of the radius of gyration, a preliminary section can be made up, its true radius of gyration determined, and the area provided checked against that required. This process will be followed out for the several members.

188. Vertical Posts.—The vertical posts Cc and Dd will be made up of rolled channels. Member Cc has a compressive stress of 283,100 lbs. and is 30 ft. long. Assuming the member to be made up of 15 in. channels, we find, by the rule given above, that the radius of gyration is approximately 5.6 ins. The allowable working stress is therefore $16{,}000 - 70 \times \frac{30 \times 12}{5.6} = 11{,}500$ lbs. per sq. in., which requires an area of $283{,}100/11{,}500 = 24.6$ sq. ins. Referring to the handbooks, we find that two 15-in. 45-lb. channels have an area of 26.48 sq. ins., and a radius of gyration for an axis perpendicular to the 15-in. face of 5.32 ins. The true working stress is then 11,260 lbs. per sq. in. and the required area is 25.1 sq. ins. The 15-in.

45-lb. channels will therefore be adopted. The channels will be placed with the flanges turned inward and the webs parallel to the axis of the truss, as shown in Fig. 13 (*a*).

It has been assumed that the radius of gyration about an axis perpendicular to the web is the least. The channels must be so spaced that this condition will be satisfied. In Fig. 13 (*a*), let x be the distance from the axis of the truss to the gravity axis a–a of the channel parallel to the web. For equal moments of inertia about axes B–B and A–A we have $I_B = I_a + A\,x^2$, where I_B = moment of inertia about axis $B-B$, I_a = moment of inertia of channels about gravity axis a–a, and A = area of channels. In terms of radii of gyration, we derive from the above expression, $r_B^2 = r_a^2 + x^2$, hence for equal moments of inertia or radii of gyration we have, in general,

$$x = \sqrt{r_B^2 - r_a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which r_B = radius of gyration of channel about axis B–B and r_a = radius of gyration about axis a–a. Both these values are given directly in the handbooks. In the present case we have $x = \sqrt{5.32^2 - .88^2} = 5.25$ ins. The distance back to back of channels must then be at least $2 \times (5.25 + 0.79) = 12.08$ ins.

This spacing of channels would be required if the post was as free to deflect laterally as it is in a plane parallel to the truss. From Fig. 13 (*b*) it is seen that the posts are rigidly supported by the floor-beam at the bottom and by the sway bracing at the top. The unsupported length, laterally, is about 20 feet. The spacing of channels then need only be great enough to make the values of l/r equal for axes A–A and B–B of Fig. 13 (*a*). If l_A and l_B be the unsupported lengths for axes A–A and B–B respectively, and r_A and r_B the corresponding radii of gyration, we have, for equal stiffness in the two planes, $l_A/r_A = l_B/r_B$, or $r_A = r_B\, l_A\,/l_B$. As shown in the preceding analysis, $r_A^2 = r_a^2 + x^2$, hence we have $x^2 + r_a^2 = r_B^2\, l_A^2/\,l_B^2$, and solving for x

$$x = \sqrt{r_B^2\left(\frac{l_A}{l_B}\right)^2 - r_a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

With l_A = 20 ft. and l_B = 30 ft. we find x = 3.44 ins. The required distance back to back of channels will be $2 \times (3.44 + 0.79) = 8.46$ ins.

The distances calculated above give the spacing of channels as governed by theoretical conditions. As the open sides of the member are to be held together by lacing bars, the flanges of the channels must not be placed so close to each other that they will interfere with the riveting machine. A clear space of at least 5½ ins. must be provided. The face of a 15-in. 45-lb. channel is 3.62 ins., so that the channels must be placed at least 12¾ ins. back to back. As this

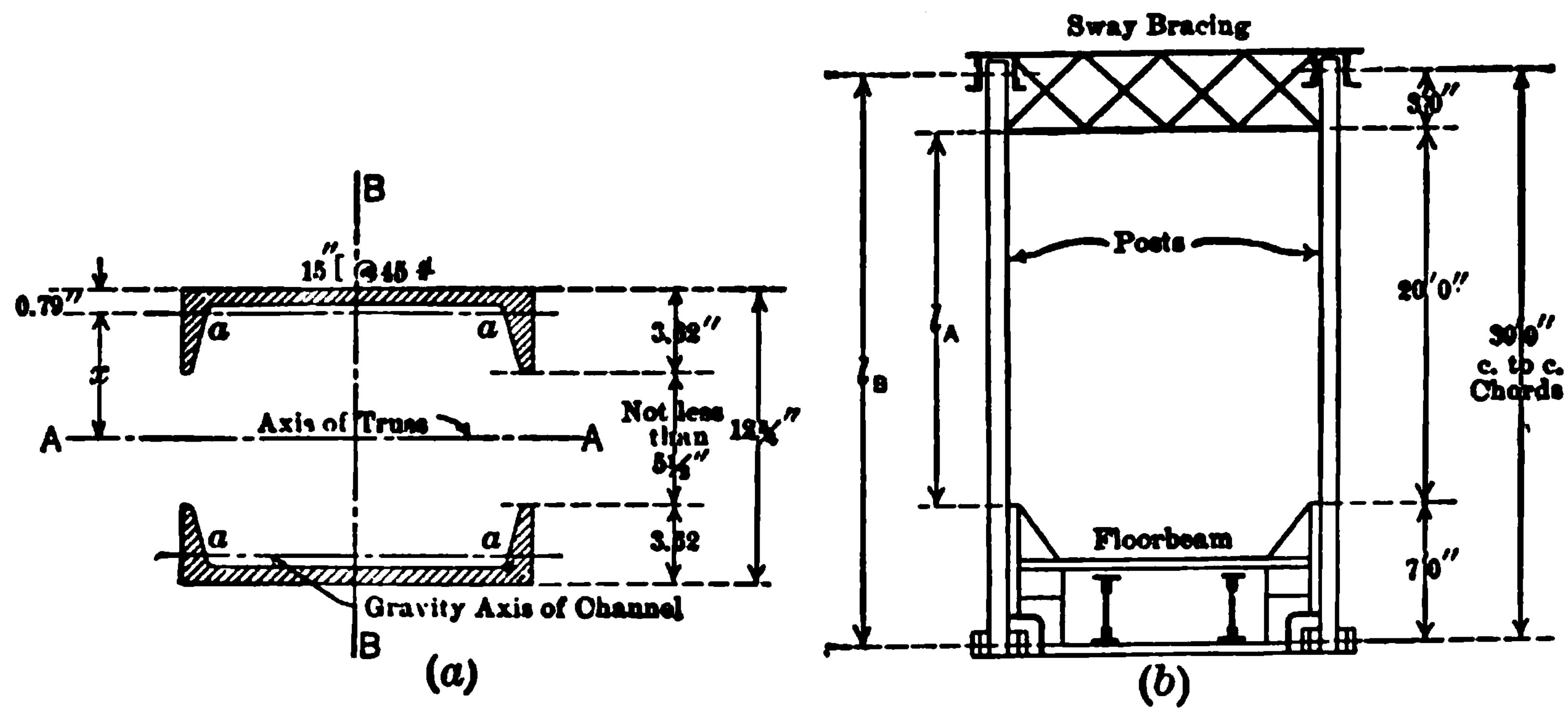


FIG. 13.

spacing of channels is greater than either of the values calculated above, it will be used later in determining the spacing to be used.

The design of member *Dd* is shown in Table C. It will be found that a 12-in. 30-lb. channel will answer for this member. A 15-in. channel is used, however, because it gives a wider face for the floorbeam connection.

189. Top Chord and End Posts.—The top chord sections will be made up to fit conditions at joint *B*, as stated in Art. 185 (*B*). We will begin with *BC*, the member having the least stress. The eye-bar *Bc* is 9 inches wide. From the table of eye-bar heads in Appendix *B* we find that the largest head for this bar is 22 inches wide. The web plates on *BC* will therefore be assumed 22 inches wide. Using the form of member and general proportions of parts shown in Fig. 10 (*a*) we arrive at a preliminary section as shown in Fig. 14. This is made up of the minimum widths and thicknesses of plates

allowed by the specifications and the general considerations given in Art. 185 (*B*). The area of this minimum section is 60.60 sq. in., as shown in Table D.

The maximum stress in member *BC* is 772,000 lbs. compression, as given in Table A. An approximate estimate of the required area can be determined by taking the radius of gyration equal to 0.4 of the depth of member, or $0.4 \times 22 = 8.8$ ins. The length of the member is 25 feet. From the column formula, $p = 13{,}620$ lbs. per sq. in. The area required is then approximately $772{,}000/13{,}620 = 56.7$ sq. ins. This area is somewhat less than the area of the section as made up of minimum size parts, but in order to answer the requirements of the specifications, it will be necessary to use excess area in this case.

The properties of the assumed section are shown in Fig. 14 and in Table D. The dimensions of the section and the location of the gravity axis of the several parts, referred to the centre line of web plates as an axis of reference, are shown in Fig. 14.

TABLE D

PROPERTIES OF MEMBER *BC*

Section	Area, Sq. Ins.	Arm (Ins.)	1st Moment	2nd Moment
Cover Plate, 28″ × 5/8″	17.50	11.56	+202	2,340
Top Angles, 4″ × 4″ x 7/16″	6.62	10.09	+ 67	672 10*
Web Plate, 22″ × 9/16″	24.76	.....	......	998
Bottom Angles, 4″ × 6″ x 5/8″	11.72	10.22	−120	1,228 15*
Totals	60.60	.....	+149	5,263

* Moment of inertia of angles about their gravity axes.

In Table D are given the necessary calculations for position of the gravity axis of the section and its moment of inertia. The "first moment" is used in the determination of e, the distance from the centre line of web plates to the gravity axis of the section, sometimes called the "eccentricity" of the section. From Table D, we have $e = 149/60.6 = 2.46$ ins. above the centre of web plates. The "second moment" is the moment of inertia of the section about axis $A-A$. The moment of inertia about the gravity axis is $I = I_A - Ae^2 = 5{,}263 - 60.6 \times 2.46^2 = 4{,}896$ in.^{4}

The correct value of the radius of gyration is $r = \sqrt{I/A} = \sqrt{4845/60.6} = 8.99$ ins. With this value of r, the correct working stress is found to be 13,660 lbs. per sq. in., and the area required is 56.6 sq. in.

The moment of inertia of the section about axis B–B, Fig. 14, must also be calculated in order to make certain that the radius of

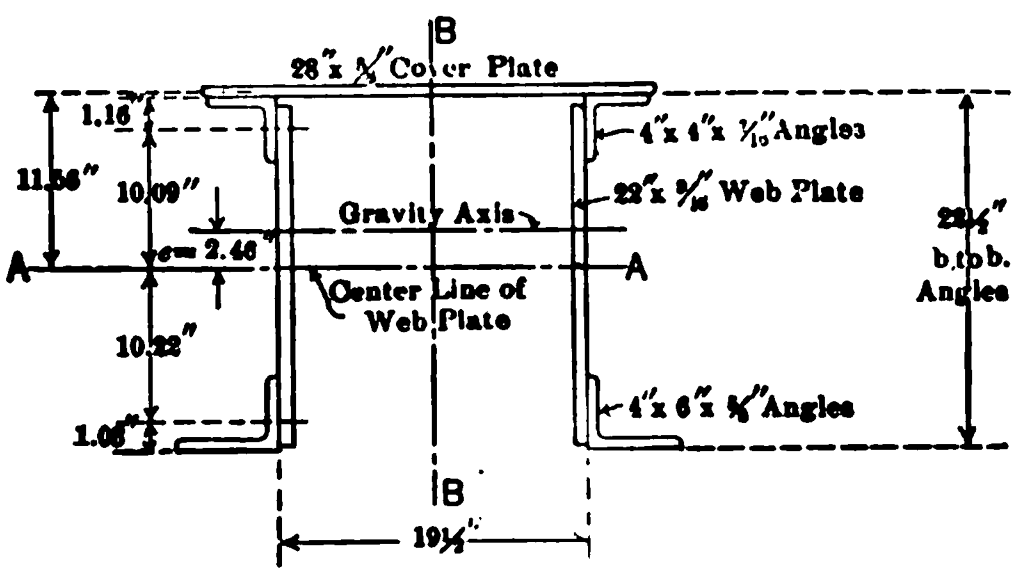


FIG. 14.

gyration determined for axis A–A is the least radius of gyration of the section. By a calculation similar to that given in Table D, the moment of inertia about the axis B–B is found to be 5,825 in.[4] As this moment of inertia is greater than that for axis A–A, we see that the proper value of r has been used in the preceding calculations.

The necessary data for the design of the top-chord member CDE, and the end post aB are given in Table C. Member CDE is made the same as BC, except that 3/4-in. web plates are used. The moment of inertia about the gravity axis is 5230 in.[4], and the corresponding radius of gyration is 8.74 in. The eccentricity of the section is 2.31 in. Member aB is made the same as BC. The greater length of the member reduces the working stress so that the required area agrees very closely with the minimum section of Fig. 14.

190. Bending Stresses in Top Chord and End Post.—In Chap. IV, Art. 64, it was shown that the top chord and end post of a pin-connected bridge are subjected to bending stresses of a secondary nature due to three principal causes: (1) the weight of the members, (2) the effect of pin eccentricity, if any, and (3) the effect of the deflection of the truss as a whole, this causing a certain deflection and bending moment in the top chord. The last mentioned effect can-

not be controlled, except, to a small degree, by varying the depth of the chord. In the truss here considered it will not amount to more than about 10 per cent and need not be further considered. The effects of weight and pin eccentricity are more readily controlled and should be limited, if practicable, to 15 per cent as a maximum, so that the total secondary stress will not exceed about 25 per cent. In considering the end post the direct effect of portal stress must also be taken into account. The question of necessary eccentricity of pin will first be examined.

191. *The Pin Eccentricity.*—The calculations of the positions of gravity axes of the compression members designed in the previous articles show that these axes are located somewhat above the centres of the web plates. The assumed eye-bar head on member *Bc* at joint *B* makes it necessary to place the pins on the centre line of the web plates, since the maximum eye-bar is of the same width as the web plate of *BC*. Considering joint *B* of the truss in question, it was found in Art. 189 that the eccentricity of member *BC* is 2.625 ins. The stress in *BC* is 772,000 lbs., which gives a moment of $772,000 \times 2.46 = 1,899,000$ in.-lbs. This moment produces a fibre stress of $1,899,000 \times 13.71/4,896 = 5,320$ lbs. per sq. in. on the compression (lower) side of the member. From Table C the axial stress in the member is $772,000/60.6 = 12,720$ lbs. per sq. in. The secondary stress due to eccentricity is then $5,320/12,720 = 41.8$ per cent of the primary stress, which is relatively high and should be reduced if possible.

The stresses due to eccentricity can be reduced by increasing the size of member *BC*, or by reducing its eccentricity. As the section adopted already contains excess area, it will be best to reduce the eccentricity. The table of eye-bars in Appendix B shows that the per cent of excess area provided in the head of the bar over that in the body of the bar is not less than 35 per cent. If the pin at joint *B* can be placed in a smaller head than that assumed, and still provide the proper excess area, it will be possible to raise the pin centre and thus reduce the eccentricity. Subsequent calculations (Art. 216) shows that a 7¾-in. pin is required at joint *B*. The excess area provided by a 20-in. head on a 9-in. bar is $\dfrac{(20 - 7\frac{3}{4}) - 9}{9} = 36.1$ per cent. For this case we can then make use of the 20-in. head

instead of the 22-inch head assumed in the preliminary calculations. This change will allow the pin centre to be placed one inch above the centre line of the web plate, thereby reducing the eccentricity to 1.46 ins. Calculations similar to those given above show that the bending moment at joint B is 1,128,000 in.-lbs., and that the resulting fibre stress is 3,160 lbs. per sq. in. This stress is 24.8 per cent of the axial stress, and the total compressive stress is 15,880 lbs. per sq. in., a value which may be permitted at the end of the member. Assuming the pin placed one inch above the centre line of the webs, we will now determine approximately the resulting effect in the entire top chord and end post.

192. *The Top Chord.*—The top chord of this truss will be made continuous from hip joint to hip joint by means of riveted chord splices. Under these conditions the entire chord becomes a continuous girder acted upon by vertical forces due to the weight of the member, and by eccentric moments such as that determined above for joint B. Similar eccentric moments are also set up at interior joints where there is a difference of chord stress, or a change in the position of the gravity axis of the chord section.

The combined effect of weight of members and eccentric moments is to be determined for the points in the members at which the buckling effect due to column action is a maximum. It may be assumed that the ⅜ point of the end segments of the chord, and the centre points of the interior chord members, are such critical points. The values of the moments at these points have been calculated in Art. 65 of Chap. III. For a 7-panel truss the formulas are as follows:

Member	Effect of Weight of Member	Effect of Eccentric Moments
BC	*$M_{\frac{3}{8}} = .072\ wl^2$	$M_{\frac{3}{8}} = -0.50\ Se$
CD	$M_c = .045\ wl^2$	$M_c = +0.06\ Se$
DE	$M_c = .045\ wl^2$	$M_c = -0.03\ Se$

* It is here assumed that the moment at the ⅜ point is the same as at the centre. It is generally slightly greater, being about .077 wl^2.

In these formulas w is the weight of the member in lbs. per ft.; l is the panel length; S is the stress in BC, and e is its eccentricity

at joint B. The product Se has already been computed above, and is $772{,}000 \times 1.46 = 1{,}128{,}000$ in.-lbs. A close estimate of the weight of the members can be obtained as soon as their area is known. Assuming steel to weigh 480 lbs. per cu. ft., the weight of the main material in member BC, which has an area of 60.6 sq. ins., is $60.6/144 \times 480 = 202$ lbs. per ft. The weight of details such as tie plates, lacing, etc., can be assumed to be about $\frac{1}{3}$ of the weight of the main material. This gives the weight of BC as $4/3 \times 202 = 270$ lbs. per ft. In the same way, CD and DE are found to weigh 305 lbs. per ft. Substituting these values in the above formulas, the moments at the several points are found to be as follows:

Member	Point	Effect of Weight	Effect of Eccentric Mom.	Total
		Inch-Pounds	Inch-Pounds	Inch-Pounds
BC	$M_{\frac{3}{8}}$	+146,000	−564,000	−418,000
CD	M_c	+103,000	+ 67,700	+170,700
DE	M_c	+103,000	− 33,850	+ 69,150

The resulting fibre stresses are to be determined for the compression side of the member, which is the bottom of the member for negative moments, and the top of the member for positive moments. From Fig. 14 it will be found that for member BC, the distance from the gravity axis to the compression fibre is 13.71 in. and the moment of inertia of the member is 4,896 in.^{4} For members CD and DE, the corresponding quantities are 9.565 ins. and 5,230 in.^{4} Substituting these values in the general formula for flexure, the fibre stresses for the several members are found to be as follows: BC, 1,170 lbs. per sq. in.; CD, 312 lbs. per sq. in.; and DE, 127 lbs. per sq. in. From Table C, the axial stresses in the members are: BC, 12,720 lbs. per sq. in.; CD and DE, 13,480 lbs. per sq. in. The secondary stresses found above are therefore a small percentage of the axial stress in the member, being about 9.2 per cent for BC, and 2.4 per cent and 0.9 per cent respectively for CD and DE, which are well within the permissible limits.

193. *The End Post.*—The end post is to be treated as an inclined simple beam between points a and B. This beam is acted upon by the weight of the member and by eccentric moments at joints a and B.

In addition to these forces, the end post is also subjected to bending in the plane of the portal due to the lateral forces brought down from the top lateral system. Considering first the effect of weight of member and eccentric moments, it will be found that the critical points are at the ends and centre of the member. The eccentricity of the section is 1.46 ins. (same as BC) and the maximum stress in the member is 748,400 lbs. compression. A negative moment of 1,093,000 in.-lbs. therefore occurs at joints a and B, causing compression on the lower fibres of the section. Due to the presence of pins at the joints a and B, the moments at these points due to weight of member are zero. The properties of the section are the same as previously given for BC, and the resulting fibre stress due to eccentric moment is found to be 3,060 lbs. per sq. in. The axial stress in the member is 748,400/60.6 = 12,350 lbs. per sq. in., and the total fibre stress at the ends is 12,350 + 3,060 = 15,410 lbs. per sq. in. The bending stress is thus 24.8 per cent of the axial stress in the member. These values are within allowable limits. At the ends of the member, where no buckling due to column action exists, the allowable fibre stress may be considered to be 16,000 lbs. per sq. in.

At the centre of the end post the effect of weight of member is a maximum. In determining the moment due to weight, the entire weight of the member can be considered as uniformly distributed over a beam whose length is equal to the horizontal projection of the end post. Expressed as a formula this reads $M = \left(\frac{wl}{d}\right)\frac{d^2}{8}$, where l = length of end post = 39 ft.; d = panel length = 25 ft.; and w = weight of member per ft., which is 270 lbs. per ft., as calculated for BC. Substituting in this formula, the centre moment is found to be a positive moment of 395,000 in.-lbs. Since the effect of eccentric moments is constant over the entire length of the member, the combined centre moment is + 395,000 − 1,093,000 = − 698,000 in.-lbs. The resulting fibre stress is found to be 1,955 lbs. per sq. in., and the sum of the axial and bending stresses is 12,350 + 1,955 = 14,305 lbs. per sq. in. In this case the bending stress is 15.8 per cent of the axial stress which is within allowable limits.

The stresses in the end post due to the lateral forces brought

down by the portal can be found from the bending moment and direct stress calculated in Art. 211. The lateral bending moment, as shown by Fig. 31, is a maximum at the base of the post and at the foot of the portal, where its value is 111,200 ft.-lbs., and zero near the centre. The direct stress is 21,850 lbs.

In calculating fibre stresses due to these moments, the moment of inertia of the section is to be taken about an axis corresponding to B–B of Fig. 14, p. 281. Since in this case the end post has the same cross-section as BC, this moment of inertia is 5,825 in.^{4}, as calculated in Art. 189. For the most remote fibre (the outside edge of the angle), which is 15¾ ins. from the neutral axis, the fibre stress at the lower end and at the base of the portal is found to be 3,610 lbs. per sq. in. The fibre stress due to direct compression is 21,850/60.6 = 360 lbs. per sq. in.

At the lower end of the post, therefore, the total combined fibre stress due to direct stress, eccentric moments, and lateral forces is 12,350 + 3,060 + 3,610 + 360 = 19,380 lbs. per sq. in.

Since this fibre stress is due to combined direct and lateral loads, Art. 25, Specifications, fixes the allowable stress at 20,000 lbs. per sq. in. The combined fibre stress at the end of the end post is, therefore, within the allowable limit. It is to be noted that the fibre stress of 3,610 lb. per sq. in., calculated above, occurs only at the extreme edge of the member. At the face of the web plate this bending stress is about 2,220 lbs. per sq. in. Also, the end of the member is reinforced by heavy pin plates, as shown in Fig. 47. These plates reduce the stresses calculated above. Even though these plates have not been taken into account, the fibre stresses are within allowable limits, and secondary stresses have been quite fully allowed for.

Near the centre of the end post, where the buckling tendency is a maximum, the moment diagram of Fig. 31 shows that the bending effect due to lateral forces is small and may be neglected. As the direct stress due to portal effect is small the total stresses in this part of the post are well within allowable limits.

At the foot of the knee brace of the portal frame the bending moment due to lateral forces is equal to that at the foot of the end post, as shown in Fig. 31. At this point the moment due to weight is not zero, as was the case at the top and bottom of the post. The

bending moment at the knee brace due to weight of member is given by the formula $M = \frac{wl}{2d}(l_1 \times l_2)\ cos^2\theta$, where w, l, and d are the same as given above; l_1 and l_2 are the two segments into which the end post is divided by the knee brace, 11 and 29.64 ft., as shown in Fig. 31; and θ is the angle which the end post makes with the horizontal.

Then $M = \frac{270 \times 39^{\cdot}}{2 \times 25}(11 \times 29.64) \times \left(\frac{25}{39}\right)^2 \times 12 = 340{,}000$ in.-lbs.

The combined moment due to eccentric moment and weight of member is 1,093,000 − 340,000 = 753,000 in.-lbs., giving a fibre stress on the compression side of 2,110 lbs. per sq. in. Adding to this the direct stress of 12,350 lbs. per sq. in., 3,610 lbs. per sq. in. due to lateral bending, and 360 lbs. per sq. in. due to direct stress, the total fibre stress is 18,430 lbs. per sq. in., which is well within the allowable stress of 20,000 lbs. per sq. in. specified by Art. 25, Specifications.

194. Design of the Stringers.—The stringers are simple plate girders equal in length to a panel of the truss under consideration. As the specifications and general considerations governing the design of stringers are the same as for the plate girder designed in Chap. VI, they will not be repeated in detail here. The computations will be given in brief.

Loads and Stresses:

Assumed weight of Stringers (See Art. 9) = 1.1(12.5 × 25 + 100).........	=	454 lbs. per ft.
Weight of floor (See Art. 176).........	=	523 " " "
Total dead load...........	=	977 lbs. per ft.
Dead load per foot per stringer taken..	=	490 " " "

Maximum Moment:

Dead load centre moment............	=	460,000 in. lbs.
Live load moment, *E*60. (See Art 169, Pt. I).........................	=	5,490,000 " "
Impact moment (l = 25 ft.)..........	=	5,067,000 " "
Total moment.................	=	11,017,000 in. lbs.

End Shear:

Dead Load . . . . = 6,130 lbs.
Live Load, *E*60. (See Art. 169, Pt. I) . . . . . = 85,200 "
Impact . . . . = 78,600 "

Total Shear . . . . = 169,930 lbs.

Depth of Stringer and Web Area.—The depth of stringers, for trusses of the size considered in this chapter, is usually taken at about one-sixth of the panel length. For this case a depth of 48¼ ins., back to back of angles, will be used. In order to comply with Art. 128, Specifications, the web plate will be made flush with the top of the top flange angles. At the bottom the angles will project ¼ in. beyond the web plate. A 48-in. web plate will be used. From Art. 18, Specifications, the allowable shear on the gross section of the web plate is 10,000 lbs. per sq. in. The web area required is 169,930/10,000 = 16.99 sq. ins. A 48 × ⅜ in. plate, which provides an area of 18.0 sq. ins., will be used.

Flanges.—Assume two 6 × 6 × $^{11}/_{16}$ in. angles.

Net area (one rivet hole from each angle) . . . . = 14.18 sq. ins.
⅛ web area . . . . = 2.25 " "

Total flange area . . . . = 16.43 sq. ins.
Effective depth = 48.25 − 2 × 1.75 . . . . = 44.75 ins.

$$\text{Required area} = \frac{11{,}017{,}000}{16{,}000 \times 44.75} \; \ldots\ldots\ldots\ldots\ldots = 15.39 \text{ sq. ins.}$$

Rivet Pitch.—Quarter point and centre live-load shear taken as ⅝ and $^{2}/_{7}$ respectively of end shear (Art. 170, Part I).

Maximum Stringer Shears
(Pounds)

Point	Dead Load	Live Load	Impact	Total
End	6,130	85,200	78,600	169,930
Quarter Point	3,065	53,200	49,200	105,465
Centre	0	24,400	22,500	46,900

Vertical load per in. due to wheel load, impact, and dead load = 1,272 lbs. (See Art. 146.)

Effective depth 44.75 ins. Value of rivet 7,880 lbs.
From eq. (27), Art. 115.

Pitch at end. . . . . = 2.25 ins.
Pitch at quarter point. = 3.28 ins.
Pitch at centre. . . . . = 5.05 ins.

Adopted spacing shown on Fig. 15.

Stiffeners.—(See Art. 149.)

Unsupported web = 48.25 − 12 = 36.25 ins.
Allowable unsupported web distance = $\frac{3}{8} \times 60 = 22.5$ ins.
Therefore, stiffeners are required.
Stiffener spacing from Art. 79, Specifications: End, 24 ins.; Quarter point, 58 ins.; Centre, 88 ins.
Maximum allowable spacing = $160 \times \frac{3}{8} = 60$ ins.
Adopted arrangement shown on Fig. 15.

Connecting Angles.—The angles connecting the stringer to the floor beam will be made $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in., with the $3\frac{1}{2}$-in. leg against

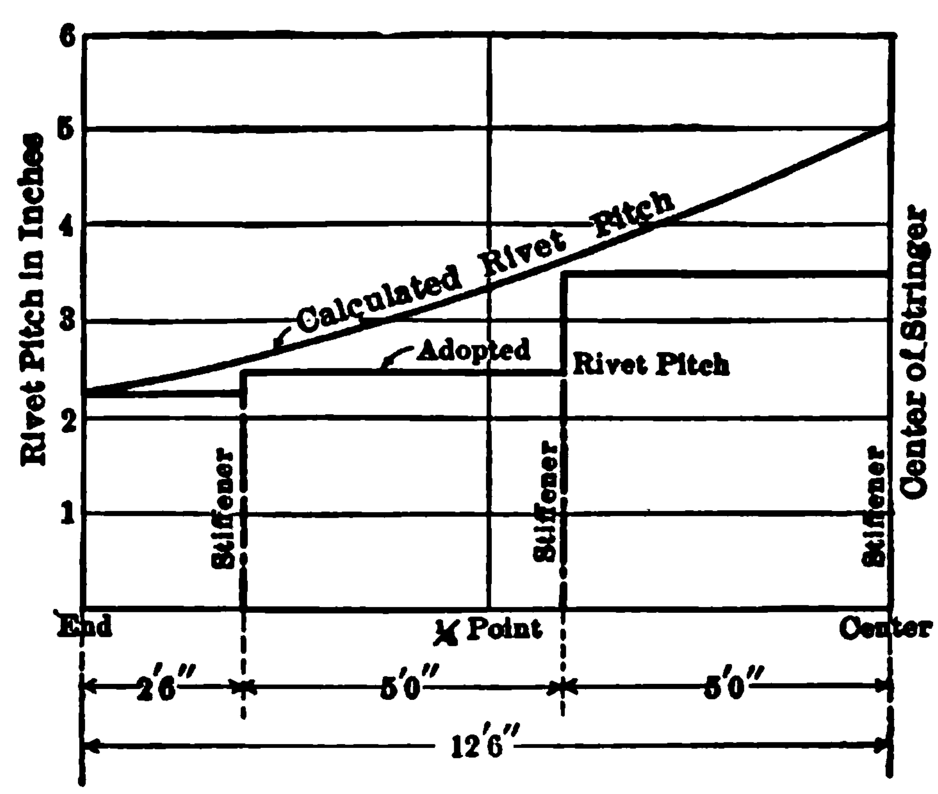


Fig. 15.

the fillers on the web plate. The rivets through the web plate are in bearing, at 7,880 lbs. per rivet, and 169,930/7,880 = 22 rivets are required. For shearing strength (double shear at 24,000 lbs./in.^{2}) 169,930/14,440 = 12 rivets are required. As shown in Fig. 16, 12 rivets are placed through the connection angles and 10 additional rivets through the filler. The 22 rivets in lines *a–a* and *b–b* are in bearing

on the web plate, and the 12 rivets in line *a–a* are in double shear, thus providing the required strength.

Lateral Bracing.—The stringers will be provided with lateral bracing consisting of single angles arranged as shown in Fig. 17. The lateral force to be carried by this bracing is due to a load of 600 lbs. per lin. ft. of bridge, as given in Art. 183. In addition to this force, the bracing must also carry a part of the 200 lbs. per ft. carried by the truss. As the stringers present a large part of the exposed truss area, we will assume that the stringer bracing must also carry one-half of the lateral force on the truss, or 100 lbs. per lin. ft. The total

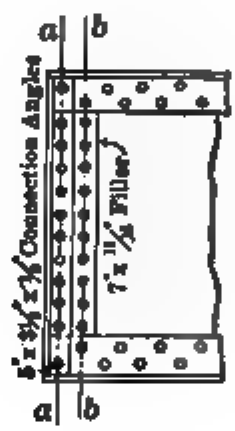


Fig. 16.

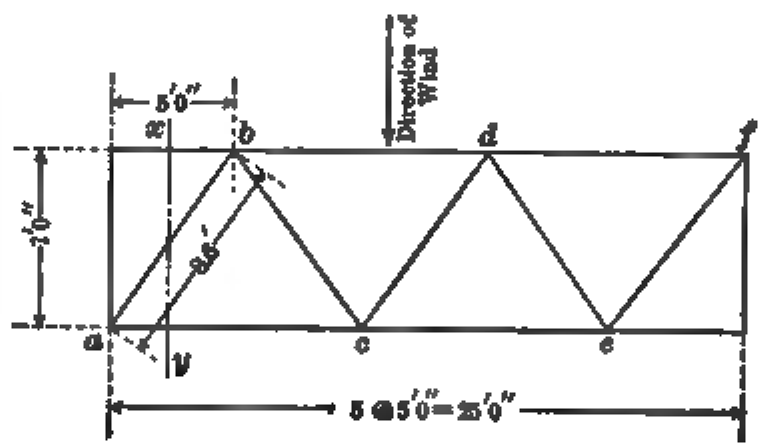


Fig. 17.

load to be carried by the stringer bracing is then 700 lbs. per lin. ft. Considering the lateral force as acting in the direction shown by the arrow in Fig. 17, member *ab* will have the maximum compressive stress. The load per panel point will be 7,000 lbs., and the shear on section *xy* is 8,400 lbs. Member *ab* will then have a stress of $8,400 \times 8.6/7.0 = 10,300$ lbs. compression.

Assuming that member *ab* is supported at each end by the flange angles and lateral plates for about 9 ins., the unsupported length of the member is found to be 85 ins. Art. 20, Specifications, states that lateral struts may have an unsupported length as great as 120 times the least radius of gyration. An angle must therefore be provided whose radius of gyration is at least $85/120 = 0.71$ ins. A $4 \times 4 \times \frac{3}{8}$ in. angle has a least radius of gyration of 0.79 ins. From the column formula of Art. 16, Specifications, the working stress for this angle is found to be $16,000 - 70 \times 85/0.79 = 8,470$ lbs. per sq. in. The area required for stress in *ab* is $10,300/8,470 = 1.22$ sq. ins. The

$4 \times 4 \times \frac{3}{8}$ in. angle provides an area of 2.86 sq. ins. As this angle is the smallest allowed by the Specifications, and provides more than enough area for the member having the greatest stress, it will be used throughout. The details are shown on Plate III.

The true weight of the stringers and bracing must now be determined and compared with the estimated weight. The weight of the stringers is found to be 11,300 lbs., or 452 lbs. per ft. of bridge. As the weight estimated by formula was 454 lbs. per ft., no revision of dead load is necessary.

195. Design of Intermediate Floor-Beams.—*Moments and Shears*—The floor-beams at panel points *b*, *c*, *d*, etc., are called intermediate floor-beams. Each floor-beam carries to the trusses the loads on the stringers in the adjacent panels. In calculating the stresses in the floor-beams the length of beam is usually taken as equal to the distance centre to centre of trusses.

The clearance diagram of Fig. 1 calls for a clear width of 15 ft. In this design the trusses have been assumed to be 17 ft. 9 in. centre to centre. From Fig. 14 the width of the end post over the lower angles is 2 ft. 7½ in. The clear distance between inside edges of end posts is then 15 ft. 1½ in., which is slightly greater than the required clearance. The assumed distance between trusses is therefore correct and will be adopted as final.

The floor-beams carry, in addition to their own weight, two concentrated loads spaced 7 ft. apart, as shown in Fig. 18. For purposes of stress calculation, the weight of the floor-beam will be assumed as 5,000 lbs. This load will be considered as uniformly distributed. The resulting dead-load moment at the centre of the beam is $\frac{1}{8} \times 5{,}000 \times 17.75 \times 12 = 133{,}000$ in.-lbs. The concentrated loads are due to the dead and live load on the stringers in the adjacent panels. From Art. 194, the dead load per foot per stringer is 490 lbs., which gives a dead-load concentration of $25 \times 490 = 12{,}250$ lbs. The live load part of the concentrated load is due to the floor-beam reaction plus impact. This floor-beam concentration has already been calculated in Art. 178 for the determination of stress in member *Bb*. From Table A, this is given as 113,500 lbs., and the corresponding impact load is 97,400 lbs. The total stringer concentration is then 223,150 lbs. From Fig. 18, the moment at the centre of the floor-beam, which is

equal to that at the stringer, is 223,150 × 64.5 = 14,400,000 in.-lbs. The total moment is 14,533,000 in.-lbs.

The shear at the truss is 223,150 + 2,500 = 225,650 lbs. Between the truss and the stringer, the shear is practically uniform, varying only by the weight of the beam between these points.

196. *Dimensions of Web and Flanges.*—The required depth of the floor-beam depends upon the size of the flange angles on the floor-

FIG. 18.

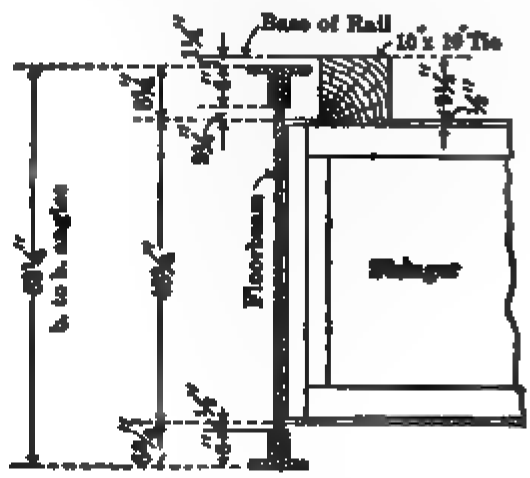


FIG. 19.

beam, the depth of the ties and the make-up of the stringer section, as shown in Fig. 19. For the arrangement shown, the depth of floor-beam is 63¼ ins. back to back of angles. The web area required for shear is 225,650/10,000 = 22.56 sq. ins. A 63 × ½ in. plate will be used, which provides 31.5 sq. ins. of area. This plate provides considerable excess area, but a thick web is desirable, as the rivet values are increased thereby, thus requiring the use of a smaller number of rivets in the floor-beam details.

The flange angles will be made of unequal leg angles, the longer leg to be placed against the web plate. This arrangement gives a relatively narrow flange and tends to prevent excessively large secondary stresses due to stringer and chord action as discussed in Art. 80. As in the case of the stringer, the top angles will be placed flush with the top of the web, and the bottom angles will project ¼ in. beyond the edge of the web plate. Assuming the flanges to be made up of 6 × 4 × ¹¹⁄₁₆-in. angles, placed as stated above, the effective depth is 63.25 − 2 × 2.06 = 59.13 ins. The flange stress is 14,533,000/59.13 = 245,500 lbs., and the area required is 245,500/16,000 = 15.32 sq. ins. Counting one-eighth of the web area as part of the flanges,

the net area required in angles is $15.32 - 3.94 = 11.38$ sq. ins. The assumed angles provide a net area of $2\ (6.41 - 0.69) = 11.44$ sq. ins., which is sufficient, and the assumed angles will be adopted.

197. *Flange Rivets.*—The number of rivets required to connect the flange angles and web plate can be determined by placing enough rivets in the flanges between the truss and the stringer connection to take up the flange stress at the stringer. The total flange stress at the stringer can be assumed to be the same as that at the centre of the floor-beam, which is 245,500 lbs. as calculated above. Part of this stress has been assumed to be carried by the web plate. Reducing the total flange stress by the ratio of net flange area to total moment-carrying area, the stress in the angles alone is found to be $245{,}500 \times 11.44/(11.44 + 3.94) = 183{,}000$ lbs. To transmit this stress requires $183{,}000/10{,}500 = 18$ rivets in bearing on the ½-in. web plate. Fig. 20 shows 19 rivets in place in the upper flange between the stringer and the end of the floor-beam. In the lower flange only 12 rivets can be placed in position, but by using a cover plate over the angles at the end of the beam, the last 6 rivets are greatly increased in value. In this case these rivets are in quadruple shear, or in bearing on the $^{11}/_{16}$-in. flange angles. For these conditions both shear and bearing give a rivet value of 28,880 lbs. per rivet. The strength provided by rivets between the stringer and end of beam at the lower flange is then $6 \times 28{,}880 + 6 \times 10{,}500$ (bearing on ½-in. web) $=$ 236,280 lbs., which is more than enough. The spacing of the flange rivets between the stringer connections is made the maximum permissible or 6 ins., as the shear is practically zero.

198. *Details of End Connections.*—The number of field rivets required to connect the floor-beam to the truss (single shear at 10,000 lbs. per sq. in.) is $225{,}650/6{,}010 = 38$ rivets. Fig. 20 shows the required number in place. The connection angles will be made of $6 \times 4 \times \frac{1}{2}$ in. angles. The rivets connecting these angles to the web of the floor-beam are in double shear and $225{,}650/14{,}440 = 16$ rivets are required. Fig. 20 shows 18 rivets in place, not counting the two rivets which pass through the top flange angles.

The end detail of the floor-beam requires careful consideration. In this design the beam is placed with its lower flange on the same level as the lower angles on member *abc*, that is, 10¼ ins. below

the centre line of the pins, as shown on Plate III. This requires the lower corner of the beam to be cut away in order to make room for the bottom chord members, thus weakening it greatly at this point. The calculations made above showed that 38 field rivets are required to connect the floor-beam to the post. Cutting away the corner of

FRONT VIEW

FIG. 20.

the beam has reduced the depth of the beam at the end so that it is impossible to get the required rivets in the space left. It will then be necessary to extend the end connection angles above the top of the floor-beam far enough to provide space for the rivets required, as shown in Fig. 20.

A rigid connection between the end connection angles and the web plate can be made by cutting off the main web plate about half way between the stringer and the truss and splicing on a plate of such shape that it will extend above the flange angles as far as the top of the end connection angles, as shown in Fig. 20. The splice plates are made of the same thickness as the flange angles in order to act as fillers for the end connection angles and for the additional

cover plates which are to be riveted on the lower flange angles. These additional plates also serve to strengthen the weak corner of the beam, and to take up the lower flange stress and carry it to the end of the beam. The curved angles give a finish to the beam, but add little to its strength.

The height to which the lower corner of the floor-beam is cut away depends upon the depth of member *abc*, and upon the size of the diagonal eye-bars. The curved angles must be placed high enough to clear the eye-bar diagonals, as shown in the end view of Fig. 20. Also these angles must not interfere with member *abc*, which in this truss requires the angles to be at least 10¼ ins. above the centre line of the bottom chord. In Fig. 20, the height of cut away was determined by placing the gauge line of the curved angles on the eighth line of rivets above the bottom of the floor-beam. It will be found that the proper clearances have been provided by this arrangement.

The floor-beam is directly connected to one channel of the post only. In order to distribute the floor-beam load evenly to the two channels, a diaphragm is placed between them. This diaphragm is made up of 4 angles and a plate, making a member of the same shape as *Bb*, shown in Fig. 12. It extends over the portion of the posts covered by the end connection angles of the floor-beam. The details of the diaphragm are shown on Plate III. The diaphragm is designed to take one-half the floor-beam shear at the post. Minimum sized angles and plates are usually sufficient. In this case $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles and a $11 \times \frac{3}{8}$ in. plate are used.

The floor-beam at joint *b* is a little longer than those at joints *c* and *d*, etc., due to the fact that the vertical member at joint *b* is narrower than the others. The difference in length is usually made up by increasing the length of the main web plate, keeping all splice and cover plates the same as for the beams at other intermediate joints.

199. *Web Splice.*—The web splice is to be calculated by the methods given in Art. 123. The moment and shear at the splice are large and should be thoroughly provided for in order to make the web plate capable of carrying a part of the flange stress, as assumed in the previous calculations. An exact location of the splice is not

possible until the arrangement of members at the lower chord joint has been determined. Subsequent calculation, Art. 220, shows that joint d is the widest bottom chord joint, and that the distance from the centre line of truss to the face of the outer eye-bar is 1 ft. $5\frac{5}{8}$ in. (See Fig. 40.) A pilot nut is placed on the pin when it is driven through the eye-bars, and a clearance of at least 6 ins. is required so that this nut can be taken off after the driving has been completed. The distance from the truss centre to the curved angle must then be 2 ft. 0 in., as shown in Fig. 20. Assuming that 4 rivets will be required to take the bottom flange stress between the splice and the end of the beam, the distance from truss to centre of splice is found to be 3 ft. 2 ins., or 38.0 ins.

The moment and shear at the splice due to the weight of the floor-beam are determined by the formulas given in Arts. 72 and 74, Chap. IV, Part I. From these formulas we find $M = \frac{5{,}000}{2 \times 213} \times 38 \times 175 = 78{,}000$ in.-lbs., and $V = 2{,}500 - 5{,}000 \times 38/213 = 1{,}610$ lbs. The moment at the splice due to the stringer concentration is $M = 223{,}150 \times 38.0 = 8{,}475{,}000$ in.-lb., and the shear is 223,150 lb. This gives a total moment of 8,553,000 in.-lbs. and a shear of 224,760 lbs. to be carried by the splice.

The resulting flange stress at the splice is $8{,}553{,}000/59.13 = 144{,}600$ lbs., of which $144{,}600 \times 11.44/15.38 = 107{,}600$ lbs. is carried by the flange angles. To transmit this stress, we must provide between the splice and the end of the beam, $107{,}600/10{,}500 = 11$ rivets in the top flange angles and $107{,}600/28{,}880 = 4$ rivets in the lower flange. Fig. 20 shows 11 rivets in the upper flange and 4 rivets in the lower flange in the required distance. The position of splice as located will be adopted as final. The rivet pitch in the splice is to be calculated by Eq. 47 of Art. 123. In this formula f is the fibre stress on the gross flange area which can be found by dividing the total flange stress by the gross area of the angles plus one-sixth of the gross web area. Then $f = 144{,}600/(12.82 + 5.25) = 8{,}000$ lbs. per sq. in. Also we have $V = 224{,}760$ lbs., $t = \frac{1}{2}$ in., $h = 63$ ins., and $r = 10{,}500$ lbs. (bearing on a $\frac{1}{2}$-in. plate). Substituting these values in Eq. 47, the required rivet pitch is found to be 1.96 ins. for a single row of rivets. By using two rows of rivets the pitch can be made

4 ins. per row. With such a spacing $\frac{1}{6}$, $\frac{p-1}{p}$ = 12½ per cent of the web area is effective as flange area. As the flange stress at the splice is low, it will not be necessary to splice for the part of the web plate under the flange angles. The excess flange area can be depended upon to carry the moment across the splice. Fig. 20 shows the details of the splice as designed.

Stringer Connection.—The rivets required to connect the stringer to the floor-beam must be determined for two cases: (*a*) End shear on a stringer, field rivets in single shear, (*b*) floor-beam reaction, field rivets in bearing on a ½-in. web plate. For this truss the end shear on a stringer is 169,930 lbs. (Art. 194.) Field rivets in single shear have a strength of 6,010 lbs. per rivet, and 169,930/6,010 = 29 rivets required for case (*a*). The floor-beam concentration is 223,-150 lbs. (Fig. 18.) Field rivets in bearing on a ½-in. web have a strength of 8,750 lbs. per rivet and 223,150/8,750 = 26 rivets required. Fig. 20 shows 30 rivets provided.

200. *Actual Weight of Beam.*—The true weight of a floor-beam is found to be 6,370 lbs., while the estimated weight used in the calculations was 5,000 lbs. It can be seen that a large part of the weight of the beam is made up of the splice and cover plates at the end of the beam. In view of the fact that the moments and shears due to the weight of the beam are a very small part of the total, the estimated weight used in the preliminary calculations is probably close enough and no revision will be made.

201. Design of End Floor-Beams.—*Types of End Floor-Beams.*—At the ends of a bridge the stringers can be supported in several different ways. In some cases, particularly in skew bridges, where a square end is desired for the track stringers, the end stringers rest on the piers or abutments, having an end detail similar to that of a deck plate girder.

The usual method of support for the end stringers in a square-ended span is by means of an end floor-beam. In some cases the end beam is made short enough to fit inside the bottom chord members of the truss, and rest on the main shoes, as shown in Fig. 23. Another type of floor-beam is shown in Fig. 25. In this case a short pair of vertical channels are provided which form a support for the

beam on the pins in the same manner as at the panel points *c* and *d*. The cover plate of the end post is notched out to allow these channels to enter the joint and bear on the pin. The methods of design are exactly the same as for an intermediate beam, except that the applied loads are as calculated for the conditions of loading shown in Fig. 21. Fig. 25 shows the sections used and the general dimensions of the various parts. The end beam may also be riveted to gusset plates on the foot of the end post.

In the present case the type of floor-beam will be used which rests on the end shoe, as the design of such a beam involves certain features not otherwise touched upon.

202. *Loads and Stresses.*—The loads to be carried by an end floor-beam consist of the weight of the beam and the concentrations brought to the beam by the stringers. For purposes of calculation the end beam will be assumed to weigh 3,000 lbs., considered as uniformly distributed. The concentrated loads brought by the stringers to the floor-beam consist of the dead- and live-load reactions for the stringers in the end panel.

From Art. 194, the stringer dead-load reaction is 6,130 lbs. The live-load portion of the concentrated load is determined from the conditions shown in Fig. 21. The last tie is carried on a bracket which is riveted to the front of the end floor-beam. Because of this, loads placed on the last tie at a distance of about one foot beyond the centre of the beam can be considered as carried by the end floor-beam. For the conditions shown in Fig. 21 the reaction of the end floor-beam is found to be 90,500 lbs. The impact allowance, loaded length 25 ft., is 83,500 lbs. Adding to these loads the stringer dead load given above, the total end floor-beam concentration is found to be 180,130 lbs. This same concentration can be taken for both types of end floor-beams mentioned above.

The type of floor-beam shown in Fig. 23 has an effective length about four feet shorter than the distance between truss centres. In order to allow the beam to rest on the sole plate, it will be made 11 ins. deeper than the intermediate beams. This distance of 11 ins. is determined by the shoe details, as shown in Fig. 55.

The moments and shear are to be calculated for the loading conditions shown in Fig. 22. With the assumed floor-beam dead load

of 3,000 lbs., the dead-load centre moment is $\frac{1}{8} \times 3{,}000 \times 165 =$ 61,900 in.-lbs., and the shear at the end of the beam is 1,500 lbs. The maximum moment due to the stringer concentration is $.180{,}130 \times 40.5 = 7{,}295{,}300$ in.-lbs., and the shear at the end of the beam 180-

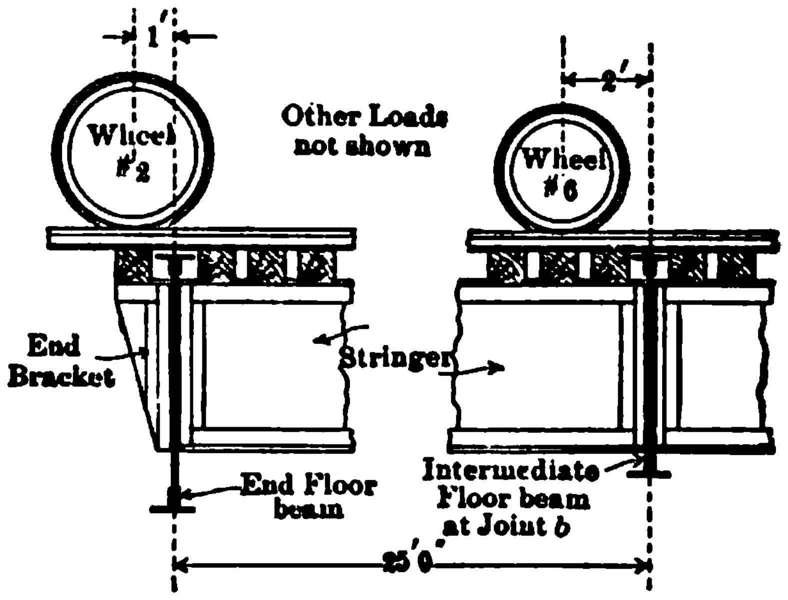


FIG. 21.

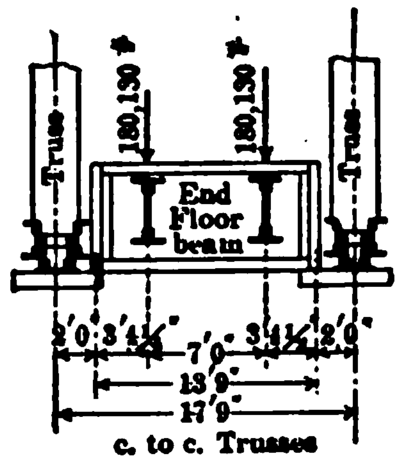


FIG. 22.

130 lbs. This gives a total bending moment due to dead and live load of 7,357,200 in.-lbs. and an end shear of 181,630 lbs. As the beam is to be 11 ins. deeper than the intermediate beams, its depth back of angle will be $74\frac{1}{4}$ ins. The web plate is to be placed flush with the backs of the top flange angles.

203. *Dimensions of Web and Flanges.*—The web area required for shear is $181{,}630/10{,}000 = 18.2$ sq. ins. A $74 \times \frac{3}{8}$ in. plate provides an area of 27.8 sq. ins. Assuming $6 \times 4 \times \frac{3}{8}$ in. flange angles, with the 6-in. leg against the web plate, the effective depth of the beam is found to be $74.25 - 2 \times 1.94 = 70.37$ ins. The flange stress is $7{,}357{,}200/70.37 = 104{,}500$ lbs. and the area required is $104{,}500/16{,}000 = 6.54$ sq. ins. Considering one-eighth the web area as part of the flange leaves $6.54 - 3.47 = 3.07$ sq. ins. to be provided by the flange angles. Allowing one rivet hole for each of the assumed angles, the area provided is found to be $2\ (3.61 - 0.38) = 6.46$ sq. ins. The assumed angles provide more than enough area, but in order to keep a uniform size of flange angles the adopted size will be used. The number of rivets required in the flange angles between the stringer and the end of the floor-beam will be calculated by the method used for the intermediate beam. As calculated above, the flange stress at the stringer is 104,500 lbs., of which $6.46/(3.47 + 6.46) \times 104{,}500 = 68{,}000$ lbs. is carried by the flange

angles. The rivets are in bearing on a $\frac{3}{8}$-in. web and 68,000/7,880 = 9 rivets are required. Fig. 23 shows 12 rivets in place.

204. *End Connections.*—The end connection angles are designed in the same manner as the end stiffener angles in a plate girder. The required bearing area is furnished by the end stiffener angles and by the $\frac{1}{2}$-in. bearing plate shown in the end view of Fig. 23. From

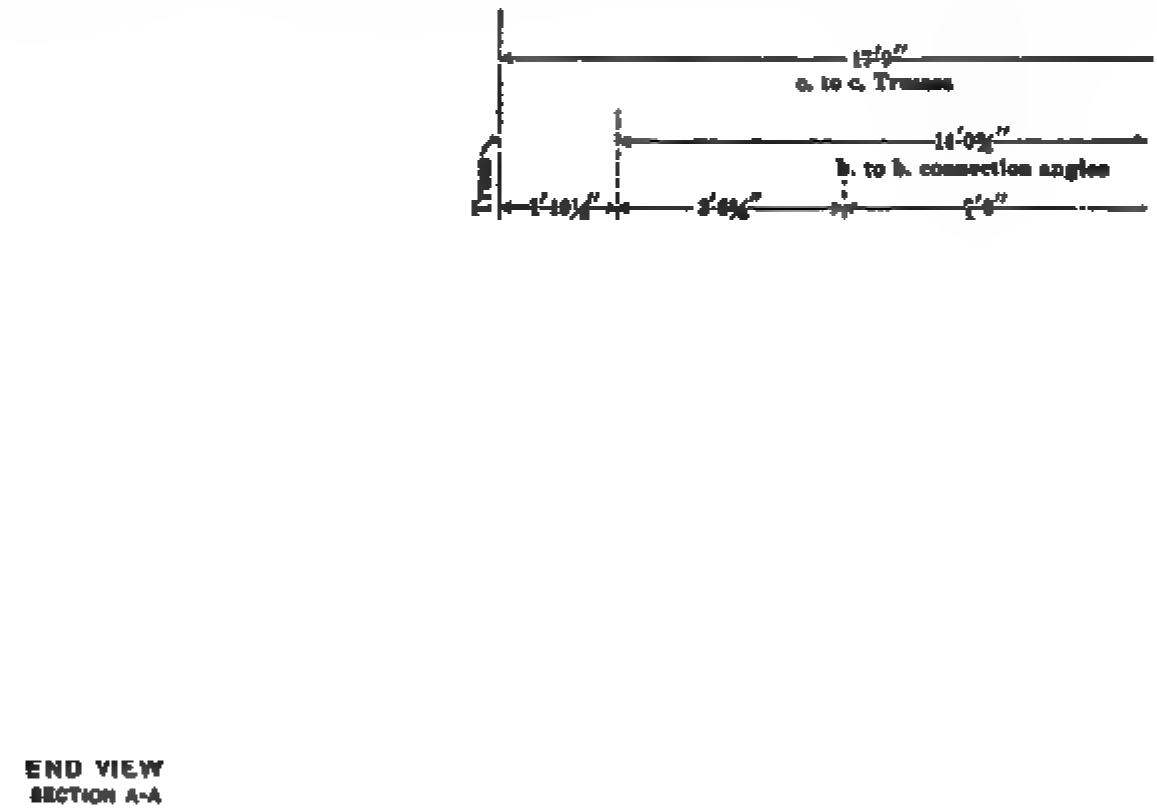


Fig. 23.

Arts. 16 and 79, Specifications, the allowable bearing stress is 14,000 lbs. per sq. in., and the area required is 181,630/14,000 = 12.95 sq. ins. Using two $6 \times 4 \times \frac{1}{2}$ in. angles, and a $\frac{1}{2}$-in. bearing plate, the proper area is supplied. The connection of the stringer to the web of the floor-beam is determined in the same manner as for the intermediate floor-beams. It was found in Art. 199 that 29 rivets were required for the maximum end shear on a stringer. For the condition of loading shown in Fig. 21 the rivets are in bearing on the web plate. These rivets are field rivets in bearing on a $\frac{3}{8}$-in. web, and 180,130/6,560 = 28 rivets are required. The same arrangement and number of rivets will be provided as for the connection on the intermediate beams.

205. *End Brackets.*—The end bracket, shown in Fig. 24, is made up as a continuation of the stringer section. This bracket acts as a

short cantilever beam carrying a moment due to the maximum driver load, plus 100 per cent impact, placed one foot in front of the end floor-beam. The moment to be carried is then $2 \times 30{,}000 \times 12 = 720{,}000$ in.-lbs.

An approximate analysis of stresses in this bracket can be made by assuming that the stress conditions are similar to those in the stringer section. Assuming the effective depth to be the same as for the stringer section, the flange stress is $720{,}000/44.75 = 16{,}100$ lbs. At the top of the bracket, the flange stress is tension. If the bracket

FIG. 24.

is fastened to the floor-beam only by rivets through the web plate, the rivets in the upper end of the bracket will be in tension. A tension plate passing through a slot in the web, and connecting the bracket angles to the main stringer angles will relieve these rivets of tension. The area required for such a plate is only about one sq. in. In Fig. 24 a $14 \times \frac{3}{8}$ in. tension plate is shown, which is connected to the angles by four rivets at each end.

206. Design of the Lateral Bracing.—The design of the lateral bracing is governed by the following articles from the Specifications:

(20) (in part). The lengths of compression members for wind and sway bracing shall not exceed 120 times their least radius of gyration.

(21) The lengths of riveted tension members in horizontal or inclined positions shall not exceed 200 times their radius of gyration about the horizontal axis. The horizontal projection of the unsupported portion of the member is to be considered as the effective length.

(70) Lateral, longitudinal, and transverse bracing in all structures shall be composed of rigid members.

(74) The minimum-sized angle to be used in lateral bracing shall be $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. Not less than three rivets through the end of the angles shall be used at the connection.

(37) The strength of connections shall be sufficient to develop the

full strength of the member, even though the computed stress is less, the kind of stress to which the member is subjected being considered.

207. *Upper Laterals.*—The stresses in the upper laterals were calculated in Art. 182 and are shown in Fig. 4. These stresses were calculated on the assumption that the diagonals take tension only. To comply with Art. 70, Specifications, these diagonals will be made of two angles laced, and placed a distance apart in a vertical plane equal to the depth of the top chord. The stress in the end diagonal

FRONT VIEW

FIG. 25.

is 17,500 lbs. tension, and the area required is 17,500/16,000 = 1.1 sq. ins. As rivets are to be placed in both legs of these angles, two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, the minimum size allowable, will be used. These angles give a net area of 2 (2.48 − 0.38) = 4.2 sq. ins., one rivet hole deducted from each angle. As the member thus formed is quite deep, the requirements of Art. 21, Specifications, are met. The rivets in the end connection required by the stress are 17,500/6,010 = 3 field rivets in single shear. Art. 37, Specifications, requires 4.2 × 16,000/6,010 = 12 rivets. As a large excess area had to be provided, the number of rivets called for by Art. 37, Specifica-

tions, is greater than any possible stress conditions can call for. It seems best in this case to depart from the specifications somewhat and use 8 rivets, 4 in each angle, as shown on Plate III. As the diagonals in the end panel, where the stress is greatest, require the minimum angles, all other diagonals will be made of the same size.

The upper lateral strut CC' of Fig. 4 has a stress of 7,500 lb. compression. The unsupported length of this member is equal to the distance between trusses minus the width of the top chord member, or $213 - 28 = 185$ inches. According to Art. 20, Specifications, the least permissible value of radius of gyration is $185/120 = 1.54$ ins. The greatest unsupported length will be in a horizontal direction as the member is supported at several places in the vertical plane by the sway bracing, as shown in Fig. 29. Two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, with the vertical legs separated by $\frac{3}{8}$-in. for a lateral plate, have a horizontal radius of gyration of 1.61 ins. For these angles so placed, the working stress is 7,950 lbs. per sq. in. and the area required is $7,500/7,950 = 0.95$ sq. ins. Two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles provide an area of 4.96 sq. ins., but as these are the minimum angles they must be used. Only 2 rivets are required in the end connections to take care of the stress, but to conform to Art. 74, Specifications, 4 rivets will be used, 2 in each angle as shown in Plate III. The same size angles will be used for all top lateral struts.

208. *Lower Laterals.*—The stresses in the bottom diagonal members were calculated in Art. 183A and are given in Fig. 5. According to the assumptions made in calculating these stresses, the shear is equally divided between the two members in the panel, one taking tension, and the other compression. The compressive stresses will govern the design.

The lateral diagonals are fastened to the floor-beams and chord members at the truss, and to each other where they cross at the centre of the panel. A light connection to the stringers is supplied at the point where the laterals cross. This connection serves to keep the laterals from vibrating. The details of this connection are shown on the general drawing. In calculating the working compressive stress in the laterals, the unsupported length of member will be taken from its connection at the truss to the centre of the panel, as shown in Fig. 26.

On Plate III and in Fig. 26, the centre lines of the lateral members are shown as intersecting the centre line of the main trusses at joints a and a', while at other joints the centre lines of the laterals intersect at a point 9 ins. inside the centre lines of the main truss. While it is desirable, in general, to have all gravity lines intersect at

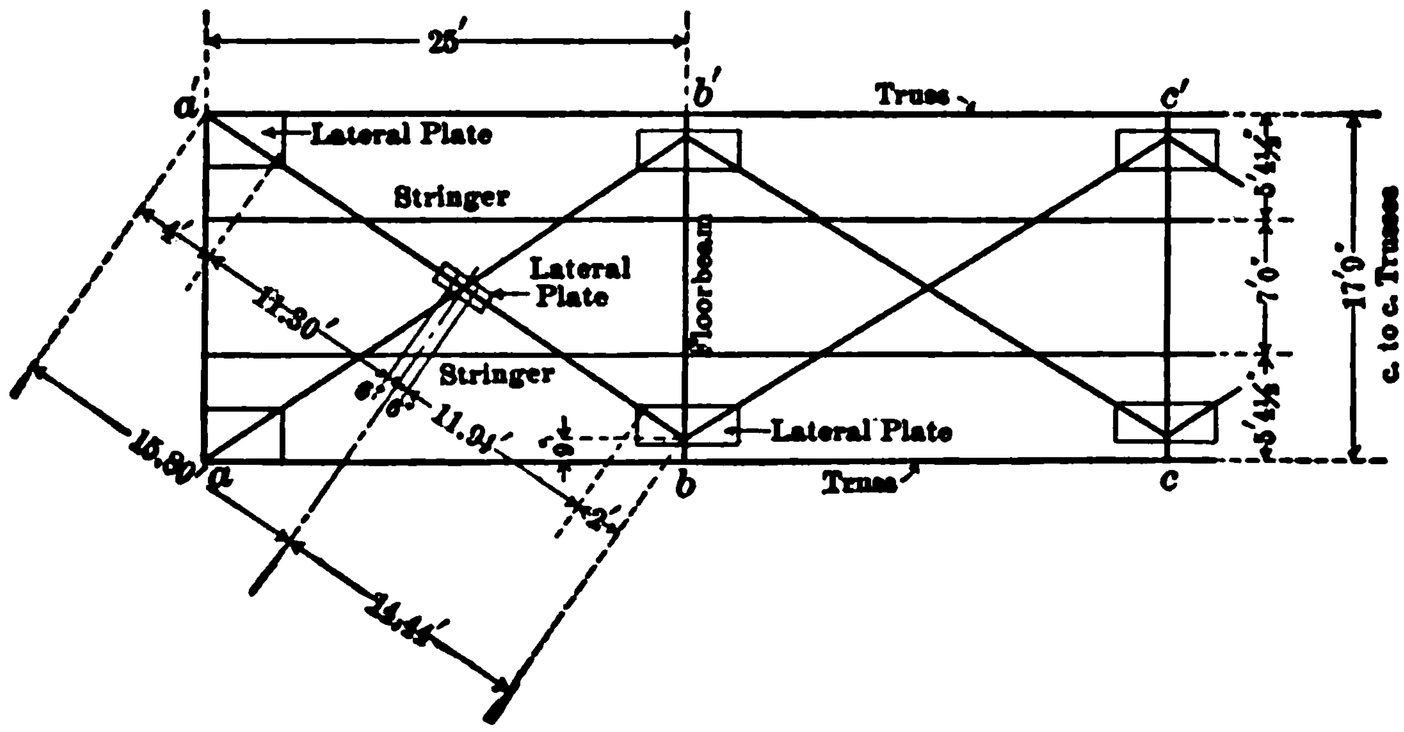


FIG. 26

a point, small eccentricity at intermediate joints is not objectionable. The resulting bending moments are resisted partly by the bottom chord, which is a very wide member, and partly by the floor-beams, which are rigidly connected to the bottom chord at the joints, so that the resulting stresses are small. At the end joints the matter is of much greater importance, owing especially to the effect of chord deformation. Any bending at this joint must be resisted partly by the member $a\ b\ c$ and partly by the shoe and by member $a\ B$, the end post. In this way $a\ B$ is subjected to a torsional moment in addition to its direct and wind loads, which is undesirable and is to be avoided if possible. At the end joint a central connection can readily be made, but at intermediate joints very large joint plates would be required. In this case the intersection of gravity lines of the laterals is made as shown in Plate III.

A rigid connection between stringer and lateral is not desirable. As shown in Art. 351, Chap. VII, Part II, the distortion of the floor-beams is such as to cause a movement of the truss with respect to the stringer. If a rigid connection has been made between lateral and stringer, large stresses will be produced in the lateral members and

connecting details. It seems best, therefore, to avoid a direct connection between lateral and stringer. This increases the unsupported length of the lateral member, and may cause the use of an excess of material in laterals near the centre of the bridge.

The stress in the laterals in the end panel is given in Fig. 5, Art. 183, as 52,500 lbs. Considering the lateral to be supported for about 4 ft. at the end of the truss, and about 6 ins. at the centre of the panel, the greatest unsupported length of member is found to be about 11.94 ft., as shown in Fig. 26. According to Art. 20, Specifications, the least allowable radius of gyration is $11.94 \times 12/120 = 1.19$ ins. Two $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, placed with the 5-in. legs back to back, and in a vertical position, have a radius of gyration of 1.34 ins. in a horizontal plane and 1.6 ins. in a vertical plane. The allowable working stress is found to be 8,500 lbs. per sq. in. The area required is $52,500/8,500 = 6.20$ sq. ins. The assumed angles provide an area of 6.10 sq. ins. To connect the laterals to the truss requires $52,500/6,010 = 9$ field rivets in single shear. Plate III shows 10 rivets in place.

In the other panels, the stresses in the members are less than in the end panel. But the unsupported length is practically the same so that it will be necessary to use the same angles throughout in order to satisfy the conditions of Art. 20, Specifications. The arrangement of members is shown on Plate III.

209. Design of the Portal and Sway Bracing.—The portal bracing is placed in the plane of the end posts, and carries the load due to lateral forces on the top chord to the abutments. From Art. 71, Specifications:

(71) Through truss spans shall have riveted portal braces rigidly connected to the end posts and top chords. They shall be as deep as the clearance will allow.

(72) Intermediate transverse frames shall be used at each panel of through spans having vertical truss members where the clearance will permit.

210. *Sway Bracing.*—The sway bracing is placed in the plane of the vertical posts of the truss. This bracing forms the struts of the top lateral system and also serves to fasten the two trusses together.

As the structure under consideration is a through bridge, the

bracing must be so arranged as not to interfere with the passage of trains. The space required by the train depends upon the extreme dimensions of the rolling stock in use on the railroad in question. In this case the clear space required is shown by the clearance diagram of Fig. 1.

The form and make-up of the sway bracing will be determined first. In any case the arrangement adopted will depend upon the distance between the top of the clearance diagram and the centre line of the top chord. Several forms in general use for trusses of the size under consideration in this chapter are shown in Fig. 27. For very shallow trusses the form shown in Fig. 27 (*a*) can be used. This form consists of laced angles placed a distance back to back

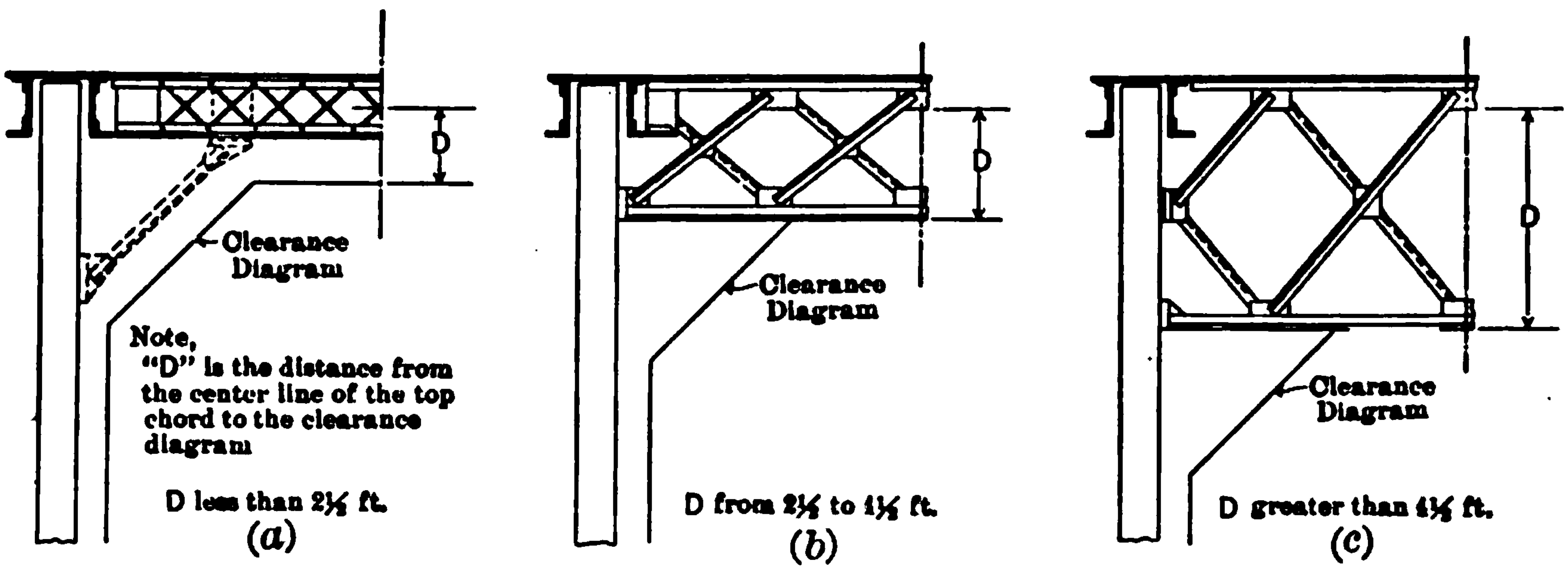


FIG. 27.

equal to the depth of the top chord. In some cases, a knee brace, shown by the dotted lines, can be added to help stiffen the frame. Figs. 27 (*b*) and (*c*) show forms of bracing used frequently in deep trusses. The limits stated for the several types are arbitrary and can be varied to suit the designer. In any case the form adopted should be such that the bracing makes angles of about 45 degrees with the horizontal. The members forming this bracing are usually not designed for any definite stress, except the top angles, which form the struts of the top lateral system, designed in Art. 207. Minimum angles allowed by the specifications, Art. 74, are $3\frac{1}{2} \times 3 \times \frac{3}{8}$ in., and are usually used for all members where rivets are driven in one leg only.

The best way to determine the form and dimensions of the sway

bracing is by means of a layout drawing, such as shown in Fig. 29. In making this layout, the distance from top of clearance diagram to centre line of top chord must be determined. The conditions for this truss are as shown in Fig. 28. From Fig. 20, the distance from the centre line of bottom chord to the bottom of the floor-beam is 10¼ ins. As the floor-beam is 5 ft. 3¼ in. deep, the distance from centre line of bottom chord to top of floor-beam is 4 ft. 5 in. The

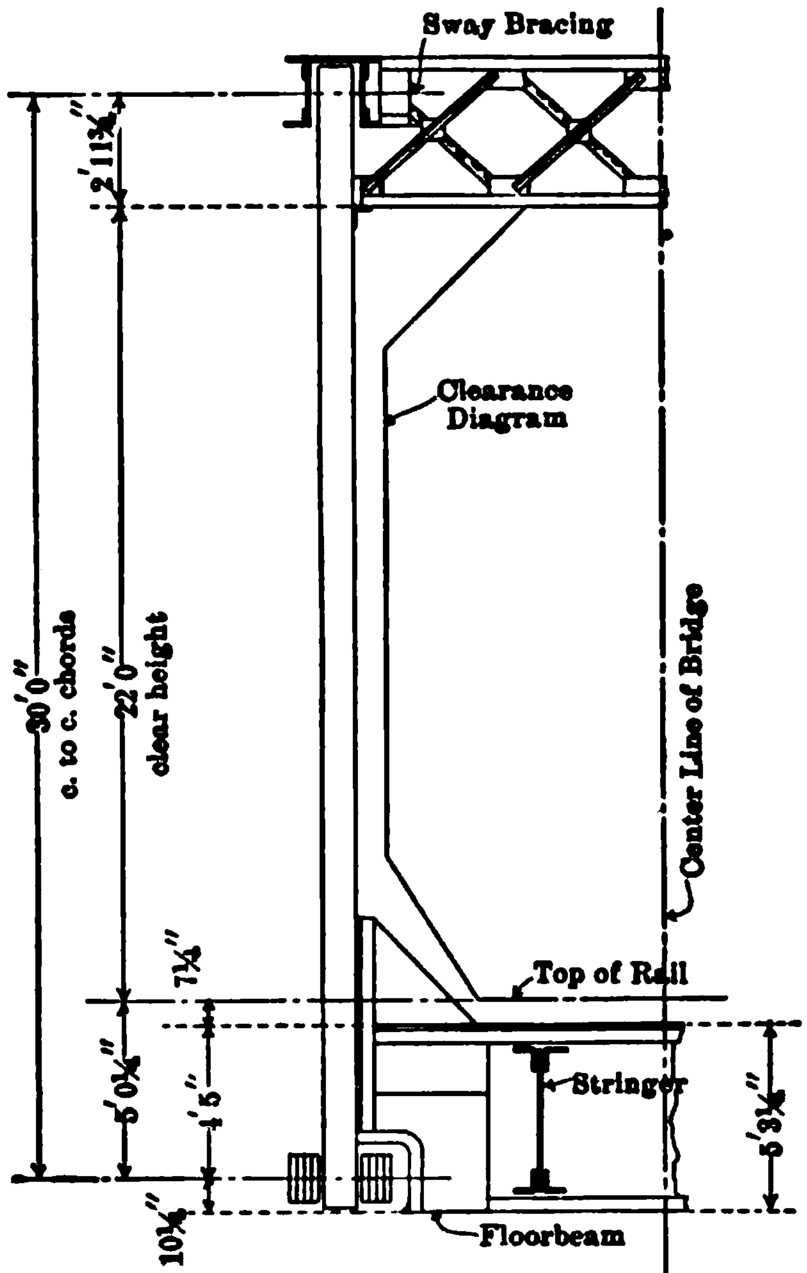


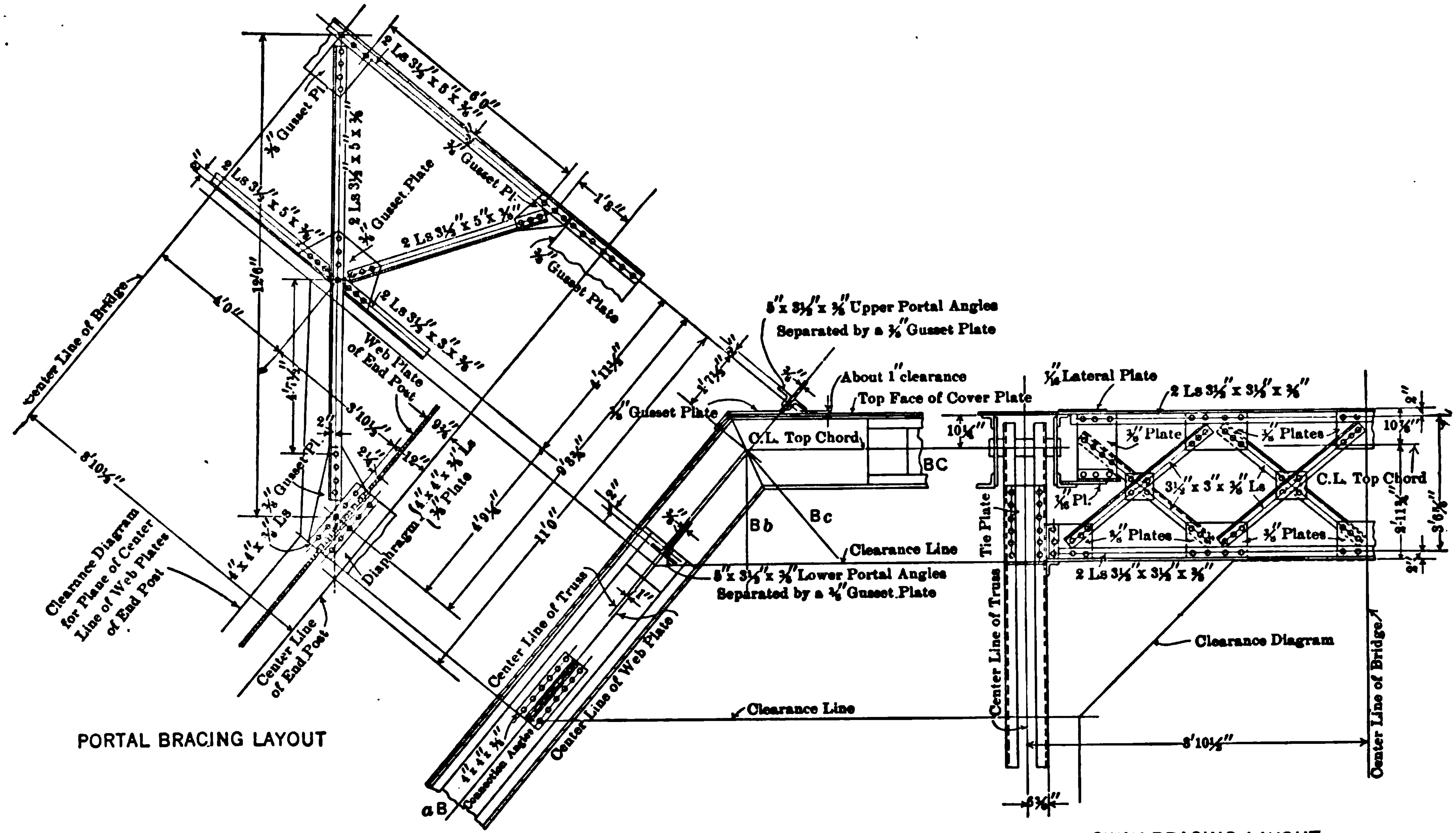
FIG. 28.

base of rail is 1¼ in. above the top of the floor-beam, as shown in Fig. 19, and from Art. 2, Specifications, the height of rail to be assumed is 6 ins. The top of rail is then 5 ft. ¼ in., above the centre line of the bottom chord. Adding 22 ft. for vertical clearance, the distance from the lowest steel work above the centre of the track to the centre line of the bottom chord is 27 ft. 0¼ in. As the truss is 30 feet deep, centre to centre of chords, this leaves 30 ft. − (27 ft. 0¼ in.) = 2 ft. 11¾ in. as the distance from the top of

the clearance diagram to the centre line of the top chord. The distance just calculated calls for sway bracing of the form shown in Fig. 27 (*b*). A layout of this bracing is shown in Fig. 29. As stated above, the top angles were designed in Art. 207. In this case $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles are used. These angles are placed with the horizontal legs on the same level as the top of the cover plate of the top chord section. The lower angles will also be two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, as rivets must be placed in the horizontal legs to form the connection to the truss. For the diagonal members $3\frac{1}{2} \times 3 \times \frac{3}{8}$ in. angles will be used with the $3\frac{1}{2}$-in. leg against the gusset plates. The scale of the layout should be large enough so that all edge distances and sizes of plates can be determined by scale, thus saving considerable calculating. For the original layout for this truss a scale of one inch to the foot was used, which proved to be large enough. Where very complicated connections are made, a larger scale should be used.

211. *Portal Bracing.*—The portal bracing which will be adopted for the truss under consideration in this chapter is known as the "A" frame type of portal, shown in its most general form in Fig. 30 (*b*). This portal is simple in form, is quite rigid, and is easily analyzed for stresses. In structures of moderate size, the members are each made up of two angles, the frame thus formed being placed in the plane of the centre of the web plates. This same form can be used for very large structures by making up two similar frames, one placed in the plane of the cover plates and the other in the plane of the lower angles of the chord section. The two parts of the frame are then connected by lacing or batten plates. Many other portal forms are used in practice, but it is impossible to consider all such forms here.

The form and dimensions of the portal depend upon the clear distance above the top of the clearance diagram, as in the case of the sway bracing. As the portal lies in the plane of the end post, the shape of the clearance diagram for this plane must be determined. This is a simple problem in descriptive geometry, and consists of finding the section cut out of the clearance diagram prism by the plane of the end posts. The necessary construction is shown in Fig. 29. It is to be noted that in this case the portal is located



PORTAL BRACING LAYOUT
SWAY BRACING LAYOUT
Center Line of Bridge
Clearance Diagram for Plane of Center Line of Web Plates of End Post
Center Line of End Post
Web Plate of End Post
Diaphragm
Center Line of Truss
Center Line of Web Plate
Connection Angles
Clearance Line
C.L. Top Chord
Top Face of Cover Plate
About 1" clearance
Upper Portal Angles Separated by a 3/8" Gusset Plate
Lower Portal Angles Separated by a 3/8" Gusset Plate
Lateral Plate
Tie Plate
Clearance Diagram
Gusset Plate
BC
Bb
Bc
aB

FIG. 29.

in a plane passing through the centre of the web plate, which is located one inch below the centre line of the pins. For conditions corresponding to Fig. 27 (*a*), it will be best to use a portal of the form shown in Fig. 30 (*a*). In the same way, use the form of Figs. 30 (*b*) and 30 (*c*) respectively, depending upon whether sway bracing of the type shown in Figs. 27 (*b*) or 27 (*c*) was used.

The layout of the portal, Fig. 29, is started by locating the top

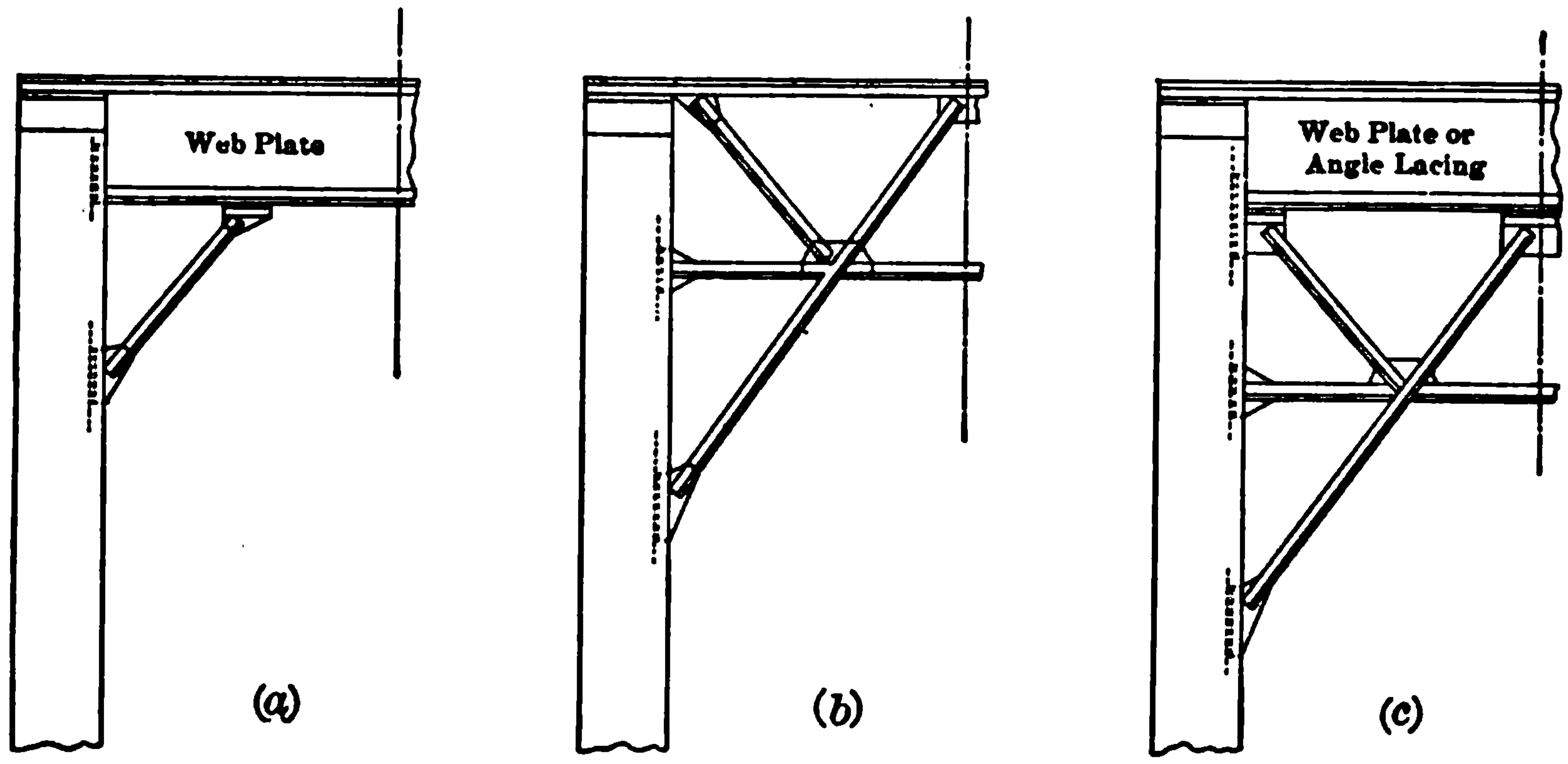


FIG. 30.

portal angles. As the stresses in the portal depend upon its final form, the size of members must be assumed, the portal formed up, and the resulting stresses calculated. For this case the main angles will be taken as $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in. with the $3\frac{1}{2}$ in. legs separated enough to take in the connecting plates. The lower edges of the top angles are placed about one inch above the top of the cover plate on the top chord, as shown in Fig. 29, in order to allow the connecting rivets to be driven. Next locate the horizontal angles just above the top of the clearance diagram. The position of these angles is to be determined in the side view of the post. As shown in Fig. 29, the lower corner of the angles clears the top of the clearance diagram. The main diagonal angles are then located. These angles must be so located that all parts of the connection at the foot of the member are outside the clearance diagram. Fig. 29 shows the sizes of members and the dimensions of the resulting portal frame.

The stresses are to be calculated for the frame shown in Fig. 31.

From Art. 183C, the load carried down the portal from the top lateral system is found to be 15,000 lbs. It will be assumed that the posts are fixed at the base, which places the point of inflection half way between the base of the post and the foot of the portal bracing—that is, points *O* and *O′* of Fig. 31. To check up this assumption we

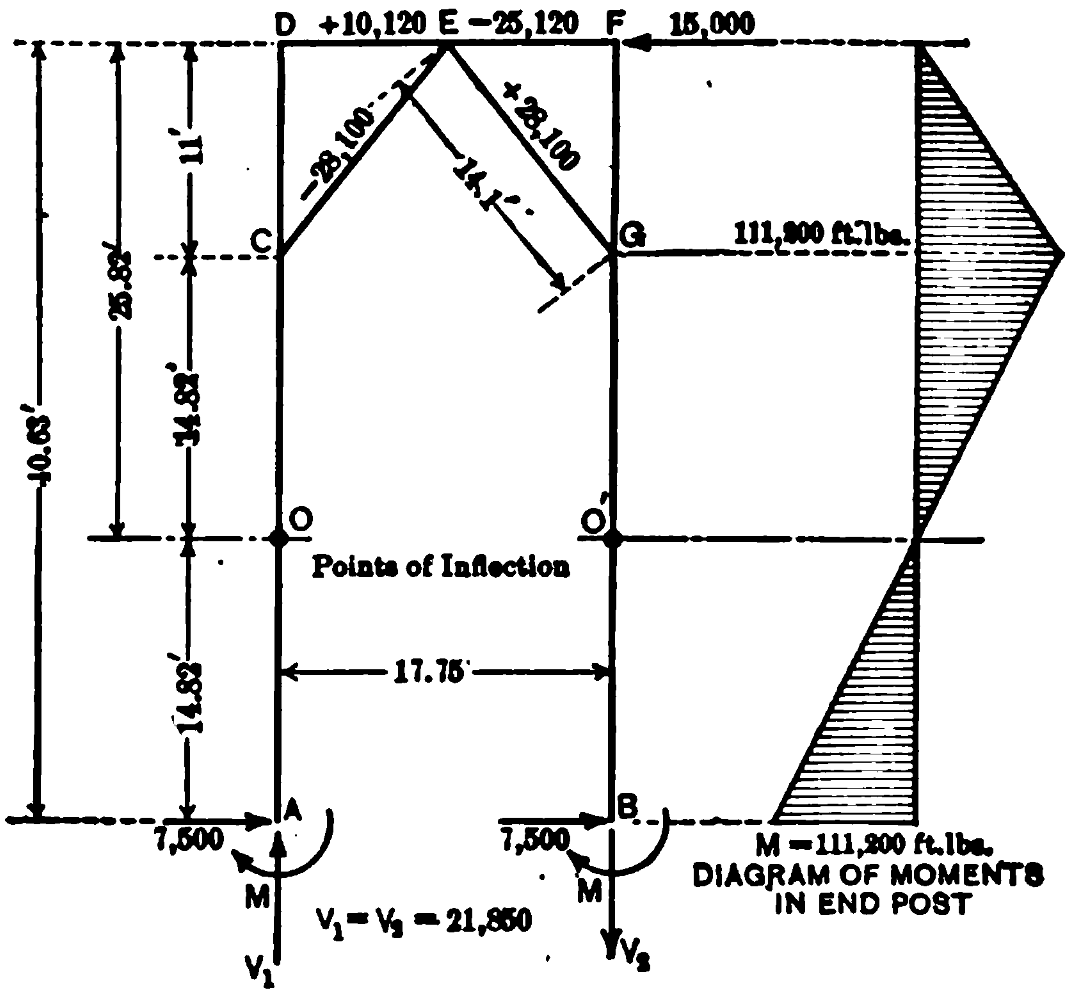


FIG. 31.

will calculate the resisting and overturning moments at the foot of the post by the method given in Art. 186, Part I. The most unfavorable conditions will be for dead load only on the structure. From Table A the dead-load stress in the end post is 140,500 lbs. Assuming the points of support of this member on the pin to be at the outside faces of the web plate, or 19½ ins. apart, as shown in Fig. 14, the resisting moment at the foot of the post is $\frac{1}{2} \times 140{,}500 \times 19\frac{1}{2}/12 = 114{,}000$ ft.-lbs. For the assumed points of inflection, as shown in Fig. 31, the overturning moment is $7{,}500 \times 14.82 = 111{,}200$ ft. lbs. The posts are therefore fixed at the base, as assumed. Proceeding with the calculations as outlined in Art. 185, Part I, the stresses are found to be as shown in Fig. 31.

The layout of the portal frame was made on the assumption that members *CE* and *EG* of Fig. 31 were made up of two $3\frac{1}{2} \times 5 \times \frac{3}{8}$ in. angles. The calculated stress for these members is 28,100 lbs.,

tension or compression, but the area of the member will be governed by the compressive stress. Fig. 29 shows that these members are supported near their centre point by a horizontal member. This member reduces the unsupported length of the member for an axis parallel to the 5-in. legs, but offers no support for the axis parallel to the 3½-in. legs. The radius of gyration of two 3½ × 5 × ⅜ in. angles with the 3½-in. legs separated by a ⅜-in. space is 1.02 ins. for an axis parallel to the 5-in. legs. From Fig. 29, the greatest unsupported length for this axis is from the horizontal member to the rivet at the end of the connection plate, or 4 ft. 7½ ins. Then $l/r = 55\frac{1}{2}/1.02 = 54.5$. For an axis parallel to the 3½-in. legs, the radius of gyration is 2.41 ins. The unsupported length for this axis must be taken as the distance between extreme rivet in the end connections, or 12 ft. 6 ins., as shown in Fig. 29. Then $l/r = 150/2.41 = 62$. This value of l/r governs the allowable working stresses for the member, which is found to be 11,660 lbs. per sq. in. The area required is then 28,100/11,660 = 2.41 sq. ins. The assumed angles provide 6.10 sq. ins., which is more than enough. These angles will be used, however, as a rigid frame is desired.

The end connections for this member are ⅜-in. gusset plates and the rivets are shop rivets in bearing. Then 28,100/7,880 = 4 rivets are required at each end of the member, the required number being shown in Fig. 29 and Plate III. The member is field riveted to the face of the end post by means of 4 × 4 × ⅜ in. connecting angles. As these field rivets are in direct tension when the stress in the member is tension, it will be best to increase the number required for shear. This shear is equal to the component of stress parallel to the end post, or 21,850 lbs., and 21,850/6,010 = 4 field rivets in single shear are required. To allow for the tension in the rivets 12 have been placed in position, as shown on Fig. 29. As the member is connected to one web plate only of the end post, a diaphragm will be placed inside the chord member in order to transfer part of the load to the other web plate. Minimum size material is used, as shown in Fig. 29.

The top member of the portal is made up of two 5 × 3½ × ⅜ in. angles, a section similar to that used for the members just designed. The greatest value of l/r occurs for an axis parallel to the 5-in.

legs, where the unsupported length is distance between outside rivets of the centre and end connections, or about 6 ft., as shown in Fig. 29. Again the angles assumed provide more than enough area, but will be used in order to provide a rigid portal. The other members of the portal have no calculated stress, and are made of the minimum sized angles. For appearance, the middle section of the horizontal angles are made of the same size as the inclined angles. All details are as shown in Plate III.

212. Design of Pins and Pin Plates—General Requirements.—In a pin-connected structure, the connection between the several members meeting at a joint is made by means of a pin. Eye-bar members have a head forged or upset on the bar, and a hole is drilled in this head for the pin, as shown in Fig. 32. Compression members and built-up tension members usually have the hole drilled in the web or side plates of the member. The load carried by other angles and plates used in making up the member must be transferred to the web plate so as to be carried directly to the pin. It therefore happens in many cases that the web plate must carry a very heavy load directly to the pin. To prevent crushing of the web plate, its thickness is increased by riveting some short plates to the web in order to increase its bearing area to the required amount. Such plates are called "pin plates."

Pins are designed with reference to shear, bearing pressure, and bending moment. The shear, however, rarely governs the design. The following articles from the specifications give the allowable working stresses, in lbs. per sq. in., to be used in designing pins.

	Lbs. per sq. in.
(17) Bending on extreme fibres of pins........	24,000
(18) Shearing, on pins........................	12,000
(19) Bearing, on pins.........................	24,000

In the case of eye-bars, the bearing on the pins will be provided for if the pin diameter is made not less then two-thirds of the width of eye-bar. To derive this rule, let the conditions be as shown in Fig. 32, where D = diameter of pin; W = width of bar; and t = thickness of bar. With a bearing stress of 24,000 lbs. per sq. in. on the diameter of the pin, and a tensile stress of 16,000 lbs. per sq. in. in the body of the bar, the conditions for equal strength in bearing and tension are

given by the expression $24{,}000 \times D \times t = 16{,}000 \times W \times t$, from which $D = \frac{2}{3} W$, as stated above.

The provision for bending moment requires the calculation of the maximum bending moment in the pins, the allowable fibre stress being 24,000 lbs. per sq. in., as stated in Art. 17, Specifications. It is

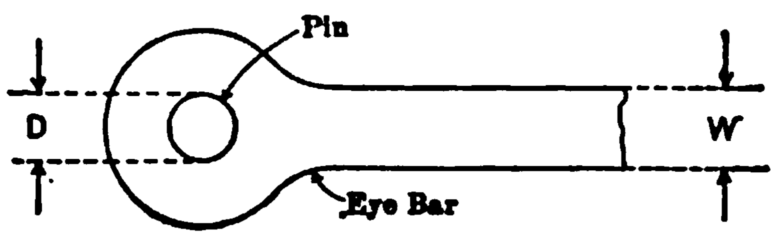


FIG. 32.

usual to assume that all pressures on pins are concentrated at the centres of bearings, an assumption which is on the safe side. Some specifications allow the assumption to be made that the pressures are distributed over a width equal to one-half the width of bearings, but this has very little effect on the computed moments.

In calculating the moments on the pin, it is necessary to make a perliminary assumption of the pin size in order to determine the necessary bearing areas for the riveted members. The moments are then calculated and the pin size determined. If necessary the bearing areas are revised, and finally the true pin size can be found.

As a rule it is desirable to use only two or three different sizes of pins in a structure, in order to reduce the machine shop work to a minimum. For trusses of the type considered in this chapter, the largest pin is usually required at the hip joint. The same size pin is generally used at the end joint of the bottom chord, although the bending moments are usually not as large as at the hip joint. For the remaining bottom-chord joints a single size of pin can usually be used, the size to be determined by conditions at joint c in this case. The smallest pins are required at the interior top-chord joints. Usually the conditions stated for Fig. 32 will determine the pin size at these joints.

In arranging the packing of the members on the pins a certain clear distance is necessary between the members in order to allow of convenient erection. This clearance is generally taken at $\frac{1}{16}$ in. between adjacent eye-bars, and about $\frac{1}{4}$ in. where a riveted member adjoins an eye-bar, or another riveted member. This clearance

implies that the rivets are countersunk and chipped. A greater clearance should be provided, if possible, and the rivets flattened. In the following discussion the design of the several pins will be taken up in detail.

213. Pin at Joint *B*, the Hip Joint.—A 7¾-in. pin will be assumed. Members *B C* and *a B* bear wholly against the pin.

214. *Pin Plates on a B and B C.*—From Table A, Art. 180, the stress in *B C* is 772,000 lbs., and that in *a B* is 748,400 lbs., both compression. Member *B C* requires a bearing area of $772{,}000/24{,}000 = 32.2$ sq. ins. This area is to be made up by pin plates fastened to each of the web plates. The thickness of bearing plates required on the pin at each web plate is $\frac{1}{2} \times 32.2/7.75 = 2.08$ ins. For member *a B*, the required thickness of plates is $\frac{1}{2} \times 748{,}400/(7.75 \times 24{,}000) = 2.01$ ins. Art. 52, Specifications, governs the general arrangement of pin plates.

(52) Pin holes shall be reinforced by plates where necessary, and at least one plate shall be as wide as the flanges will allow and be on the same side as the angles. They shall contain sufficient rivets to distribute their portion of the pin pressure to the full cross-section of the member.

The adopted arrangement of plates is shown in Fig. 33, which is a horizontal section through the joint, showing the members on one-half of the pin; and in Fig. 34, which is a side view of member *B C*. The web plates of members *a B* and *B C* are shown hatched in Fig. 33. Their outside faces are in the same plane, which is 9¾ ins. from the centre line of the truss, as determined from Fig. 14. The outside pin plate on *B C* and the inside plate on *a B* extend around the pin, forming a "hinge plate." This is done in order to hold the members on the pin and to aid in driving the pin. Fig. 34 shows clearly the hinge plate on *B C*. Plates *a* and *g* are made of the same thickness as the bottom angles on the chord member, ⅝ in. thick in this case, in order to act as fillers for other plates. Since the top angles are only $\frac{7}{16}$ in. thick, a $\frac{3}{16}$ in. filler, shown by the hatching in Fig. 34, must be used in order to make an even surface at the top of the member. This filler is very thin and in no way affects the strength of the connection at the top angle. Plate *h* on *a B*, extending over the legs of the angles as required by Art. 52, Specifications, is then added. This plate is made $\frac{7}{16}$ in. thick, as some of

the rivets must be countersunk. From tables giving the dimensions of countersunk rivet heads, it will be found that such a head is $7/16$ ins. high. Therefore the minimum thickness of plates containing

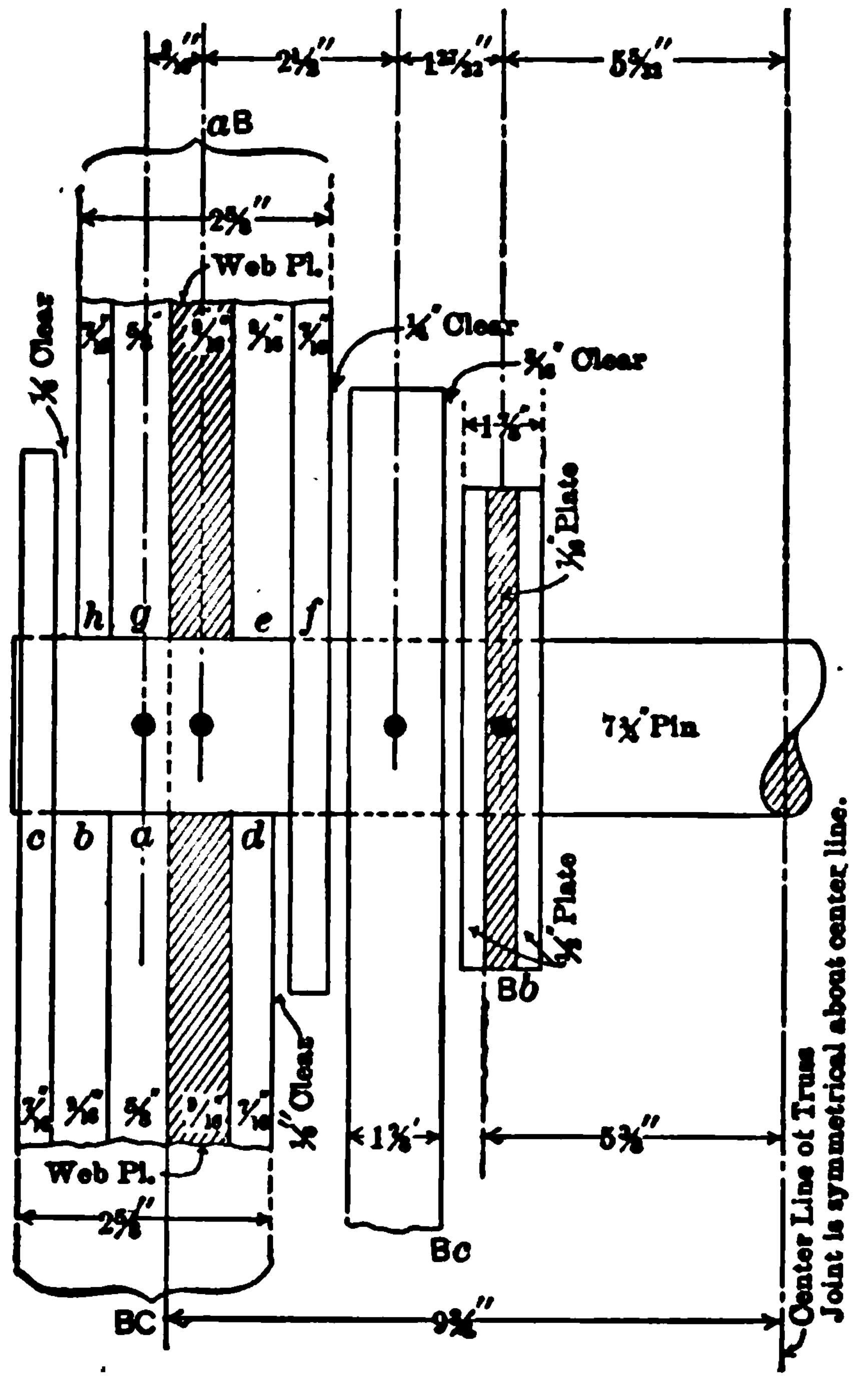


FIG. 33.

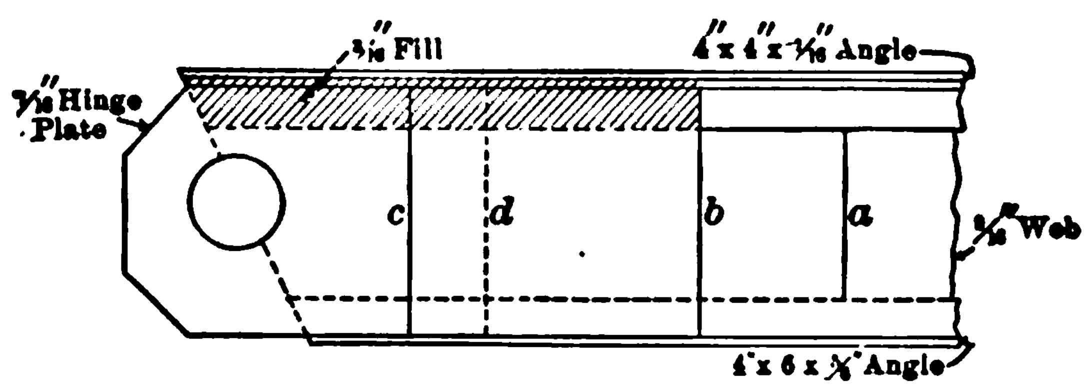


FIG. 34.

countersunk rivets is $7/16$ ins. On the outside of member *B C* are placed plate *b*, $9/16$ ins. thick, and plate *c*, the hinge plate, $7/16$ in. thick. Plate *b* is made $9/16$ in. thick in order to provide $1/8$ in. clearance between the adjacent parts of member *a B* and *B C*, as shown in Fig.

33. On the inside of the members, the plates are made of the thickness shown in Fig. 33, again providing 1/8 in. clearance between the two members. The total thickness of bearing on each member as made up is 2 5/8 ins., which is slightly more than necessary. This is the least thickness obtainable with the number of plates used. A less number is undesirable, as the arrangement shown brings the rivets that connect the angles to the web plate into double shear or bearing, thereby increasing their value.

215. *Pin Plates on B b.*—Member *B b*, whose maximum stress as given in Table A, Art. 180, is 234,900 lbs. tension, requires a width of bearing for each half of the member of $\frac{1}{2} \times 234{,}900/(24{,}000 \times 7\frac{3}{4}) = 0.63$ ins. The end details of this member are subject to the conditions of Art. 28, Specifications, which will determine the design in this case.

(28) Pin-connected riveted tension members shall have a net section through the pin-hole at least 25 per cent in excess of the net section of the body of the member, and the net section back of the pin-hole, parallel with the axis of the member, shall be not less than the net section of the body of the member.

From Table C, Art. 186, the net area of the body of the member is 16.71 ins., which requires a net area of $16.71 \times 1.25 = 20.88$ sq. ins. through the pin-hole. The arrangement of pin plates on member *B b* is shown in Fig. 35. The four angles and one plate which make up the body of the member are cut off 12 ins. below the centre line of the pin. Plates are then riveted over the outstanding legs of the angles in order to connect the member to the pin. In Fig. 33, the hatched plate on member *B b* is of the same thickness as the angles of the main member, 7/16 in. in this case. The position of this filler is shown more clearly by the hatched plates in the front view of Fig. 35, and by *abcd* of the side view. On each side of this filler, and riveted to the legs of the main angles, are placed two 1/2-in. pin plates, shown by *aoef* of Fig. 35. The inner of these plates is slotted to pass around the toes of the main angles. These plates provide a total thickness on each half of the member of 1 7/16 ins. As calculated above, the net area on section $p-q$ of Fig. 35 must be $\frac{1}{2} \times 20.88 = 10.44$ sq. ins. Since the plates are 1 7/16 ins. thick, the width of plates outside the pin must be $10.44/1.4375 = 7.28$ ins. Adding to

this the pin diameter, the total width on line *p–q* must be 7.28 + 7.75 = 15.03 ins. A plate 16 ins. wide will be used.

On the line *r–s*, the specifications require a net area equal to that in the body of the member, which is 8.35 sq. ins. for each half of the member. As the plates are $1\frac{7}{16}$ ins. thick, the net length back

Fig. 35.

of the pin must be at least 8.35/1.4375 = 5.80 ins. Adding to this half the pin diameter gives the required gross length on *r–s* from pin centre to end of member as 5.80 + 3.875 = 9.675 ins. Fig. 35 shows a length of $9\frac{3}{4}$ ins. This provides $\frac{1}{2}$ in. clearance between the top of the member and the cover plate on *BC*, which is $10\frac{1}{4}$ ins. above the pin centre. It will be found that two $\frac{7}{16}$-in. pin plates used in place of the $\frac{1}{2}$-in. plates will provide sufficient net area at section *p–q*, but will not answer the requirements at section *r–s* because of interference with the cover plate of the top chord.

The true width of member *B b*, back to back of angles, can now be determined, as the width used in the preliminary design had to be assumed. Allowing $\frac{1}{4}$ in. clearance on one side of member *B c* and $\frac{3}{16}$ in. on the other, as shown in Fig. 33, the distance from the back of angles on *B b* to the centre line of truss is found to be $5\frac{3}{8}$ ins., which makes the width of member *B b* $10\frac{3}{4}$ ins. back to back of

angles. A plate 10 ins. wide is then the correct size of plate for the body of the member. As this is the size assumed in Table C, no revision of the area of the member is necessary.

216. *Moment in Pin.*—The pin will receive its maximum bending moment when the stresses in both *B b* and *B c* are large. While the maximum stresses do not occur simultaneously, it will be found that the position of loads causing a maximum stress in *B b* also causes a stress in *B c* not much below its maximum. This position of loads will therefore be taken as giving the maximum pin moment. For the maximum stress in *B b*, wheel 4 was placed at joint *b*, which also gave the maximum stress in *a B*. From Table A, the live-load stresses in these members are 113,500 lbs. tension for *B b*, and 370,400 lbs. compression for *a B*. Knowing the live-load stresses in these members, the simultaneous stresses in *B C* and *B c* can be calculated from the equilibrium of joint *B*.

The impact percentage to be used must evidently be the same for all members, as the pin is in equilibrium under applied forces. This percentage will be taken as 70 per cent., which in this case is roughly an average for all members entering the joint.

TABLE E

FORCES ACTING ON PIN AT JOINT *B*

Member	Vertical Components				Horizontal Components
	Live Load	Impact	Dead Load	Total	
aB...........	284,900	199,400	96,000	580,300	483,000
Bb...........	113,500	79,500	24,000	217,000	0
Bc...........	171,400	119,900	72,000	363,300	302,500
BC...........	......	......	.....	......	785,500

The dead-load stresses for members *B b* and *B c* will be taken as computed in Table A, and the simultaneous stresses in *a B* and *B C* will be determined from the equilibrium of joint *B*. It will be noted that the dead-load vertical component for *a B*, as given in Table E, is less than would be obtained from the dead-load stress for this member given in Table A. This is due to the fact that in considering the equilibrium of joint *B*, the dead joint load at this point has been neglected. As the dead-load stresses are a relatively small

proportion of the total stress, this discrepancy is of no practical consequence.

Table E gives in convenient form the partial vertical components in the several members, and also the total vertical and horizontal components. Fig. 36 shows the direction in which the several components act on the pin.

The maximum resultant bending on the pin is to be calculated

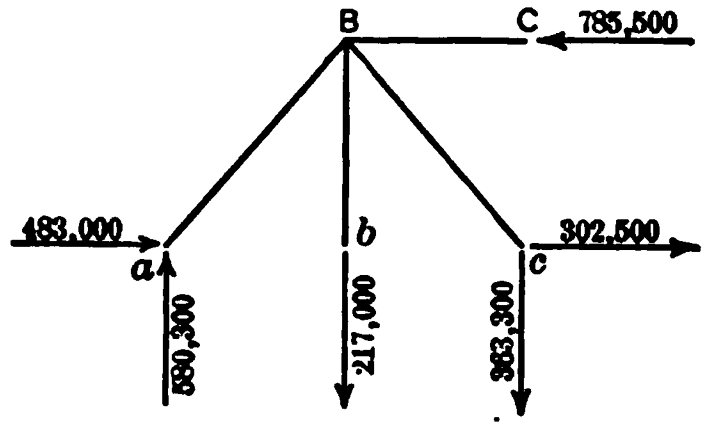


FIG. 36.

for the forces given in Table E. These forces are applied at the centres of bearings of the several members, as shown in Fig. 33. The

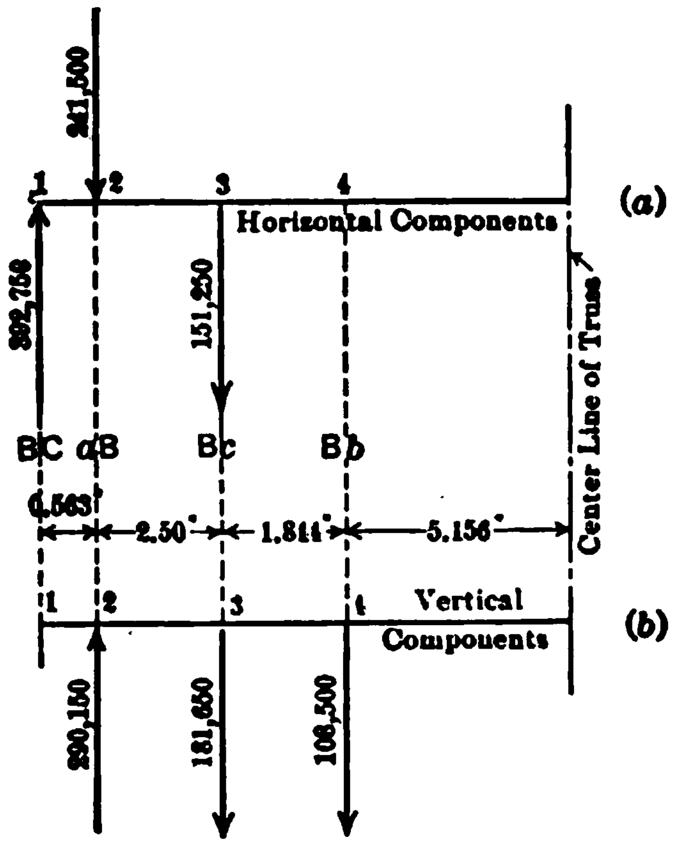


FIG. 37.

forces acting on one-half of the pin, taken from Table E, and the lever arms of the forces, taken from Fig. 33, are shown in Fig. 37. By calculating the resultant moment for each of the points 2, 3, and 4, and comparing results, the maximum moment can be determined.

Experience, however, will usually enable the point of greatest moment to be determined by inspection. In this case the maximum moment occurs at point 4, and only the moment at this point will be calculated. The moment due to the two sets of components will be calculated separately. The horizontal moment at 4 is equal to that at 3, since the resultant of the horizontal components of $B\,C$ and $a\,B$ and $B\,c$ is a couple. This moment is equal to 392,750 (0,563 + 2.50) − 241,500 × 2.50 = 599,000 in.-lbs. For the vertical components, the moment at point 4 is 290,150 (2.50 + 1.844) − 181,650 × 1.844 = 926,000 in.-lbs. The resultant of the moments is $\sqrt{(599{,}000)^2 + (926{,}000)^2} = 1{,}103{,}000$ in.-lbs. From a table of bending moments on pins for a fibre stress of 24,000 lbs. per sq. in., it will be found that a 7¾-in. pin can carry a bending moment of 1,096,800 in.-lbs. As the actual moment exceeds the allowable moment by only 0.55 per cent., the assumed pin will be adopted as final. If the assumed and calculated pin sizes do not check, the bearing areas and calculated moments must be revised.

In case a table of bending moments on pins is not at hand, the general formula for flexure, $f = Mc/I$ can be used. Substituting the value of section modulus of a circle, and fibre stress $f = 24{,}000$ lbs. per sq. in., the formula becomes $d = (0.000426\ M)^{\frac{1}{3}}$, where d is the diameter of the pin in inches and M is the bending moment in in.-lbs. For the moment calculated above, the required pin diameter is 7.78 ins.

217. Pin at Top-Chord Joint C.—The pin moment at this joint will be a maximum when the stresses in $C\,d$ and $C\,c$ are a maximum. As the top chord is continuous over the joint, the chord members do not bear wholly on the pin, as at joint B. The chord stress taken by the pin is equal to the difference between the stresses in the adjacent chord members. This difference in stress is a maximum when the diagonal $C\,d$ has its maximum stress. The components of all forces acting on the pin can then be determined from the maximum stress in $C\,d$, which from Table A, Art. 180, is 352,400 lbs. tension. Considering the joint in question to be in equilibrium, the various components are as given in Table F. Again, as in the case of joint B, it will be noticed that the stress in $C\,c$ when joint C is in equilib-

rium differs from the maximum stress given in Table A by the dead joint load at joint *C*.

TABLE F

FORCES ACTING ON PIN AT JOINT *C*

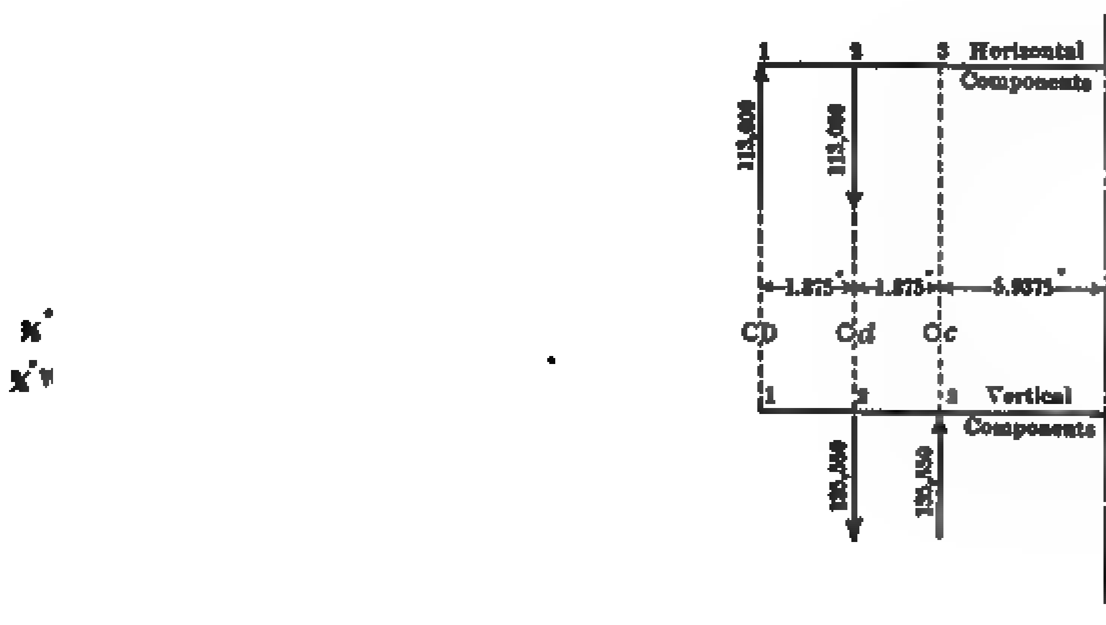
Member	Vertical Components	Horizontal Components
CD	0	226,000
Cd	271,100	226,000
Cc	271,100	0

A $5\frac{1}{2}$-in. pin will be assumed for joint *C*. From the discussion given in Art. 212, the least pin diameter allowable at any joint is two-thirds of the width of eye-bar at the joint. In this case *Cd* is a 7-in. bar. The least allowable pin is then $\frac{2}{3} \times 7 = 4\frac{2}{3}$ ins. in diameter.

The necessary thickness of bearing plates on each web of *C D* is $\frac{1}{2} \times 226{,}000/(24{,}000 \times 5\frac{1}{2}) = 0.856$ ins. As the web plate on *C D* is $\frac{3}{4}$ in. thick, a $\frac{3}{8}$-in. pin plate will be added to make up the necessary bearing area. From Table A, the maximum stress in *C c* is 283,100 lbs. This requires a bearing thickness for each half of the member of 1.07 ins. Member *C c* is made up of two 15-in. 45-lb. channels, as given in Table C. These channels have a 0.62-in. web. A $\frac{1}{2}$-in. pin plate, placed inside the channel, as shown in Fig. 38, is required to make up the necessary bearing area.

The spacing back to back of channels for member *C c* has not been definitely determined. From calculations made in Art. 188, it was found that the least spacing of channels was $12\frac{3}{4}$ ins. This spacing will be adopted as final. The clear distance between the faces of the eye-bar *C d* will be $\frac{1}{2}$ in. on each side, as shown in Fig. 38. The forces acting at the centre of the bearings of the members on one-half the pin are shown in Fig. 38(*b*). Point 3 is the point of maximum moment, and the moment of horizontal components is $113{,}000 \times 1.875 = 212{,}000$ in.-lbs.; the moment of vertical components is $135{,}550 \times 1.875 = 254{,}000$ in.-lbs.; and the resultant moment is $\sqrt{(212{,}000)^2 + (254{,}000)^2} = 331{,}000$ in.-lbs. A $5\frac{1}{2}$-inch pin will carry a moment of 392,000 in.-lbs. The assumed pin is a little too large, but as it is a convenient size, it will be used.

218. Pin at Top Chord Joint *D*.—In order to keep the number of pin sizes small, a 5½-in. pin will also be assumed for joint *D*. As is the case of joint *C*, the width of bearing required on the web plates of the top chord is determined by the horizontal component of the



(a) (b)

FIG. 38.

maximum stress in the diagonal. From Table A this maximum stress is 181,900 lbs. tension. Its horizontal component is found to be 116,500 lbs., and hence the width of the bearing for each half of the top chord member is 0.441 ins. Since the web plate on this member is ¾ in. thick, no pin plates are required.

Member *D d* has a maximum stress of 151,900 lbs. compression, which requires a width of bearing of 0.575 in. for each half of the member. As the web of the 15-in. 33-lb. channels, of which the member is composed, is only 0.40 in. thick, pin plates must be added to make up the required bearing area. A ⅜-in. plate will be added, placed inside the channels, as shown in Fig. 39(*a*). The distance back to back of channels in member *D d* will be made 12¾ ins., the same as for *C c*. Fig. 39(*a*) shows the arrangement of members, lever-arms and clearances provided.

In calculating the maximum bending moment on the pin, two

cases must be considered, due to the presence of a counter at the centre of the pin, as shown in Fig. 39(*a*). When the stress in the diagonal $D\,e$ is a maximum, which will be called Case A, the components of forces on one-half the pin are as shown in Fig. 39(*b*); and when the stress in the counter $D\,c$ is a maximum, which will be called Case B, the components are as shown in Fig. 39(*c*). These components are calculated by the method used in Art. 217 for joint C, using in each case the maximum stress for the diagonal member in question, as given in Table A. From these components, the resultant moment for Case A is found to be 154,500 in.-lbs., and that for Case B is 221,000 in.-lbs. Case B governs the design of the pin. The assumed pin is somewhat larger than required to carry the moment, but it will be used for reasons stated at the beginning of this article.

219. General Arrangement of Lower Chord Members.—The eye-bars or built-up channels making up the bottom chord of a pin-connected truss must be so arranged that the bending moments on the pins are as small as practicable, and so that the requirements of Art. 83, Specifications, are fulfilled.

(83) The eye-bars comprising a member shall be so arranged that adjacent bars shall not have their surfaces in contact; they shall be as nearly parallel to the axis of the truss as possible, the maximum inclination of any bar being limited to one inch in 16 feet.

In order to realize all of the above conditions, it is best to study each joint individually, paying particular attention to the packing of members on the pins for least bending moment. After this has been done, the several joints must be studied collectively in order to make certain that the maximum allowable inclination of bars between adjacent joints has not been exceeded. It will probably be found that several or all of the joints will have to be rearranged. The problem is then to make the required changes and still keep the bending moments as small as possible.

In trusses of the size considered in this chapter, all of the members at any point should be packed outside of the channels comprising the vertical compression members. An exception to this general rule is the case of a counter member made up of a single bar. Such a member can be placed at the pin centre. A little study of

Fig. 39 will show that bars placed at the pin centre cause large bending moments unless the stress in the bar is small. This arrangement should therefore be used only when the stress in the counter

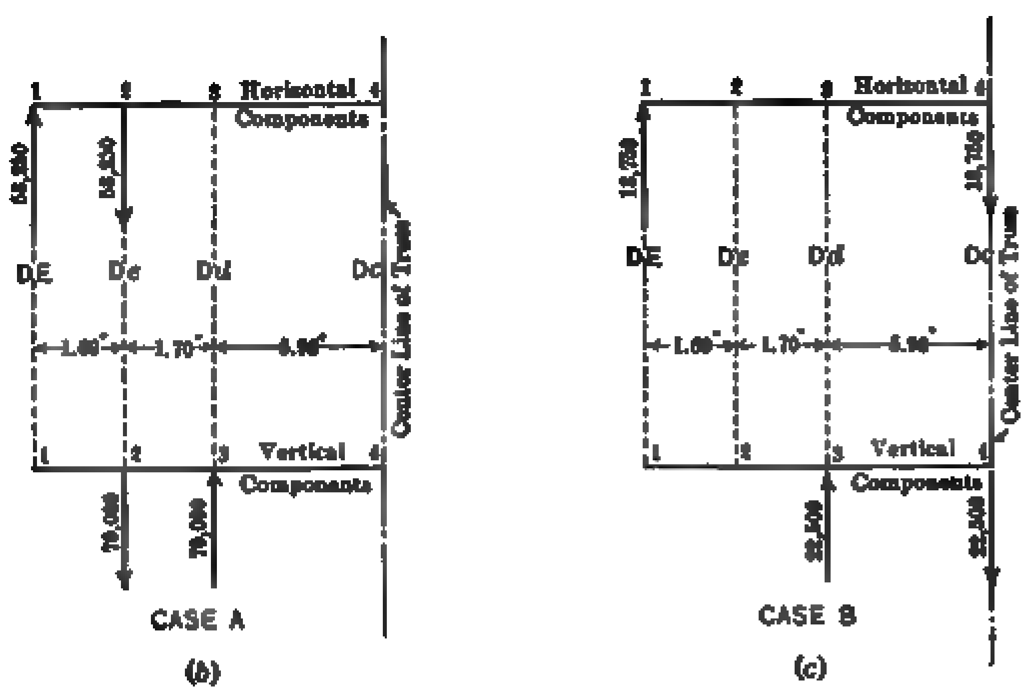


FIG. 39.

is small. If a large stress exists in the counter, two bars should be used, even though excess area is provided.

When bars in pairs are placed between the channels of the posts,

it will be found that the conditions for least bending moment require a bar to be placed as close to each channel as possible. This usually means that the flanges of the channel must be clipped away in order to allow the bars to enter the joint. The channels are thereby greatly weakened at the foot, where very heavy concentrations exist. Such clipping away of the flanges of the channels is not advisable, a consideration which leads to an arrangement with all members placed outside the channels, as stated above.

In determining the arrangement of bottom-chord members for the truss considered in this chapter, it was found best to start with joint *d*, which is the widest bottom-chord joint. A layout of this joint was made using the least possible clearance between bars in order to obtain as narrow a joint as possible. The proper bars were then projected to joint *c*, and the members at this joint arranged. It was found that large clear distances between the bars would be required if the members were run truly parallel to the truss axis. In order to reduce these clearances the outer bars were given the maximum allowable slope across the panel. As member *a b c* is a built-up member, the conditions at joints *a* and *c* had to be considered together in order to make certain that the members would clear the end post. The working out of the adopted arrangement will be taken up in detail in the following articles.

220. Pin at Lower Chord Joint *d*.—A 7-in. pin will be assumed. The bearing areas of all of the eye-bars will be properly taken care of since the assumed pin is more than two-thirds of the width of any of the bars. Member *D d* must have sufficient bearing area to take care of its maximum stress, as given in Table A, Art. 180, and the simultaneous floor-beam load at point *d*, since the floor-beam is riveted directly to the foot of the post, as shown in Fig. 20 and Plate III. It can be shown, however, that the greatest bearing pressure on the pin for *D d* occurs when the diagonal *C d* has its maximum stress. From Table A, Art. 180, the maximum stress in *C d* is 352,400 lbs. tension. The vertical component of the stress, which is 271,000 lbs., is taken by member *D d*, and requires a width of bearing for each half of the member of 0.806 ins. Member *D d* is made up of 15-in. 33-lb. channels, which have 0.40-in. webs. A 7/16-in. pin plate will be added, placed outside the channel, as shown in Fig. 40.

The arrangement of members at joint *d* will be as shown in Fig. 40. As stated in Art. 219, it will be best to place all members outside of the member *D d*. The channels composing *D d* are to be spaced 12¾ ins. back to back, as determined in Art. 218 for joint *D*.

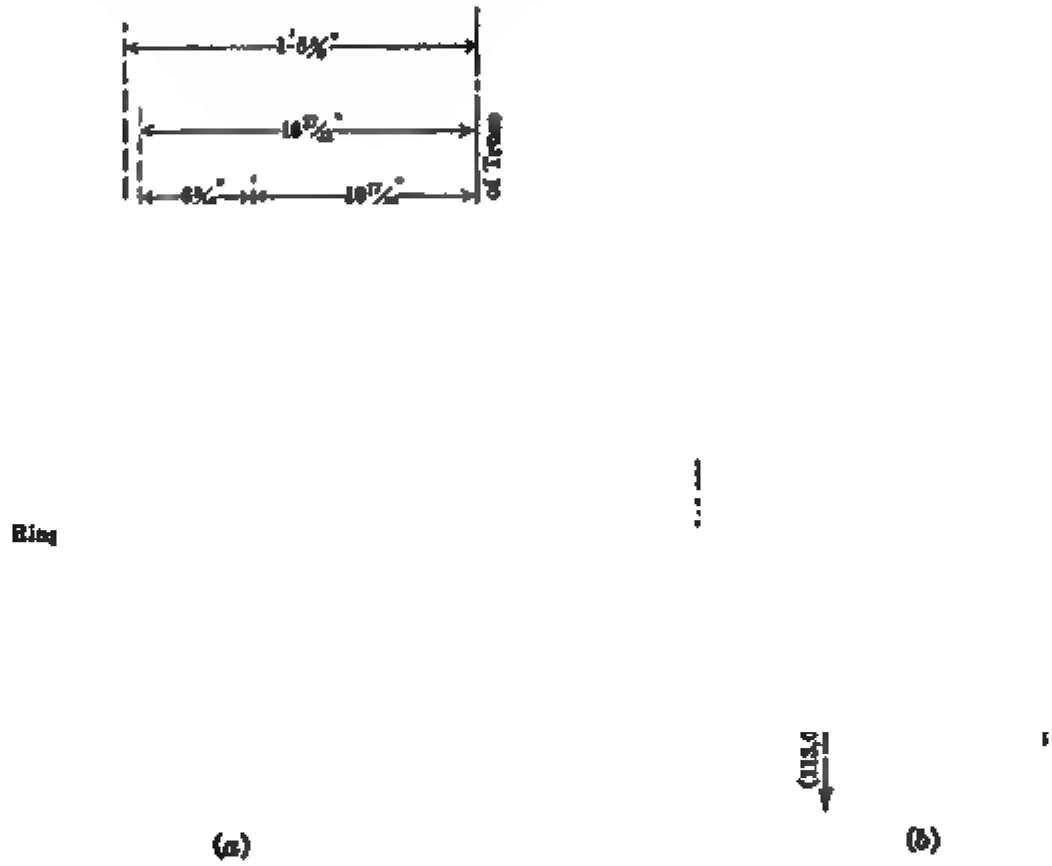


FIG. 40.

In order to keep the lateral inclination of the diagonal bars between joints within the limits specified in Art. 83, Specifications, these bars will be placed just outside the post. As the diagonals are 39 ft. long, the allowable inclination between joints *C* and *d* is $2\frac{7}{16}$ ins. Placing the bar of *C d* next to the post, with clearances as shown on Fig. 40(*a*), the allowable limit will not be exceeded. Outside these members, the bars of *c d* and *d e* are placed as shown in Fig. 40(*a*). The arrangement here shown will produce the least bending moment on the pin and, at the same time, answer the requirements of Art. 83, Specifications, regarding the maximum allowable slope of bars between panel points. It will be found that a smaller bending moment on the pin is produced when the bars are arranged as shown in Fig. 22(*b*), page 249. But as this arrangement leads to slopes in excess of the allowable, it will be necessary to adopt that shown in Fig. 40.

Fig. 40(*b*) is a diagram of the forces acting on one-half of the pin. From this diagram it can be seen that the moment of the horizontal components about point 6 is equal to the moment of the resultant of the stresses in members *c d* and *d e*, shown by the dotted arrow, about this same point. It is evident from the diagram that this resultant is equal to the difference between stresses in *c d* and *d e*. Therefore the maximum horizontal bending moment occurs when the difference in stress between *c d* and *d e* is a maximum, which occurs when the stress in diagonal *C d* is a maximum. The stresses shown for *c d* and *d e* on Fig. 40(*b*) have been calculated for the loading conditions which gave the maximum stress in *C d*. In this particular case, however, it is not necessary to calculate the stresses in *c d* and *d e*, for the position of their resultant can be determined by inspection. Since the bars of *c d* are spaced equal distances on each side of those of *d e*, and since equal stresses exist in each pair of bars, it is evident at once that the resultant lies half way between points 2 and 3 of Fig. 40(*b*). Also, from equilibrium of horizontal forces, it can be seen at once that the resultant of the stresses in *c d* and *d e* is equal to the horizontal component of maximum stress in *C d*. The dotted arrow in Fig. 40(*b*) represents this resultant force in position and direction.

The maximum moment in the pin occurs at point 7. From Fig. 40(*b*) the moment of horizontal forces is $113{,}000 \times 5.88 = 665{,}000$ in.-lbs.; the moment of vertical forces is $135{,}500 \times 1.42 = 192{,}500$ in.-lbs.; and the resultant moment is 692,000 in.-lbs. As the 7-in. pin assumed will carry a bending moment of 808,200 in.-lbs. it will be adopted.

The moment on the pin was also calculated for maximum stress in the member *d E*. It was found that the resulting moment was very much smaller than that given above.

221. Pin at Lower Chord Joint *c*.—A 7-in. pin will also be assumed for this joint. The general arrangement of members is as shown in Fig. 41(*a*), which is a horizontal section through joint *c*. The channels of member *C c* are to be spaced 12¾ ins. back to back, as determined for joint *C* in Art. 217. As in the case of member *D d*, the bearing area on *C c* must be made up to take care of the vertical component of the maximum stress in the diagonal member

entering the joint. From Table A, Art. 180, the maximum stress in diagonal *B c* is 540,900 lbs. tension, and its vertical component is

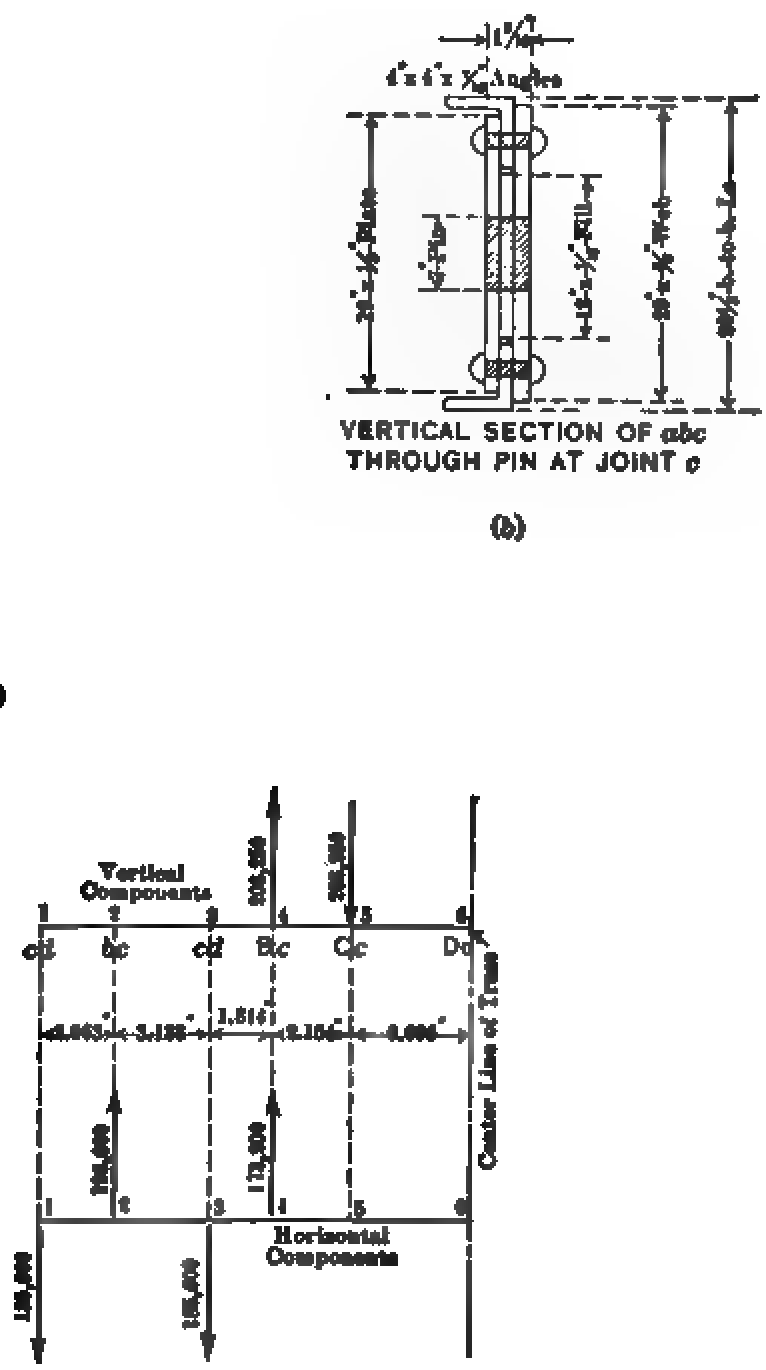


FIG. 41.

416,500 lbs. The bearing area required for each half of the member is then 1.24 ins. As the web of a 15-in. 45-lb. channel is 0.62 in. thick, the necessary bearing area will be provided by placing a ⅝-

in. plate inside the channels, and a $^{7}/_{16}$-in. plate outside, as shown in Fig. 41 (*a*).

The pin plates on the built-up tension member *abc* will be determined by the requirements of Art. 28, Specifications. From Table C, the net area in the body of the member is 31.49 sq. ins. The net area through the pin-hole must then be $\frac{1}{2} \times 31.49 \times 1.25 = 19.68$ sq. ins. for each half of the member. Fig. 41 (*b*) shows a section of the member cut through the pin. The angles and plates shown in the figure provide a gross area of 33.87 sq. ins. Allowing for the assumed 7-in. pin and the rivets shown by the hatched areas, the net area of the member is found to be 19.80 sq. ins.

In order to keep the inclination of *B c* within the specified limits, it will be placed just outside of the vertical posts, a clearance of $\frac{5}{8}$ in. being provided. This clearance will probably not require the flattening of rivet heads through the pin plates on *C c*, as the standard $\frac{7}{8}$-in. rivet head is $^{39}/_{64}$-in. high. The rivet heads will act as a separator between the adjacent members.

The position of the bars of member *c d* must be determined with respect to their effect on the pin moment at joint *c*, and also with reference to the position of the same bars at joint *d*, in order to keep the inclination of the members between joints within the allowable limits. As the panels are 25 ft. long, the allowable slope is $1^{9}/_{16}$-ins. per panel. Placing the outer bar of *c d* $15^{11}/_{32}$-ins. from the centre line of the truss, as shown in Fig. 41 (*a*), its slope across the panel is found to be $1\frac{1}{2}$ ins.

From the force diagram for horizontal forces of Fig. 41 (*c*), it can be seen that with the outer bar of *c d* placed as indicated above, the least moment on the pin will occur when *a b c* is placed close to the outer bar of *c d*, and when the inner bar of *c d* is placed close to *B c*. Accordingly, the inner bar of *c d* will be placed with a clearance of $\frac{1}{8}$ in. between it and *B c*, and *a b c* will be placed with a clearance of $\frac{1}{2}$ in. between it and the outer bar of *c d*. This arrangement locates the outer face of the web plate $13\frac{1}{8}$ ins. from the centre line of the truss, as shown in Fig. 41 (*a*). Before this spacing of *a b c* can be accepted as final, the conditions at point *a* must be investigated. As member *a b c* is a built-up tension member, the web plates must be parallel to the centre line of the truss for the full length of the

member. Referring to Fig. 42 it can be seen that *a b c* clears the end post at joint *a*. A further discussion of the conditions at joint *a* is given in Art. 222.

The greatest moment on the pin at joint *c* will be found to occur when member *B c* has its maximum stress. Fig. 41 (*c*) shows the forces acting on one-half of the pin for maximum stress in *B c*, which occurs when wheel 3 is at joint *c*. The stresses shown for *c d* and *a b c* are the simultaneous stresses for these members. In this case the short-cut described for joint *d* cannot be used, as the bars of *c d* are not symmetrically placed about *a b c*. The maximum resultant moment, as calculated for the forces shown in Fig. 41 (*c*), occurs at point 5, and its amount is 799,000 in.-lbs. As the assumed 7-in. pin can carry a bending moment of 808,200 in.-lbs., it will be adopted.

The counter member *D c* is placed on the centre line of the truss to conform to the arrangement adopted at joint *D*. When the stress in *D c* is a maximum, the greatest moment on the pin is found to be 248,000 in.-lbs. It will usually not be necessary to calculate the moment for this stress condition, as the counter has little effect on the pin moment, due to its small stress, and due to the fact that the simultaneous stresses in the other members on the pin are also small.

222. Pin at Lower Chord Joint *a*.—The arrangement of members at joint *a* will depend upon the type of end floor-beam used. In Arts. 201 to 204 two types of end floor-beams were designed. The beam shown in Fig. 23 rests directly on the shoe. When this type of beam is used, the pin moment is due to the stresses in the end post, bottom chord, and shoe. In Fig. 25 the end floor-beam bears on the pin which must be designed to carry the floor-beam load in addition to the stresses from the truss member. In this article the end pin will be designed for both types of floor-beam.

A 7¾-in. pin will be assumed for this joint. This is the same size as adopted at joint *B* at the other end of the end post. When the floor-beam rests directly on the shoe, as shown in Fig. 23, the arrangement of members is as shown in Fig. 42 (*a*), which is a half-vertical section through the joint. The width of bearing required for member *a B* is determined from the maximum stress in the member as given in Table A. This requires a width of $748{,}400/(2 \times 7\frac{3}{4} \times 24{,}000) = 2.01$ ins. for each half of the member. The arrange-

ment of plates shown in Fig. 42 (*a*) provides 2½ ins. of bearing. In this form of end detail, the shoe web plates must provide a bearing area great enough to take the vertical component of the maximum stress in the end post, which is found to be 575,000 lbs. This requires a width of bearing of 1.55 ins. for each half of the shoe. Fig. 42 (*a*) shows three ⅝-in. web plates and a ⅜-in. hinge plate, which provide a total thickness of 2¼ ins. As the shoe is rather deep, the thickness of plates provided is made greater than that required in order to increase the lateral rigidity.

The pin plates on member *a b c* are determined by the requirements of Art. 28, Specifications. Proceeding as in the design of pin plates for this member at joint *c*, given in Art. 221, it will be found that the net area required through the pin-hole is 19.68 sq. ins. for each half of the member. Fig. 42 (*a*) shows an arrangement of plates for *a b c* which provides a gross area of 38.62 sq. ins. and a net area of 20.94 sq. ins.

The distance back to back of angles for member *a b c* was determined in Art. 221 for conditions at joint *c*. This same spacing must be used at joint *a*, as *a b c* is a built-up tension member. Placing the back of the angles on *a b c* 13⅛ ins. from the centre line of truss, as shown in Fig. 42 (*a*), it will be found that the clearance between the hinge plate on *a B* and the web plate on *a b c* is 1¼ ins.

If the clearance between *a B* and *a b c* is much larger than in Fig. 42 (*a*), say 1¾ ins. or more, a rearrangement of members must be made in order to reduce this clearance. This can be done in several ways. As the width of *a b c* has been fixed by conditions at other joints, it may be possible to reduce the width of this member by making changes at other joints, such as reducing clearances. In the present case no such changes can be made, for the minimum clearances have been used at joint *d*. This fixes the width of *a b c* in this case, so that the changes must be made elsewhere. It is possible to increase the distance between the centre line of truss and the angles of the chord section, thus reducing the clearance. This change would require a wider cover plate on the chord section. It would also make necessary a revision of all joints of the top chord. Except in extreme cases such a change is not advisable. A somewhat better method of reducing the clearance is to place the shoe

web plates outside the end post. In order to make this change, the hinge on the end post must be placed inside the member, and that on the shoe must be placed outside the web plates of the shoe. This change places the hinge plate of the shoe between *a b c* and

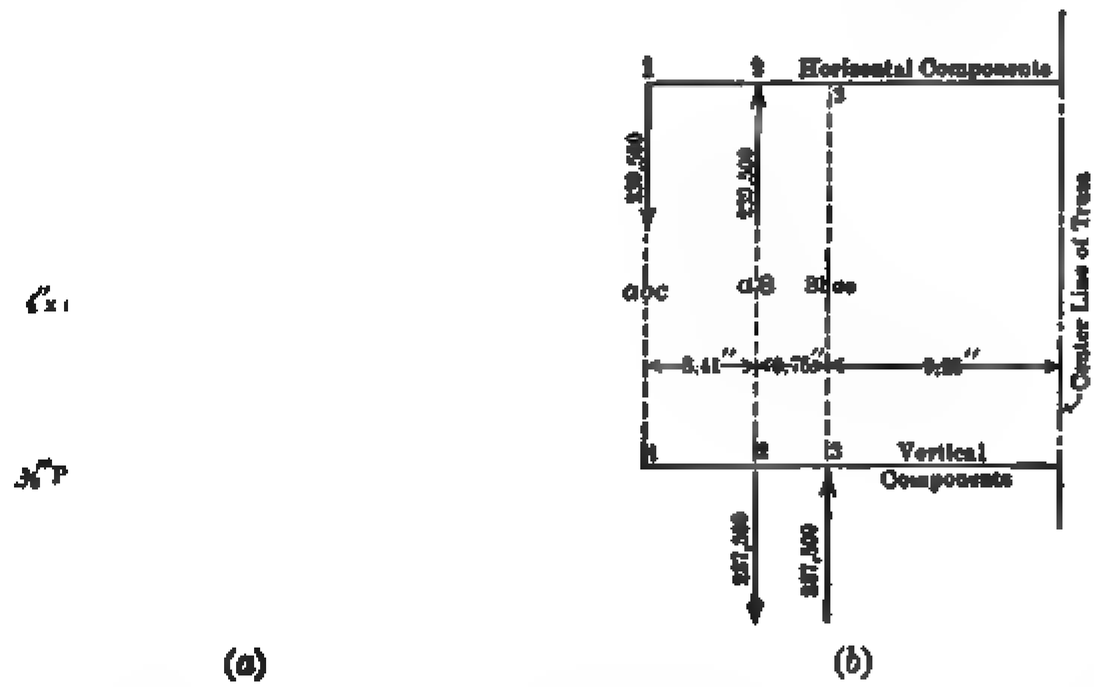


FIG. 42.

a B, thus dividing the clearance into two parts. The objection to this arrangement is that the bending moment on the pin is somewhat greater than for the arrangement shown in Fig. 42(*a*), which gives the least bending moment.

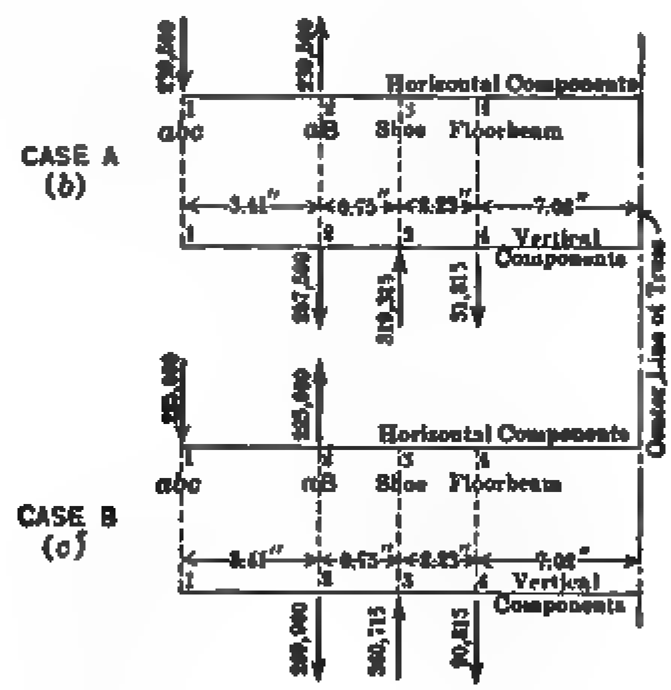
The forces acting on one-half of the pin, when the stress in the end post is a maximum, are shown in Fig. 42(*b*). From this diagram, the resultant moment on the pin is found to be 845,000 in.-lbs. A 7¾-in. pin can carry a moment of 1,096,800 in.-lbs. The assumed pin will therefore be adopted.

Fig. 43(*a*) shows the arrangement of members when the end floor-beam rests directly on the pin, as shown in Fig. 25. This detail differs from that shown in Fig. 42(*a*) only in that a pair of channels have been added to carry the floor-beam load to the pin. From Art. 202, this floor-beam load is found: 181,630 lbs. The width of bearing required for each half of the member is then 0.488 in. As the web of the 15-in. 33-lb. channels is 0.40 in. thick, a pin plate must be added. This plate will be made 7/16 in. thick, so that the

rivets can be countersunk. It will be placed on the outside of the channel, as shown in Fig. 43(*a*). A convenient spacing for the channels is found to be 14 ins. back to back, which provides a clearance of $^{11}/_{16}$ in. between the channels and the hinge plate of the shoe.

The bending moment on the pin must be calculated for two loading conditions, which are: Case A, stress in the end post a maximum; and Case B, the load on the end floor-beam a maximum.

For Case A, the stress in the end post is a maximum, and the components for *a B* and *a b c* are the same as shown on Fig. 42(*b*).



(*a*)

FIG. 43.

The loads to be carried by the end floor-beam and shoe remain to be determined. When the end post has its maximum stress, wheel 4 is at point *b*. The reaction for each stringer at the end floor-beam, due to the loads in panel *a b*, is 28,800 lbs., and the impact allowance is 27,200 lbs., the loaded length being 18 ft. From Art. 202, the dead-load reaction at the end of the floor-beam is 6,130 + 1,500 = 7,630 lbs. The total load on the pin due to the end floor-beam is then 63,630 lbs. As the pin is in equilibrium, the load on the shoe must be the sum of the vertical component of stress in the end post and the load on the floor-beam. This load on the shoe is found to be 638,630 lbs. Fig. 43(*b*) shows the forces acting on one-half of the pin. The greatest moment on the pin will be found to occur at point 3, and the resultant moment at this point is 845,000 in.-lbs. It will be

noted that this is exactly the same as the moment determined for the same point for the forces shown in Fig. 42(*b*).

The conditions of loading for Case B are such as to produce the maximum end floor-beam reaction. As shown in Fig. 21 of Art. 202, wheel 2 is placed one foot in front of the end floor-beam. The resulting end floor-beam reaction is given in Art. 202 as 181,630 lbs. In members *a B* and *a b c*, the simultaneous stresses are found to be 702,000 lbs. compression for *a B* and 450,000 lbs. tension for *a b c*. These stresses include allowances for impact, which are determined for the loaded length causing the stress. Thus for the end floor-beam reaction the loaded length is 25 ft., a panel length, while for *a B* and *a b c*, the full length of the truss is used. The load carried by the shoe will be taken as equal to the sum of the end floor-beam reaction plus the vertical component of the simultaneous stress in the end post, a total load of 721,430 lbs. Fig. 43(*c*) shows the components of forces on one-half of the pin. The maximum resultant moment occurs at point 3 and its amount is 790,000 in.-lbs.

The foregoing calculations show that the assumed 7¾-in. pin is large enough to care for the greatest moment for either of the types of floor-beam.

223. Detail at Lower Chord Joint *b*.—The function of the connection at joint *b* is to transfer to member *B b* part of the weight of member *a b c*. Several methods of making this connection are in common use. In some cases a small-sized pin is used.

The riveted joint shown in Fig. 44 provides a simple and rigid connection. This is made by extending the angles of member *B b* down between the side plates of member *a b c*. At the foot of the member the web plate is cut away and a short, wide plate riveted between the angles of *B b*. This plate is then connected to *a b c* by four angles of minimum size. The details and sizes of material for this case are as shown in Fig. 44. In riveting the connection angles to the web plates of *a b c*, care must be taken to preserve the net section of the members, as shown in Fig. 11, Art. 186. The rivets shown on Fig. 44 are more than enough to provide for the load to be transferred to member *B b*.

224. The Adopted Pin Sizes.—From the calculations in the preceding articles, the pin sizes adopted for the several joints are as

follows: Joints *a* and *B*, 7¾ ins.; joints *c* and *d*, 7 ins.; joints *D* and *C*, 5½ ins. Before these pin sizes can be adopted as final, it is necessary to examine each joint in detail in order to make certain that the eye-bar heads will enter the joints without interfering with other members.

The conditions at joint *B* are shown in a general way by Fig. 9, Art. 185 (*B*). From Art. 191 it will be found that a 20-in. eye-bar

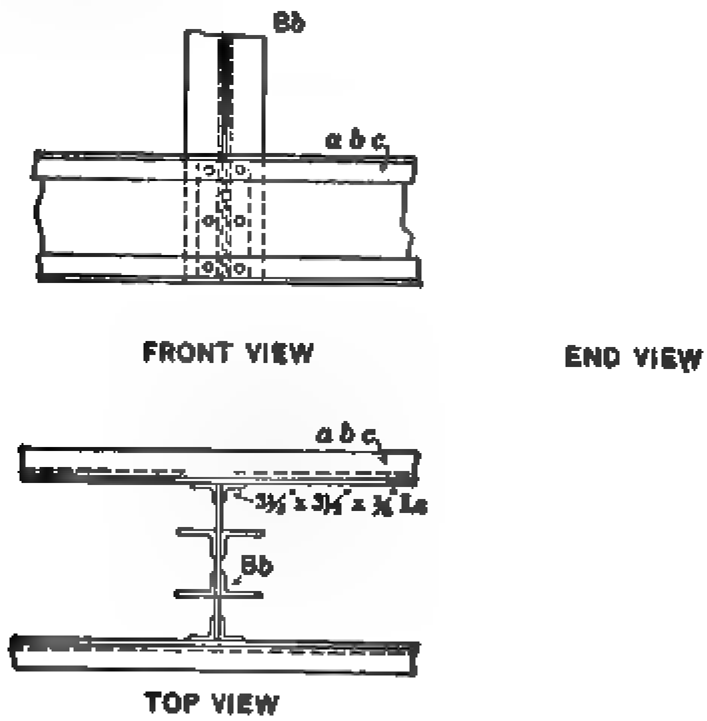


Fig. 44.

head was decided upon for member *B c*, and that the pin centre was fixed at one inch above the centre line of web plates. Fig. 14 shows that the under side of the cover plate is 10¼ ins. above the centre of the pin. The clearance between the head of the eye-bar and the cover plate is then ¼ in., which is sufficient.

The clearance at the other top chord joints will also be found to be sufficient. As at joint *B*, the distance from the pin centre to the under side of the cover plate is 10¼ ins. At joint *C*, the largest bar is 7 ins. wide. Since the pin is 5½ ins. in diameter, the table of eye-bars in Appendix B shows that a 16½-in. head is required. At joint *D*, the eye-bar is 6 ins. wide and the pin is 5½ ins. in diameter, requiring a 14-in. head. The clearance at joint *C* is then 2 ins., and that at joint *D* is 3¼ ins.

The conditions at lower chord joint *d* are shown in Fig. 40. As

the largest bar at this joint is 8 ins. wide and the pin is 7 ins. in diameter, an 18-in. eye-bar head is required. Fig. 20 shows that the distance from centre line of pins to the top of lateral plates is $10\frac{1}{4}$ ins. This provides a clearance of $1\frac{1}{4}$ ins.

The general arrangement of members at lower chord joint c is shown in Fig. 8, Art. 185 (A). Member Bc is 9 ins. wide and the pin at c is 7 ins. in diameter. The eye-bar head is then 20 ins. wide. As at joint d, the distance from centre of pin to top of lateral plate is $10\frac{1}{4}$ ins. A clearance of $\frac{1}{4}$ in. is thus provided. As shown in Fig. 8, the eye-bar of cd must fit between the toes of the angles on abc. Member cd is an 8-in. bar, which for a 7-in. pin requires an 18-in. head. From Fig. 41(b) it can be seen that the distance between the inside edges of the angles is $19\frac{5}{8}$ ins. The eye-bar head will therefore not interfere.

All joints have thus been found to give a proper clearance between eye-bars and other members. The pin sizes as stated at the beginning of this article can be adopted as final.

225. Attachment of Pin-Plates; General Requirement.—In the preceding articles we have determined the thickness of pin-plates required at the several joints in order to prevent crushing of the main members at the pins. These plates must be attached to the main members by means of sufficient rivets to take up the stress carried by each pin-plate. The bearing pressure on the pin has been assumed as uniformly distributed over all the plates bearing on the pin. On this assumption, the stress in any plate can be found from the formula $s = St/T$, which may be stated in the form of a proportion, as follows: s, the stress in any pin-plate, is to S, the total stress on all of the plates, as t, the thickness of the pin-plate in question, is to T, the total thickness of bearing.

The attachment of pin-plates must be designed with particular reference to the distribution of the loads received by the several plates at the pin to the various parts of the main member. This distribution of load must take place without overstressing the pin-plates, rivets, or any part of the main member. Each pin-plate must be long enough to transfer the stress in the plate to the main member. This stress may be only the stress belonging to the particular plate or may be the accumulated stress in a number of plates outside of

the plate in question, but which must pass through this plate to reach the main member. Where there are a number of plates, the stress per square inch on those nearest the main section will be greater than the average. It is, therefore, always better to have the thickest plates nearest the main section. In case the rivets connecting the plates to the main members are very long, their number must be increased in order to take care of the bending on the rivets. The required increase in number is governed by Art. 42, Specifications:

(42) Rivets carrying calculated stress whose grip exceeds four diameters shall be increased in number at least one per cent for each additional $^1/_{16}$ in. of grip.

As $\frac{7}{8}$-in. rivets are used in this design, their length can be as much as $3\frac{1}{2}$ ins. before addition must be made to the required number.

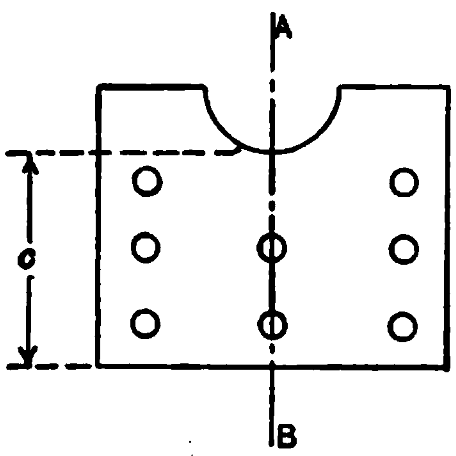


FIG. 45.

The distance from the edge of the pin-hole to the end of the plate must always be great enough to offer sufficient resistance against splitting on the line AB, of Fig. 45, the dangerous section being the one which has the least net area. This distance c can be determined by considering the plate as a beam balanced over the centre of the pin and acted upon by forces each equal to the value of one rivet applied at the centres of the rivet-holes. It is a safe rule to make the length of the pin-plate not less than three-fourths of the width. This point is often overlooked in designing. Even as a question of appearance alone it deserves attention.

The detail calculation of the pin-plate attachment for the various members will be given in the following articles:

226. Pin-Plates on Member BC at Joint B.—Fig. 46 shows the arrangement of pin-plates on member BC at joint B. The thickness of pin-plates was determined in Art. 214, and is shown in Fig. 33. The thicknesses are: a, $\frac{5}{8}$ in.; b, $\frac{9}{16}$ in.; c and d, each $\frac{7}{16}$ in.; and the web plate, $\frac{9}{16}$ in. This gives a total thickness of bearing on each half of the pin of $2\frac{5}{8}$ ins.

From Table A of Art. 180, the maximum stress in member BC is 772,000 lbs. The amount of load transmitted to each half of the

pin is 386,000 lbs., which is distributed among the several plates in proportion to their thicknesses, as given in Table Ga.

TABLE Ga

STRESSES IN PLATES AT PIN-HOLE

Plate	Thickness	$s = St/T$	Stress
a	10/16	$386{,}000 \times \frac{10}{42}$	92,000
b	9/16	$386{,}000 \times \frac{9}{42}$	82,700
c	7/16	$386{,}000 \times \frac{7}{42}$	64,300
d	7/16	$386{,}000 \times \frac{7}{42}$	64,300
Web	9/16	$386{,}000 \times \frac{9}{42}$	82,700
Totals	42/16	....	386,000

These stresses must be so transmitted to the main member that it may safely be assumed that the stresses on section p–q, Fig. 46(a),

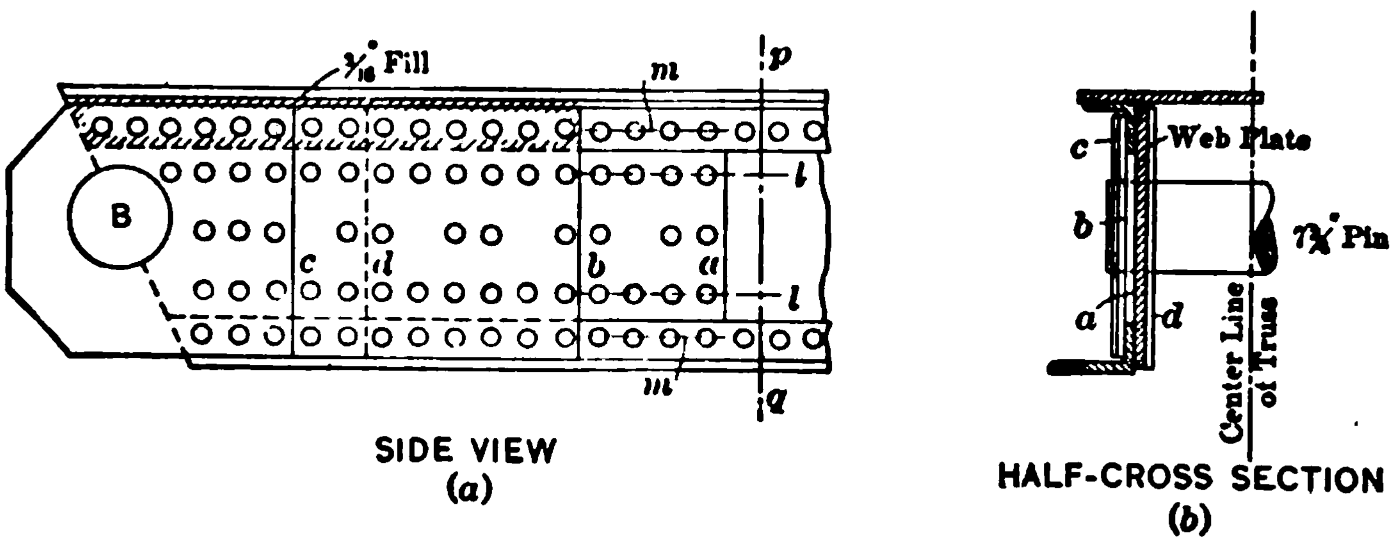


FIG. 46.

beyond the last pin-plate, are uniformly distributed over the section. On this assumption, the stresses at section p–q for the several parts of the main member are as given in Table Gb.

In designing the pin-plate attachments so that the stresses given in Table Ga for the several plates may be safely transferred to section p–q, the problem may be divided into three parts, as follows:

(a) To transmit from the pin-plates to the vertical leg of the top angle, the stress carried by the top angle and half of the cover plate. (b) To transmit from the pin-plates to the vertical leg of the lower angle, the stress carried by that angle. and (c), To transmit

from the pin-plates to the web plate the difference between the stress given over to the web plate directly by the pin and the stress in the web at section p–q.

TABLE Gb

STRESSES ON SECTION p-q

Section	Area	$s = St/T$	Stress
½ Cover Plate.....	8.75	$386{,}000 \times \frac{8.75}{30.30}$	111,400
Top Angle.........	3.31	$386{,}000 \times \frac{3.31}{30.30}$	42,200
Web Plate.........	12.38	$386{,}000 \times \frac{12.38}{30.30}$	157,800
Bottom Angle......	5.86	$386{,}000 \times \frac{5.86}{30.30}$	74,600
Totals........	30.30	....	386,000

In working out the design along these lines, the rivets used in the attachment of the plates must be so arranged that all the plates acting as a unit will take the total stress, and, at the same time, the several plates acting alone will each take their proportionate part of the total stress.

The stress to be transmitted from the pin-plates to the vertical leg of the top angle is the stress in the top angle and one-half of the cover plate, which from Table Gb is 42,200 + 111,400 = 153,600 lbs. Fig. 46(b) shows that the top angle lies between the plates b and c on the outside of the member, and plate d and the web plate on the inside. The rivets through the upper angle are then in bearing or double shear, as the case may be. It can readily be seen that it is advantageous and economical to use as many rivets in bearing or double shear as possible, in order to transfer the given stresses in as short a distance as possible. This can be done by making all of the plates except the filler, a, wide enough to take the rivets through the top angles. For the conditions shown in Fig. 46, these rivets will be in bearing on the angles. In this case the top angles are only 7/16 in. thick, while the filler a and the lower angles are 5/8 in. thick. As shown in Figs. 34 and 46, a 3/16-in. filler is used to make up for the difference in thickness. It is also evident that with a

filler, such as plate *a*, extending beyond the other plates, it is possible to rely upon the indirect transmission of stress through the web plate as from the four rivets on the line *l* in the filler to the four rivets on line *m* in the top and bottom angles.

The four rivets in line *m*, mentioned above, are in single shear, and their value is $4 \times 7{,}220 = 28{,}880$ lbs. Of the 153,600 lbs. to be carried by the top angles, there remains $153{,}600 - 28{,}880 = 124{,}720$ lbs. to be carried by the rivets through the pin-plates and the top angle. These rivets are in bearing on the $\frac{7}{16}$-in. angles, as stated above, and their value is 9,190 lbs. per rivet. The number required is $124{,}720/9190 = 14$ rivets. Fig. 46(*a*) shows the required number in place. It is to be noted that the six rivets in plate *b* beyond the end of plate *d* are not strictly in bearing on the angles, as they do not also pass through plate *d*. But we can again take advantage of the fact that there is an indirect transmission of shear by the web plate, the six rivets in plate *b* in the second row, from the top and bottom of the member assisting the web plate in transmitting stress, and thus placing the rivets in bearing.

The stress in the lower angle, as given in Table Gb, is 74,600 lbs. As for the top angle, four rivets in line *m* are available in single shear, leaving $74{,}600 - 28{,}880 = 45{,}720$ lbs. to be taken by rivets through the pin-plates and the lower angle. These rivets are in bearing on the $\frac{5}{8}$-in. lower angle, and the member required is $45{,}720/13{,}130 = 4$ rivets. Fig. 46(*a*) shows 11 in place. This large excess of rivets is due to the fact that good practice makes it necessary to maintain the same rivet spacing in the top and bottom angles, and also the plates must be cut off square.

The number of rivets calculated above are required for the plates acting as a unit. It now becomes necessary to calculate the number required in the individual plates. The placing of these rivets will determine the lengths of the several plates. The stresses in the several plates are determined on the assumption that the total load is divided among the plates in proportion to their thickness. On the vertical leg of the top angle, the stress to be carried was found to be 153,600 lbs. The thicknesses of the plates are as follows: *a*, $\frac{5}{8}$ in.; *b*, $\frac{9}{16}$ in.; *c* and *d*, each $\frac{7}{16}$ in.—a total of $\frac{33}{16}$ in. Table Gc gives the stresses in the several plates at the top angle.

TABLE Gc

STRESSES IN PLATES AT TOP ANGLE

Plate	Thickness	$s = St/T$	Stress
a	10/16	$153{,}600 \times \frac{10}{33}$	46,600
b	9/16	$153{,}600 \times \frac{9}{33}$	41,800
c	7/16	$153{,}600 \times \frac{7}{33}$	32,600
d	7/16	$153{,}600 \times \frac{7}{33}$	32,600
Totals	33/16	....	153,600

TABLE Gd

STRESSES IN PLATES AT LOWER ANGLE

Plate	Thickness	$s = St/T$	Stress
a	10/16	$74{,}600 \times \frac{10}{33}$	22,600
b	9/16	$74{,}600 \times \frac{9}{33}$	20,400
c	7/16	$74{,}600 \times \frac{7}{33}$	15,800
d	7/16	$74{,}600 \times \frac{7}{33}$	15,800
Totals	33/16		74,600

The stress to be carried by the vertical leg of the lower angle has been found to be 74,600 lbs. This stress is divided among the several plates, as shown in Table Gd.

Plate *c* alone has a stress of 15,800 lbs. at the lower angle, and 32,600 lbs. at the upper angle. As the rivets are in single shear, 15,800/7220 = 3 rivets are required at the lower angle, and 32,600/7220 = 5 rivets are required at the upper angle. Fig. 46(*a*) shows 3 rivets in place on the lower line and 6 on the top line. The angle of cut-off of member *B C* is such that it will be necessary to put an extra rivet in the top row.

Taken together, plates *c* and *d* have a stress of 65,200 lbs. at the top angle and 31,600 lbs. at the lower angle. Fig. 46(*a*) shows that the 6 rivets in plate *c* at the top angle, and the 3 rivets at the lower angle, also pass through plate *d*. These rivets are, therefore, in bear-

ing on the angles. At the top angle, a rivet in bearing has a value of 9,190 lbs. The 6 rivets which pass through both c and d then have a value of 55,140 lbs. This leaves 65,200 − 55,140 = 10,060 lbs. to be provided for. By extending plate d far enough beyond the end of plate c so that it will take in two rivets, which will be in single shear, a strength of 14,440 lbs. is added. The total value of rivets in plates c and d on the top angle then becomes 55,140 + 14,440 = 69,580 lbs., which is sufficient. When the plates c and d are squared off, it will be found that at the lower angles there are 3 rivets in bearing on the ⅝-in. lower angles and 2 rivets in single shear, giving a total strength of 3 × 13,130 + 2 × 7220 = 53,830 lbs. As the load to be carried is 31,600 lbs., the rivets provided are more than enough.

Considering plates b, c, and d acting together, the stress to be carried at the top angle is 41,800 + 32,600 + 32,600 = 107,000 lbs., and at the lower angle the total stress is 52,000 lbs. All of the rivets passing through these plates are to be considered as in bearing on the angles. As stated above, the six rivets between the ends of plates d and b can be considered in bearing due to the indirect transmission of shear by the web plate. The value of the rivets in place on the top line, as shown in Fig. 46(a), is 14 × 9,190 = 128,660 lbs., and on the lower line 11 × 13,130 = 144,430 lbs. As the number of rivets placed in position through plate b and the angles has been fixed by calculations previously made, no reduction can be made in the number of rivets in position in plates b, c, and d.

The case of all plates taken together has already been considered, and will not be repeated here.

The next step in the design of pin-plate attachment is to transmit from the pin-plates to the web plate the difference between the stress given over to the web directly by the pin and the stress in the web plate at section p–q of Fig. 46(a). From Tables Ga and Gb this difference in stress is 157,800 − 82,700 = 75,100 lbs. As before, this stress is assumed to be divided among the several plates in proportion to their thickness. The stresses in the plates are as given in Table Ge.

It will be assumed that these stresses are taken care of by the row of rivets which is located on the centre line of the web plate. The rivets in plate a in the lines l are not to be counted on as taking

any part of these stresses, as these rivets have already been counted upon to transmit indirect shear from lines l to m. Plate c alone has a stress of 15,900 lbs. As the rivets are in single shear, the number required is 15,900/7220 = 3 rivets. Fig. 46(a) shows 3 rivets provided. The stress in plates c and d, taken together, is 31,800 lbs.

TABLE Ge

STRESSES IN PIN-PLATES ON CENTRE LINE OF WEB PLATE

Plate	Thickness	$s = St/T$	Stress
a....................	10/16	$75{,}100 \times \frac{10}{33}$	22,800
b....................	9/16	$75{,}100 \times \frac{9}{33}$	20,500
c....................	7/16	$75{,}100 \times \frac{7}{33}$	15,900
d....................	7/16	$75{,}100 \times \frac{7}{33}$	15,900
Totals...........	33/16	....	75,100

To carry this stress Fig. 46(a) shows 3 rivets in bearing on the 9/16-in. web plate, and one rivet in single shear. These rivets have a value of $3 \times 11{,}810 + 1 \times 7220 = 42{,}650$ lbs. Plates c, d, and b, taken together, have a total stress of 52,300 lbs. The rivets provided are 4 in bearing and 4 in single shear, giving a total strength of $4 \times 11{,}810 + 4 \times 7220 = 76{,}120$ lbs. Plates a, b, c, and d, taken together, carry the total stress of 75,100 lbs. Fig. 46(a) shows 4 rivets in bearing and 7 rivets in single shear, giving a total strength of 97,780 lbs. The arrangement shown in Fig. 46(a), therefore, provides sufficient strength.

The final step in the design of pin-plate attachment consists in checking up the arrangement of rivets shown in Fig. 46(a) in order to make certain that the stresses given in Table Ga for the several plates can be safely carried. In estimating the strength of attachment of plates, the rivets to be counted as effective in resisting stress are those in the vertical legs of the top and bottom angles, and the row of rivets along the centre line of the web plate. The second row of rivets from the top and bottom of the member are to be considered as binding the plates together, and transferring indirectly some

of the stress from the pin-plates to the web plate and thence to the angles. From Table Ga, plate c alone has a stress of 64,300 lbs. Fig. 46(a) shows 12 rivets in single shear in place, giving a strength of $12 \times 7220 = 86{,}640$ lbs. Plates c and d, taken together, have a stress of 128,600 lbs. The rivets provided are 6 in bearing on the $\frac{7}{16}$-in. top angles, 3 in bearing on the $\frac{9}{16}$-in. web plate, 3 in bearing on the $\frac{5}{8}$-in. lower angles, and 5 in single shear. These rivets provide a total strength of 166,060 lbs. In the same way, plates b, c, and d, taken together, have a total stress of 211,300 lbs., and the rivets provided have a strength of 301,930 lbs., and when all plates are taken together the total stress is 303,300 lbs. and the rivets provided have a strength of 381,350 lbs. As the strength provided in each case is in excess of the stress, the attachment of pin-plates as shown in Fig. 46(a) is correct, and will be taken as final.

The spacing of rivets in pin-plates should be made rather small, say 3 ins., in order to keep the plates as short as possible. This spacing should be continued to a point about a foot beyond the end of the last pin-plate, where the spacing can be increased to a maximum of about 5 or 6 ins. These general directions, if followed out, will answer all conditions of Art. 43, Specifications:

(43) The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivets, for a length equal to one- and one-half times the maximum width of member.

This article allows the spacing in pin-plates to be as much as $3\frac{1}{2}$ ins., but the spacing recommended above will result in shorter plates.

227. Attachment of Pin-Plates on Member aB.—The design of pin-plate attachment for joints a and B of member aB is carried out along the same lines as that given in Art. 226, for member BC at joint B. Fig. 47 shows the arrangement of plates and the rivets required for both ends of member aB. At joint a, the number and general arrangement of plates are the same as for joint B on member BC. As the design of pin-plate attachment for the two joints differs only in the amount of stress to be carried, and the thickness of a few of the plates, the calculations will not be repeated here. This will be left as a problem for the student.

The arrangement of pin-plates on member aB at joint B differs

slightly from that on member BC at the same joint. Comparing Fig. 46(a) and Fig. 47, joint B, it will be found that BC has only one plate inside the web plate, while aB has two plates inside. This difference in arrangement of plates will make no difference in the design until the lengths of the several plates are determined.

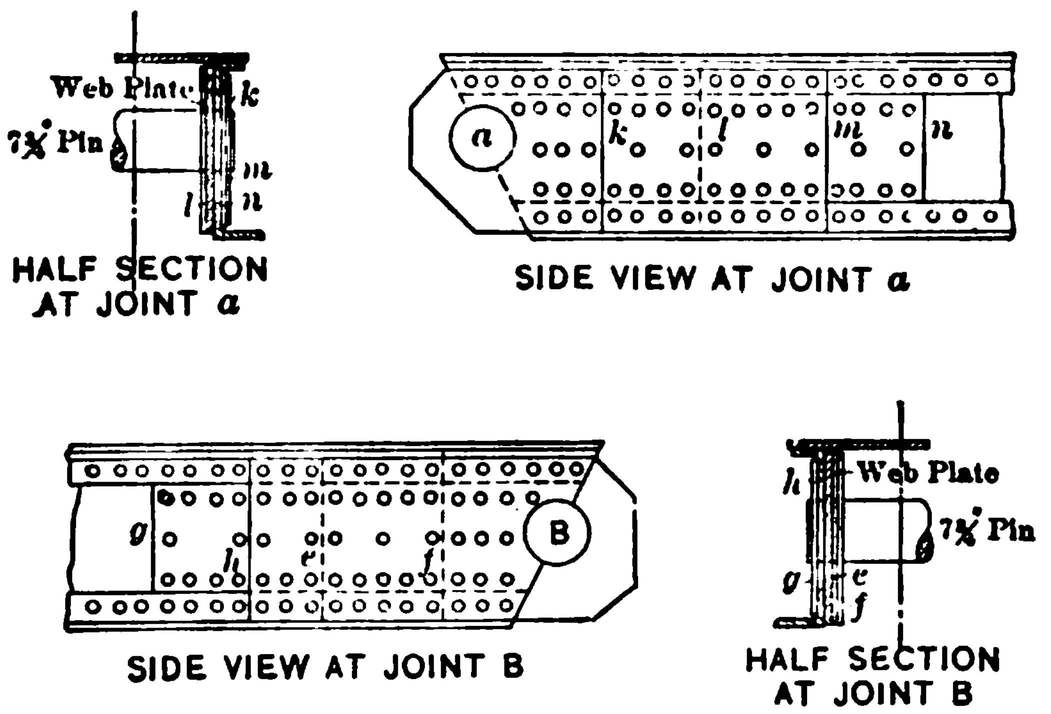


FIG. 47.

In this case, when plate f is considered alone, and when plates e and f are considered together, the rivets are in single shear. It is only when plates e, f, and h are considered together that part of the rivets can be considered as bearing on the angles. With these hints the remainder of the problem will be left to the student, who can easily check the design given in Fig. 47.

228. Pin-Plates at Joint C.—At joint C, pin-plates are required on the web plate of the top chord member and on the web of the channels forming member Cc, as shown in Fig. 38 of Art. 217. In each case the stress to be carried at the pin is assumed to be divided between the web plate and pin-plate in proportion to their thickness.

The stress to be taken care of on the web of the top chord member is the difference in stress between member BC and CD, which is a maximum when Cd has its maximum stress. This difference in stress was used in the design of the pin at C in Art. 217. From Table F, this difference in stress is 113,000 lbs. for each half of the member.

Fig. 48(a) shows the arrangement of plates on the top chord member. The pin-plate is usually made wide enough to fit between the

points of the chord angles, which in this truss requires a plate 14 ins. wide. The web plates of members BC and CD differ in thickness, but it is usual to run the web plate of CD about two or three feet to the left of joint C and there make a butt splice, as shown in Plate III. The web plate at C is, therefore, to be taken as for member $C\,D$, that is $\frac{3}{4}$ in. thick. From Fig. 38 the pin-plate at C is found to be $\frac{3}{8}$ in. thick. The total thickness of bearing is then $\frac{9}{8}$ ins.

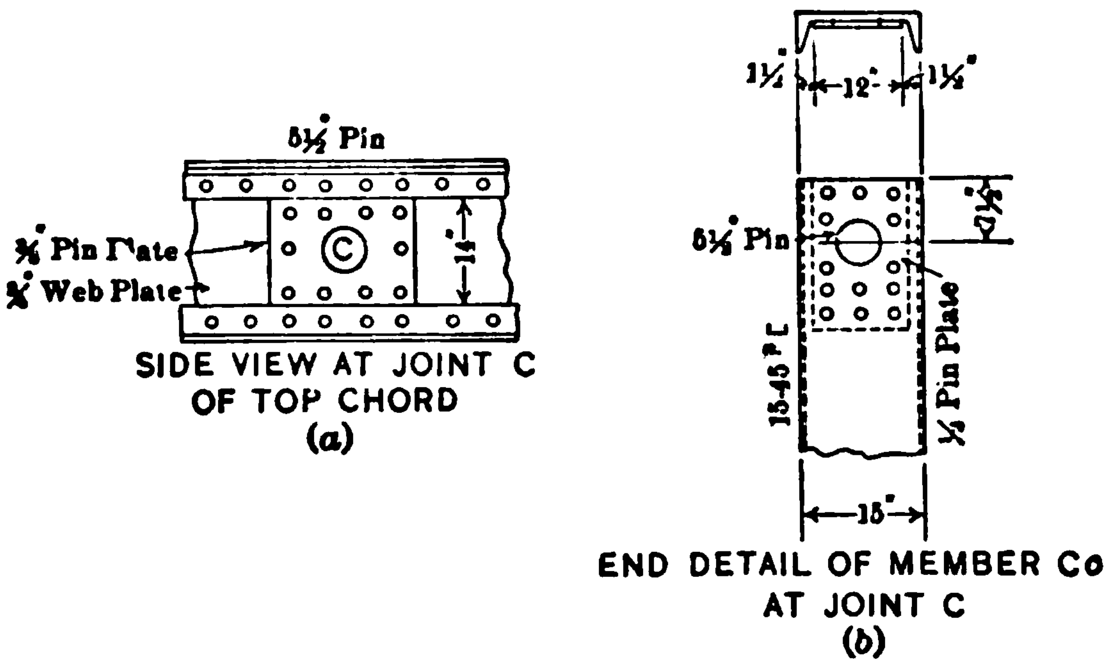


FIG. 48.

Of the 113,000 lbs. to be carried at the pin, the web plate takes $113{,}000 \times \frac{6}{9} = 75{,}300$ lbs., and the pin-plate takes $\frac{3}{9} \times 113{,}000 = 37{,}700$ lbs. The rivets connecting the pin-plate to the web plate are in single shear and the number required is $37{,}700/7220 = 6$ rivets. Fig. 48(a) shows 10 rivets in place. This is more than required by the stress, but it is hardly possible to make a neat and rigid connection with the use of a smaller number.

The maximum stress in member $C\,c$ is given in Table A as 283,100 lbs., or 141,550 lbs. for each half of the member. Fig. 38(a) shows that the channels composing member $C\,c$ have a 0.62-in. web. and that a $\frac{1}{2}$-in. pin-plate is used, giving a total bearing of $0.62 + 0.50 = 1.12$ ins. The stress carried by the pin-plate is then $141{,}550 \times 0.50/1.12 = 63{,}200$ lbs. As the rivets connecting the pin-plate to the channel are in single shear the number required is $63{,}200/7220 = 9$ rivets. Fig. 48(b) shows 13 in place.

The width of plate which will fit inside the flanges of a 15-in. channel can be determined from the tables of standard sections in a rolling-mill hand-book. A 12-in. plate will be used in this case.

As member $C c$ is in compression, the stress is resisted on the under side of the pin. For this reason most of the rivets have been placed below the pin, as shown in Fig. 48(*b*). Only enough rivets are placed above the pin to bind the plates firmly together.

229. Pin-Plates on Member $D d$.—Fig. 39(*a*) shows that no pin-plates are required on the web of the top chord members, and that a 3/8-in. plate is required on the webs of the channels. As the design of the pin-plate attachment is exactly similar to that given in Art. 33 for joint C, the calculations will not be repeated here. Fig. 49 shows the details of the joint as designed.

At joint d of the lower chord, Fig. 40(*a*) of Art. 220 shows that member $D d$ is the only one requiring pin-plates. A 7/16-in. pin-plate is called for, placed outside the channel, as shown in Fig. 50. As the web of the 15-in. 33-lb. channel composing member $D d$ is 0.40 in. thick the total thickness of bearing for each half of the pin is $0.40 + 0.44 = 0.84$ in. The stress to be carried by one-half of member $D d$ is given in Art. 220 as 135,500 lbs. Of this stress the pin-plate must carry $0.44/0.84 \times 135{,}500 = 71{,}000$ lbs. As the rivets are in single shear the number required is $71{,}000/7220 = 10$ rivets. Fig. 50 shows 12 rivets in place through the pin-plate, not counting the two rivets in the top row, which serve to hold the diaphragm in place until the floor-beam field rivets are driven.

The distance from the centre of the pin-hole to the top of the pin-plate is limited in this truss to about 15 ins., as the pin-plate must not interfere with the floor-beam. From Fig. 20, the distance from the centre of the pin to the lower edge of the curved angles is 15 3/8 ins. Allowing 3/8 in. for clearance, the plate shown in Fig. 50 will not interfere.

The angles shown at the foot of member $D d$ are intended for the lateral plate connection for the bottom lateral system. These angles must be placed with the horizontal legs on the level of the bottom of the floor-beam, or in this truss 10 1/4 ins. below the centre of the pin.

230. Pin-Plate Attachment on Members at Lower Chord Joint c.—At lower chord joint c, members $C c$ and $a b c$ require pin-plates. The design of pin-plate attachment on member $a b c$, which is a riveted tension member, brings up some new points in design which will be considered in detail in the next article.

Fig. 41(*a*) of Art. 221 shows that two pin-plates are required at the foot of the 15-in. 45-lb. channels composing member $C\,c$. The arrangement of plates is as shown in Fig. 51. A $\frac{3}{8}$-in. plate is provided on the inside of the channel, and a $\frac{7}{16}$-in. plate on the outside.

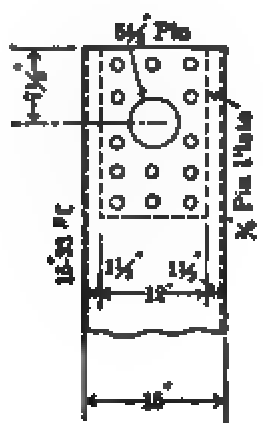


Fig. 49. End detail of member Dd at joint D.

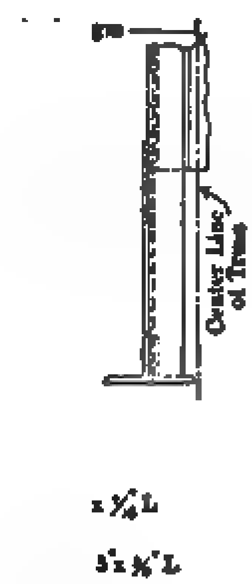


Fig. 50. End detail of member Dd at joint d.

As the web of the channel is 0.62 in. thick, the total thickness of bearing on the pin is $0.38 + 0.62 + 0.44 = 1.44$ ins. In Art. 221 it was found that the load to be carried by the pin-plates on one-half of the member was 208,250 lbs. If this load is assumed to be carried by the several plates in proportion to their thickness, the $\frac{3}{8}$-in. plate must carry $0.38/1.44 \times 208{,}250 = 55{,}000$ lbs., and the $\frac{7}{16}$-in. plate must carry $0.44/1.44 \times 208{,}250 = 63{,}700$ lbs. Considering the plates as acting alone, the rivets being in single shear, the number required in the $\frac{3}{8}$-in. pin-plate is $55{,}000/7220 = 8$ rivets, and the number required in the $\frac{7}{16}$-in. plate is $63{,}700/7200 = 9$ rivets. When both plates are considered as acting together, the total stress is $55{,}000 + 63{,}700 = 118{,}700$ lbs. As the rivets are in bearing on the 0.62-in. web of the channel, their value is $0.62 \times 0.875 \times 24{,}000 = 13{,}000$ lbs. per rivet, and the number required is $118{,}700/13{,}000 = 10$ rivets. Fig. 51 shows 13 rivets in place.

At joint *c* the lateral plate is riveted to member *a b c* so that connection angles are not necessary at the foot of the post, as they were

at joint d. The distance from the centre of the pin to the top of the pin-plate is governed by the same conditions as at joint d.

231. Pin-Plates on Member $a\ b\ c$.—The arrangement of pin-plates at joints a and c of member $a\ b\ c$ will differ slightly, due to the difference in pin sizes at the two joints. As the method of calculation of pin-plate attachment is the same at both joints, the detail work will be given only for joint c.

Fig. 52 shows the arrangement of plates at joint c. The thickness of these plates was determined in Art. 221. (See Fig. 41(b).)

Table C, Art. 186, gives the maximum direct stress in member

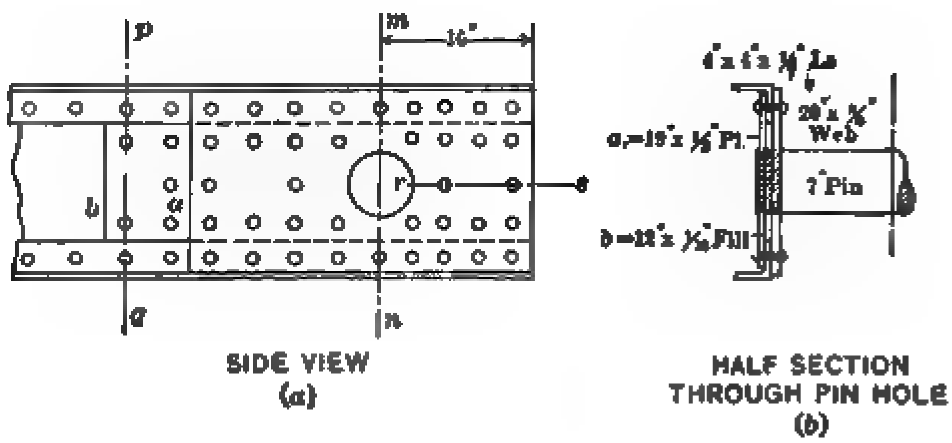


Fig. 51. End detail of member $C\ c$ at joint c.

Fig. 52. End detail of $a\ b\ c$ member at joint c.

$a\ b\ c$ as 477,400 lbs. tension, and the net area of the member as 31.49 sq. ins. In designing the end connection for this member, the stress used in designing is subject to the conditions of Art. 37, Specifications, which requires that the connection be able to develop the full strength of the net area of the member. The full strength of the member is then $31.49 \times 16{,}000 = 504{,}000$ lbs. Also, Art. 28, Specifications, states that the net section through the pin-hole for riveted tension members must exceed the net area in the body of the member by 25 per cent. In designing the end connection, this same increase in strength of member should be made. This is equivalent in this case to designing the pin-plate attachment for a total load

of $1.25 \times 504{,}000 = 630{,}000$ lbs., or 315,000 lbs. for each half of the member.

The general principles governing the design of pin-plate attachment on a riveted tension member are the same as for compression members. As *a b c* is a tension member, the pressure on the pin is applied at point *r* of Fig. 52(*a*). The total load is assumed to be divided among the several plates in proportion to their thickness, as given in Table Ha. These loads must be distributed to the several parts of the net section on the pin, *m–n* of Fig. 52(*a*), in proportion to the areas of the parts. Table Hb gives these stresses. Finally, the stresses existing on section *m–n* must be transmitted to the body of the main member at section *p–q*, in order to realize the uniform distribution of stress assumed in the design of the member in Art. 186.

Proceeding with the design of pin-plate attachment in order to realize the conditions outlined above, the first step in the design is to adjust the various differences in stress for members on the right of the pin-hole, as given in Table Ha and Hb.

Comparing the stresses given in Table Ha for plates *a*, *b*, and the web, with the values given in Table Hb for these same plates at section *m–n*, it can be seen that the plates have delivered to them in bearing on the pin a stress which is greater than they are able to transfer back across the net section at the pin. It can also be seen that this excess stress is transmitted to the angles, and is carried across the net section by them, each angle having a stress of 45,700 lbs., as shown by Table Hb. The rivets which transmit this stress to the angles also pass through the web and plate *a*, which are on opposite sides of the angles. This arrangement places the rivets in bearing on the angle, which is $\frac{7}{16}$ in. thick, thus requiring $45{,}700/9190 = 5$ rivets, the number shown in Fig. 52(*a*). This stress of 45,700 lbs. comes partly from the web-plate, and partly from plates *a* and *b*. The parts coming from the web-plate and plate *a* are transmitted directly through these plates to the angle. Plate *b* transmits its stress to the angles indirectly, as this plate is not fastened directly to the angles. Its stress is first transmitted to plate *a* and the web, and then to the angles. The amount of stress thus transmitted is found, from Tables Ha and Hb, to be $88{,}200 - 34{,}800 = 53{,}400$ lbs.

As the rivets are in bearing on plate b, which is $\frac{7}{16}$ in. thick, the number required is $53,400/9190 = 6$ rivets. Fig. 52(a) shows 10 rivets in place. The number in place is largely determined by the number required in the angles. Under the conditions it is hardly possible to use a smaller number than shown.

TABLE Ha

BEARING STRESSES IN PLATES ON PIN AT r OF FIG. 52(a)

State	Thickness	$s = \frac{St}{T}$	Stress
a	8/16	$315 \times \frac{8}{25}$	100,800
b	7/16	$315 \times \frac{7}{25}$	88,200
Web	10/16	$315 \times \frac{10}{25}$	126,000
	25/16	....	315,000

TABLE Hb

STRESSES IN PLATES ON NET SECTION THROUGH PIN-HOLE. SECTION m–n OF FIG. 52(a)

Section	Net Area	$s = \frac{Sa}{A}$	Stress
Top L	2.87	$315 \times \frac{2.87}{19.80}$	45,700
a	5.00	$315 \times \frac{5.00}{19.80}$	79,600
b	2.19	$315 \times \frac{2.19}{19.80}$	34,800
Web	6.87	$315 \times \frac{6.87}{19.80}$	109,200
Bottom L	2.87	$315 \times \frac{2.87}{19.80}$	45,700
	19.80	....	315,000

TABLE Hc

STRESSES ON NET SECTION OF MAIN MEMBER SECTION p–g OF FIG. 52(a)

Section	Net Area	$s = \frac{Sa}{A}$	Stress
Top L	2.87	$315 \times \frac{2.87}{15.74}$	57,500
Web	10.00	$315 \times \frac{10.00}{15.74}$	200,000
Bottom L	2.87	$315 \times \frac{2.87}{15.74}$	57,500
	15.74	....	315,000

The next step in the design is to transmit the stresses given in Table Hb for section *m–n* to section *p–q* so that the final stresses in the several parts of the section of the body of the member shall be as given in Table Hc.

This can be accomplished by transferring to the angles and web-plate of the main section the stresses given in Table Hb for plates *a* and *b*, by means of rivets placed to the left of the pin-hole. The part of these stresses which is transmitted to the angles is the difference in the stresses given for the angles in Tables Hb and Hc, or 57,500 − 45,700 = 11,800 lbs. for each angle, and 23,600 lbs. for both angles. Since plate *a* covers the angles, while plate *b* is only a filler, it will be assumed that all of the 23,600 lbs. transmitted to the angles comes from plate *a*. The balance of the stress in plate *a* and all of the stress in plate *b* will be assumed as transferred directly to the web. On this assumption, the stress transferred by plate *a* to the web is found from Table Hb to be 79,600 − 23,600 = 56,000 lbs. As the rivets are in single shear, the number required is 56,000/7220 = 8 rivets. Fig. 52(*a*) shows 10 rivets connecting plate *a* to the web. To transmit the stress of 11,800 lbs. which plate *a* carries to each angle requires 11,800/7220 = 2 rivets in single shear. Fig. 52(*a*) shows 4 rivets in place in each angle. Plates *a* and *b* together transfer to the web a stress of 34,800 + 56,000 = 90,800 lbs. on the assumption made above. To transmit this stress requires 90,800/7220 = 13 rivets. Fig. 52(*a*) shows 15 rivets in place through plates *a* and *b* and the web on the left of the pin-hole. In the last vertical row of rivets in plate *b* only two rivets have been used. This arrangement makes the effective net area as assumed in the calculation of net effective area for *a b c* in Art. 186.

The net length of the pin-plates on the line *r–s* of Fig. 52(*a*) is determined by the requirements of Art. 28, Specifications. According to the specifications "the net section back of the pin-hole, parallel with the axis of the member, shall be not less than the net section of the body of the member." As the net area for each half of the member is 15.74 sq. ins., and the thickness of bearing plates on the pin is 1-$\frac{9}{16}$ in., the net length required on line *r–s* is 15.74/1.5625 = 10.1 ins. Fig. 52(*a*) shows that provision must be made for the half diameter of a 7-in. pin and two rivets. This requires

the gross length on line r–s to be at least $10.1 + 3.5 + 2 = 15.6$ ins. The arrangement shown in Fig. 52(a) provides a gross length of 16 ins.

Fig. 53 shows the arrangement of plates at joint a. The design of pin-plate attachment is exactly the same as explained for joint c and is left as a problem for the student.

232. Pin-Plates on Member $B\,b$ at Joint B.—Fig. 54 shows the arrangement of pin-plates on member $B\,b$ at joint B. The thickness and width of these plates were determined in Art. 215 (see Fig. 35.)

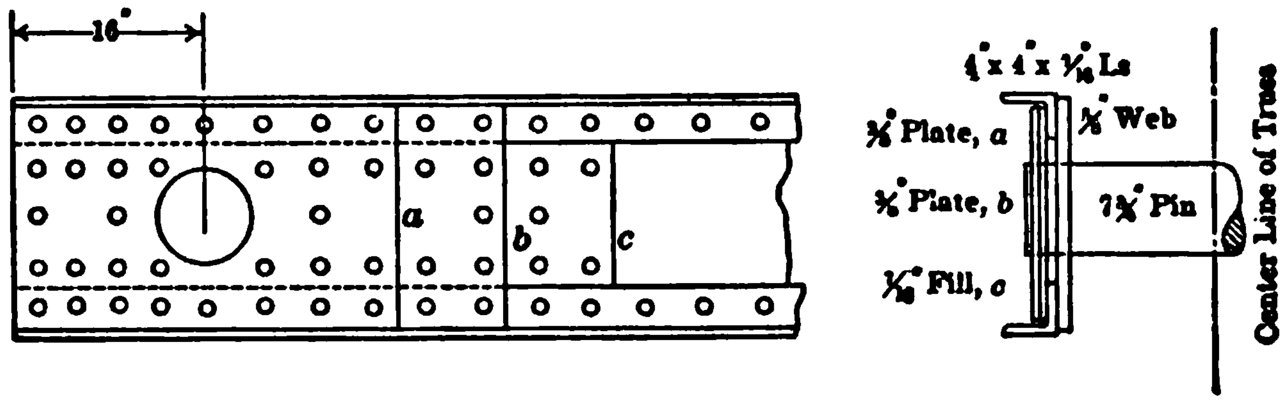


FIG. 53.

As in the preceding article, the stress for which the pin-plate attachment is to be designed is determined from the requirements of Arts. 28 and 37 of the Specifications. Since the net area of the member, as given in Table C, is 16.71 sq. ins., the plate attachment must be designed for a total load of $\frac{1}{2} \times 16.71 \times 16{,}000 \times 1.25 = 167{,}100$ lbs. for each half of the member. This load is to be transmitted to the several plates at point r, Fig. 54(a), in bearing on the pin, each plate having a stress which is in proportion to its thickness. On this assumption the $\frac{7}{16}$-in. filler has a stress of $167{,}100 \times 0.4375/1.4375 = 51{,}000$ lbs., and each $\frac{1}{2}$-in. side plate has a stress of $167{,}100 \times 0.50/1.4375 = 58{,}100$ lbs. At section p–q each plate has a stress which is in proportion to its area on that section. Since the proportionate net areas of the three plates on section p–q are the same as their proportionate bearing areas at point r, the stresses in the plates at these two places are equal. Therefore, no rivets are required above the pin to equalize stresses. The two rivets shown have no definite stress and simply serve to bind the plates together.

Below the pin the centre plate is a filler, and extends only to the top of the angles of the main member, which is at line a–b of Fig.

54(*a*). The stress in the filler must be transmitted to the outside plates and thence to the main member. As the rivets connecting the filler to the side plates are in bearing on the $^7/_{16}$-in. filler, the number required is 51,000/9190 = 6 rivets. Fig. 54(*a*) shows 8 rivets in place.

The pin-plates are attached to the main members by rivets passing through the two outside pin-plates and the outstanding legs of

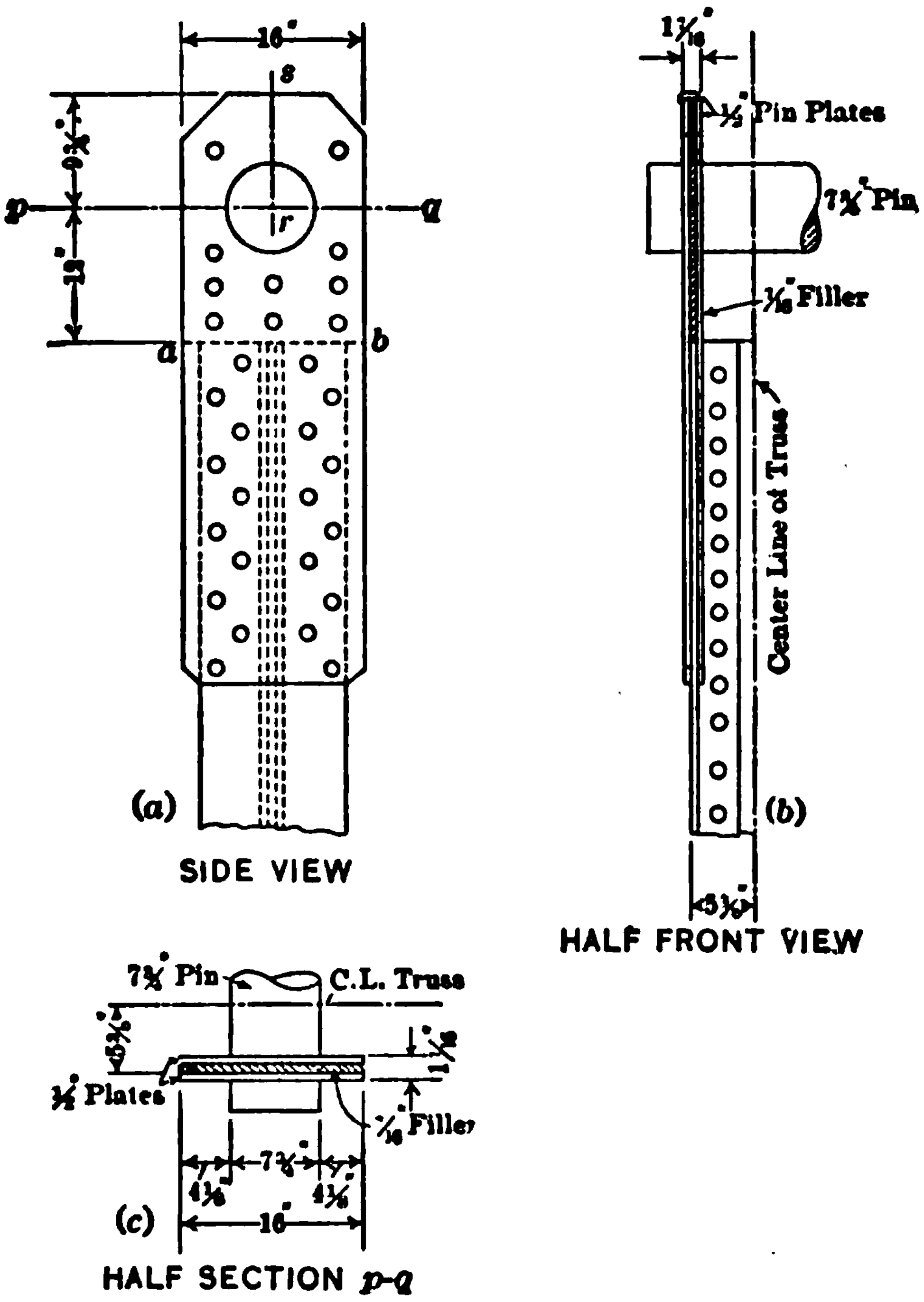


FIG. 54. End detail of member *Bb* at joint *B*.

the main angles, as shown in Fig. 54. To provide a connection which will develop a strength equal to that at the section *p–q* will require 167,100/9190 = 20 rivets, the number shown in place in Fig. 54(*a*).

The length required on line *r–s* in order to meet the requirements of Art. 28, Specifications, has already been determined in Art. 215. Fig. 54 shows the required length.

233. Design of End Bearings.—The end bearings of a truss span

must be so designed that the maximum end reaction can be transmitted to the masonry without exceeding the allowable working pressures on the material composing the abutments or piers. The articles from the specifications relating to the design of end bearings are as follows:

(19) (in part). Bearing on expansion rollers; in pounds per linear inch, $600d$, where d is the diameter of roller in inches. Bearing on masonry, 600 pounds per square inch.

(59) Provision for expansion to the extent of ⅛ in. for each 10 ft. shall be made for all bridge structures. Efficient means shall be provided to prevent excessive motion at any one point.

(60) Spans of 80 ft. and over resting on masonry shall have turned rollers or rockers at one end; and those of less length shall be arranged to slide on smooth surfaces. These expansion bearings shall be designed to permit motion in one direction only.

(61) Fixed bearings shall be firmly anchored to the masonry.

(62) Expansion rollers shall be not less than 6 ins. in diameter. They shall be coupled together with substantial side bars, which shall be so arranged that the rollers can be readily cleaned. Segmental rollers shall be geared to the upper and lower plates.

(63) Bolsters or shoes shall be so constructed that the load will be distributed over the entire bearing. Spans of 80 ft. or over shall have hinged bolsters at each end.

(64) Wall plates may be cast or built up; and shall be so designed as to distribute the load uniformly over the entire bearing. They shall be secured against displacement.

To comply with the above articles from the specifications, the end bearings of truss spans usually consist of a built-up shoe or bolster which rests on a cast or built-up bed-plate. The loads are brought from the truss to the shoe by means of the end pins of the bottom chord member, thus meeting the requirements of Art. 63, Specifications. At one end of the truss, the bed-plate is so arranged that it forms a fixed bearing, while at the other end a set of rollers or rockers is provided in order to allow free movement of the end of the truss due to temperature changes or elongation of the lower chord under live load. Details of fixed and expansion bearings in general use are given in Chap. VII.

The dimensions of the base plate and shoe will depend upon the bearing area required on the masonry, and on the number and length of rollers required. For the type of floor-beam shown in Fig. 23, where the beam rests directly on the sole plate of the shoe, the loads

from the end post and floor-beam are carried to the rollers along separate paths. The rollers provided under the shoe and under the end of the floor-beam must each be able to take the maximum loads brought to these members, and at the same time the total length of rollers must be able to take the sum of the end post and floor-beam loads when their combined effect is a maximum. In the design shown in Fig. 25, where the floor-beam rests directly on the end pin, the total load is carried to the rollers along one path. The rollers are then to be designed for the maximum end reaction. These principles will now be applied to the design of the shoe and rollers for the truss in question.

The loads for the type of floor-beam resting directly on the sole plate, as shown in Fig. 23, have already been calculated in previous articles. From Art. 222, the maximum vertical component of the end post is found to be 575,000 lbs., and from Art. 202 the maximum end floor-beam reaction is 181,630 lbs. Also, from Art. 222, p. 335, the maximum sum of end post and floor-beam loads is 721,430 lbs. The specifications (Art. 62) allow the use of 6-in. rollers as the minimum size. From Art. 19, Specifications, the allowable bearing stress for a 6-in. roller is $6 \times 600 = 3600$ lbs. per lin. inch. For the loads given above, $575{,}000/3600 = 160$ lin. ins. of roller are required under the shoe, $181{,}630/3600 = 50.5$ lin. ins. are required under the floor-beam, and $721{,}430/3600 = 200$ lin. ins. under the shoe and floor-beam taken together.

The bearing area required on the masonry is governed by Art. 19, Specifications, which calls for a bed-plate area of $721{,}430/600 = 1200$ sq. ins.

Fig. 55 shows the details of the rollers and bed-plate. The rollers rest on a cast bed whose upper surface is formed by a series of flat horizontal surfaces running parallel to the bridge axis. These surfaces are separated by grooves about one inch wide to allow for drainage. In some cases this bearing surface is made of rails planed to an even surface. At the truss centre grooves have been cut in the upper and lower surface of the rollers, which engage tongues cut on the bearing plates. This arrangement prevents lateral motion of the shoe on the rollers.

The net bearing of the portion of the rollers under the shoe is

shown on Fig. 55(*a*) to be 28 ins., not counting the centre tongue, which requires 160/28 or 6 rollers to furnish the required bearing for the maximum load on the shoe. For the portion of the rollers under the floor-beam a bearing of 10 ins. is provided, requiring 50.5/10 or 6 rollers. The net length of bearing for the whole roller is 38 ins., requiring 200/38 or 6 rollers to support the maximum end reaction. Fig. 55 shows 6 rollers provided, the number called for by these calculations.

The width of the cast base is shown to be 12 ins. greater than the length of rollers. This added length is provided to make room for

END VIEW
(*a*)

FIG. 55. Details of end shoe expansion end.

anchor bolts by means of which the bed-plate is fastened to the masonry, in order to comply with Arts. 61 and 64, Specifications. In order to reduce the length of the shoe, it is usual in rollers of this size to cut away the sides, forming a "segmental" roller, as shown in Fig. 56. The amount cut away must be such that the rollers will not tend to turn over on the flat side during changes in position of the end of the span. Assuming the rollers do not slip, a forward motion of B (Fig. 56) causes the vertical axis of the roller to turn through an angle $\theta = \frac{2B}{D}$ radians $= 114.6 \frac{B}{D}$ degrees, where D is the

diameter of the roller. As can be seen from Fig. 56 the limiting value of θ is α, which has a value of $\alpha = \sin^{-1}\frac{d}{D}$, where d is the width of the flat portion of the roller. Placing $\alpha = \theta$, we find $d = D \sin\left(114.6\frac{B}{D}\right)$, which gives the limiting width of the flat portion of a roller for any given diameter of roller, and a stated forward

FIG. 56.

motion. The distance between adjacent rollers, shown by A in Fig. 56, must be such that the rollers will not come in contact before the required forward motion has been completed. If b is the least allowable perpendicular distance between the faces of adjacent rollers in their revolved positions, it can be shown that

$$A = (d + b)\sec\left(114.6\frac{B}{D}\right).$$

With the values here given it is possible to determine the required length of the shoe.

The movement of the end of the span is to be calculated subject to the requirements of Art. 59, Specifications. Since the span is 175 ft. long the movement to be provided for is $175/(10 \times 8) = 2\frac{3}{16}$ in. If it be assumed that this movement takes place one-half on each side of the vertical position of the roller, we find $B = 0.547$ in. If the width of the flat portion of the roller is taken as equal to two-thirds of the diameter of the roller, or $d = 4$ ins., we find that $\theta = 10.5°$ and $\alpha = 42°$. There is then no danger of overturning the roller. Assuming that the rollers approach within ¼ in. of each other in the

revolved position, or $b = \frac{1}{4}$ in., and with $d = 4$ ins. and $\theta = 20.9°$, we find the required distance between rollers to be

$$A = (4 + 0.25) \sec 10.5 = 4.33 \text{ ins.}$$

As shown in Fig. 55, the rollers are spaced $4\frac{3}{4}$ ins. centres. The resulting length of shoe then becomes 32 ins.

The rollers are connected by means of two horizontal bars, as shown in Fig. 55, in order to comply with Art. 62, Specifications. If W = width of side bars, E = distance between centres of bars, and e = clear distance to be provided between bars in the revolved position, it can be shown from Fig. 56 that

$$W = (E \cos \theta - e)$$

with $E = 3$ ins., $e = \frac{1}{2}$ in., and $\theta = 10.5°$ as calculated above, it will be found that $W = 2.45$ ins. Fig. 55 shows the adopted arrangement.

Art. 62, Specifications, requires segmental rollers to be geared to the upper and lower plates. Fig. 55 shows a tooth on the upper and lower face of one of the centre rollers engaging a notch cut in the upper and lower plates.

From the calculations given above, the width of the bed-plate is 58 ins. and its length is 32 ins., giving a base area of 1856 sq. ins., which is more than enough, 1200 sq. ins. being required.

If a floor-beam of the type shown in Fig. 25 is used, resting directly on the shoe, a slightly different arrangement of parts becomes necessary. As calculated in Art. 222 the maximum end reaction is 721,430 lbs. The length of rollers required is 721,430/3600 = 200 lin. ins. If the details of the rollers be taken as symmetrical about the centre line of the truss and similar to the left half of Fig. 55(a), the net bearing per roller is 28 ins. and 200/28 = 7 rollers are required. This will require a shoe 36 ins. long. The general details are similar to those shown in Fig. 55.

The shoe, or bolster, is built up of plates and angles. In Art. 222, the web-plates of the shoe were designed so as to furnish the proper bearing on the end pin. Fig. 42(a) shows three $\frac{5}{8}$-in. web-plates and a $\frac{3}{8}$-in. hinge plate. The web-plates are fastened to the shoe by means of $6 \times 6 \times \frac{3}{4}$-in. angles. As a rule these angles are

not designed. They are made very heavy in order to form a rigid connection between the web- and sole-plate. The web-plates are usually made quite deep. A common rule is to make the depth of plates below the pin at least equal to one-half the length of the shoe. Fig. 55(*b*) shows a depth of 17⅜ ins. provided. As these webs are quite deep, it is best to tie them together by means of a diaphragm consisting of a ⅜-in. plate and four 3½ × 3½ × ⅜-in. angles, as shown in Fig. 55. Added rigidity is also gained by means of a tie-plate placed across the front of the webs.

All details are fully shown on the general drawing of Plate III.

234. **Minor Details.**—Under the head of minor details will be considered such items as the tie-plates on members; lacing of compression members; splicing of top-chord members; and the camber of the truss. Most of these details do not depend upon calculations, but are determined from the results of practical experience. The size of parts and details of construction are usually outlined in the specifications. These details will now be taken up for the truss under consideration.

235. *Lacing of Members.*—The tie-plates on compression members are governed by Art. 46, Specifications:

(46) The open sides of compression members shall be provided with lattice and shall have tie-plates as near each end as practicable. Tie-plates shall be provided at intermediate points where the lattice is interrupted. In main members the tie-plates shall have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than one-half this distance. Their thickness shall not be less than one-fiftieth of the same distance.

As shown in Fig. 14, Art. 189, the top chord and end post form open compression members on the under side. The tie-plates are to be determined subject to the conditions stated above. If the rivets in the lower angles are placed 3 ins. from the backs of the angles, the distance between rivet-lines will be 25½ ins. The tie-plates shown at the ends of the end post and chord members were made 30 ins. long, which is a little in excess of the required length. At intermediate points, where the vertical posts enter the top-chord section, tie-plates 15 ins. long are used, or one-half the length of the end plates. The thickness of the tie-plates is made $25.5/50 = 0.51$ or ½ in.

The rivet-lines in the vertical compression members are about 9 ins. apart. As shown in the general drawing, the tie-plates used are longer than required by the specifications, in order to tie the members securely together. At the lower end of the posts, the plates are made 15 ins. long, and at the top the plates are made 2 ft. long. Due to the form of the sway-bracing in this truss, it seems best to use one long plate at the top of the post, in order to tie the parts of the member together from a point near the pin to a point at the sway-brace connection. In case deeper sway-bracing is used, two tie-plates would be used, one at the top of the post and the other at the sway-brace connection, each plate being about 15 ins. long. The thickness of these need be only $9/50 = 0.18$ ins., but to meet the requirements of other sections of the specifications a $\frac{3}{8}$-in. plate must be used.

The specifications make no mention of the tie-plates to be used on riveted tension members, such as *a b c* of the end panels of the bottom chord. Tie-plates on such tension members are used in order to make the two parts of the member work together, and to stiffen the member against vibration, and, in some cases, to provide a member which will be able to take a small amount of compression due to a possible reversal of stress under wind load, as mentioned in Art. 161. As shown in the general drawing, tie-plates about 2 ft. long are provided at the ends of member *a b c*, and $2\frac{1}{2} \times \frac{1}{2}$ in. double lacing is provided for the space between tie-plates. A safe rule for the dimensions of tie-plates on tension members is that the length of end plates is to be about two-thirds of the distance between rivet lines, and the intermediate plates to be about one-half of this distance. The thickness can be made about $\frac{1}{60}$ of the distance between rivet lines, or $\frac{1}{2}$ in. for this truss. The distance between plates should be such that the value of l/r for each web on a length equal to the unsupported distance between plates shall not exceed 200, as called for by Art. 21, Specifications, for the member as a whole. In spacing the rivets in these plates, care must be taken so that the net section of the member will not be reduced.

Lacing of compression members is provided to make the several parts of a member act as a unit. Two different arrangements of lacing in general use are shown in Fig. 57. A set of single lacing is

shown in Fig. 57(*a*). The bars are so arranged as to make an angle of about 60° with the axis of the member. Standard practice usually limits the use of single lacing to cases where distance d between rivet lines is less than 15 ins. When d is greater than 15 ins., double

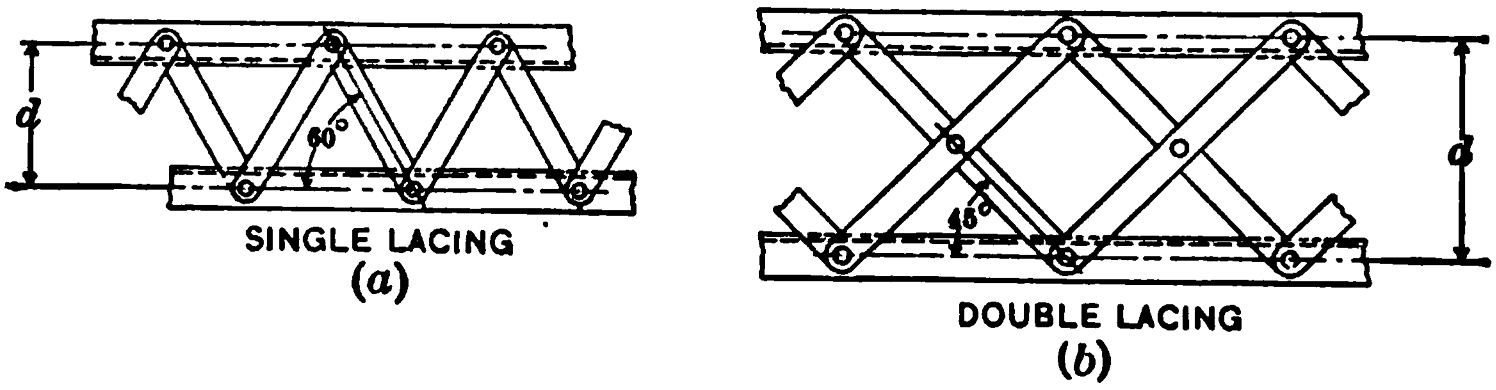


FIG. 57.

lacing, shown in Fig. 57(*b*) is used. The angle between the lacing bars and the axis of the member is usually taken as 45 degrees.

The articles from the Specifications relating to lacing are as follows:

(47) The latticing of compression members shall be proportioned to resist the shearing stresses corresponding to the allowance for flexure for uniform load provided in the column formula in paragraph 16 by the term $70\,l/r$. The minimum width of lattice bars shall be 2½ in. for ⅞-in. rivets, 2¼ in. for ¾-in. rivets, and 2 in. if ⅝-in. rivets are used. The thickness shall not be less than one-fortieth of the distance between end rivets for single lattice, and one-sixtieth for double lattice. Shapes of equivalent strength may be used.

(48) Three-fourths-inch rivets shall be used for latticing flanges less than 2½-in. wide, and ¾-in. for flanges from 2½ to 3½ in. wide; ⅞-in. rivets shall be used in flanges 3½-in. and over, and lattice bars with at least two rivets shall be used for flanges over 5 in. wide.

(49) The inclination of lattice bars with the axis of the member shall be not less than 45 degrees, and when the distance between rivet lines in the flanges is more than 15 in., if single rivet bar is used, the lattice shall be double and riveted at the intersection.

(50) Lattice bars shall be so spaced that the portion of the flange included between their connections shall be as strong as the member as a whole.

As the angles on the open side of the top chord and end post have 6-in. legs, two rivets must be used in the end of each lattice bar. This requires a bar 5 ins. wide. For bars of this size single lacing is generally used, although the distance between rivet lines is 25½ ins., as calculated above. Assuming the lacing to make an angle of 45° with the axis of the member, the length of each bar is $25.5 \times 1.41 = 36.1$

ins. Since the lattice bars are rigidly fastened at the ends, the unsupported length can be considered as $\frac{2}{3}$ of the total, or 24 ins. The thickness required is $\frac{24}{40} = 0.60$ or $\frac{5}{8}$ ins. Rivets $\frac{7}{8}$ in. in diameter are to be used. This lattice bar is the minimum size allowed by the specifications. It must be able to take care of the loads specified by Art. 47, Specifications. Since the end post is the longest of the main chord members, its value of l/r, and therefore the load to be carried by the lacing, will be larger than the other chords. If the $5 \times \frac{5}{8}$-in. bars will carry this load, they will be used throughout. Art. 47, Specifications, requires that the lacing bars be able to carry a shear such as would be produced by a uniform load causing a fibre stress of $70\,l/r$ on the extreme fibres of the chord section. Expressed as a formula, this shear is given by $V = \dfrac{280\,A\,r}{c}$ where V = shear to be carried, A = area of chord section, r = radius of gyration for an axis perpendicular to the plane of the lattice bars, and c = distance from axis of chord to most remote fibre. From Art. 189 the area of the end post section is 60.6 sq. ins. and its moment of inertia about a vertical axis is 5,825 ins.[4] The corresponding value of r is then 9.82 ins. From Fig. 14, p. 281, the value of c is $15\frac{3}{4}$ ins. We then have $V = \dfrac{280\,A\,r}{c} = 280 \times 60.6 \times 9.82/15.75 = 10{,}550$ lbs. This shear can be considered as carried by the lacing and by the cover-plate of the chord section, each taking half the shear, or 5,275 lbs. The bars slope at an angle of about 45 degrees to the axis of the member, and the stress in each bar is $5{,}275 \times 1.41 = 7{,}440$ lbs. per bar, tension or compression. This stress is for a lattice bar located at the end of the member. The general drawing shows that the centre of the first set of bars is located about 6 ft. from the end of the end post, the remaining distance being covered by tie-plates. The stress in the first set of lattice bars will then be less than calculated. Since the shear calculated above has been assumed to be due to uniform load conditions, it varies uniformly from the maximum value calculated above for the end of the post to a value of zero at the post centre. As the

post is 39 ft. long, the stress in the lattice bars 6 ft. from the end of the post is $7,440 \times 13.5/19.5 = 5,160$ lbs.

As stated above, the unsupported length of a lattice bar is 24 ins. The least radius of gyration for a $5 \times \frac{5}{8}$-in. rectangle is 0.18 ins. Therefore, $l/r = \frac{24}{0.18} = 133$, and from the column formula of Art. 16, Specifications, the working stress is 6,700 lbs. per sq. in. For a $5 \times \frac{5}{8}$-in. bar, the total load which can be carried is 21,000 lbs. The assumed bar is sufficient and will be used throughout.

Art. 50, Specifications, limits the spacing between lattice bars to a distance which will make l/r for the flanges not greater than that for the end post as a whole, which, from Table C of p. 275 is 52.3. From the handbook of rolled shapes, the radius of gyration of a $4 \times 6 \times \frac{5}{8}$-in. angle placed as shown in Fig. 14, p. 281, is 1.9 ins. for an axis parallel to the web plates of the chord section. Since the lattice bars make an angle of $45°$ to the axis of the chord member, the unsupported length between lattice bars along the line of the lower angles is equal to twice the distance between rivet lines in the two lower chord angles, or 51 ins. Then $l/r = \frac{51}{1.9} = 27$, a value well within the required limits called for by the specifications.

The lacing for the vertical posts *Cc* and *Dd* is to be determined in the same manner as for chord members. From the handbook it will be found that $\frac{7}{8}$ in. rivets can be driven in the flanges of the channels used for these members. The width of the lattice bar for $\frac{7}{8}$-in. rivets is fixed by Art. 47, Specifications, at $2\frac{1}{2}$ ins. It will be found that $2\frac{1}{2} \times \frac{3}{8}$-in. lattice bars will answer all other requirements of the specifications.

236. *Top Chord Splice.*—For convenience in shipping and erecting, the top chord of trusses of the size considered in this chapter is broken up into pieces of a length about equal to a panel of the truss. These parts are then joined together by means of a field-splice. The design of this splice is governed by the conditions of Art. 51, Specifications.

(51) Abutting joints in compression members when faced for bearing

shall be spliced on four sides sufficiently to hold the connecting members accurately in place. All other joints in riveted work, whether in tension or compression, shall be fully spliced.

Thus by providing a milled joint between the sections of the top chord, the splice to be provided is intended only to hold the parts to a firm bearing. If this milled joint is not provided the joint must be fully spliced. Due to the heavy stresses in the top chord members, these splices would be very large and also very expensive.

In making these top chord splices, it is usual to place the splice at a point on the side of the joint nearer the end of the truss. The web plate of the chord member for the panel nearer the centre of the truss is extended over the joint to the point of splice, as shown on the general drawing.

The cover plate of the chord section and lower chord angles are spliced by means of plates of the size required for tie-plates at these points. These plates have been mentioned at the beginning of this article under the head of Tie-Plates. The web of the chord section is spliced by means of plates placed on both sides of the web. The inside splice-plates are made of a depth equal to the depth of the web plate, and the outside splice-plates are made of a depth such that they will fit in between the points of the angles on the chord section. The length of these plates is such that two rows of rivets can be placed each side of the splice.

The splice is usually located after the size of the lateral plates has been determined. The upper and lower splice-plates are then placed as close to the lateral plate as convenient. All details are shown on the general drawing.

237. *Camber.*—As actually erected, the panel-points of a truss are raised a certain distance above a horizontal line through the supports. This is done so that when the truss is fully loaded it will deflect to a true horizontal position. Such an adjustment of the position of the panel-points is known as putting a camber in a truss. From the specifications:

(81) Truss spans shall be given a camber by so proportioning the length of the members that the stringers will be straight when the bridge is fully loaded.

The proper camber for a truss can be determined in two different

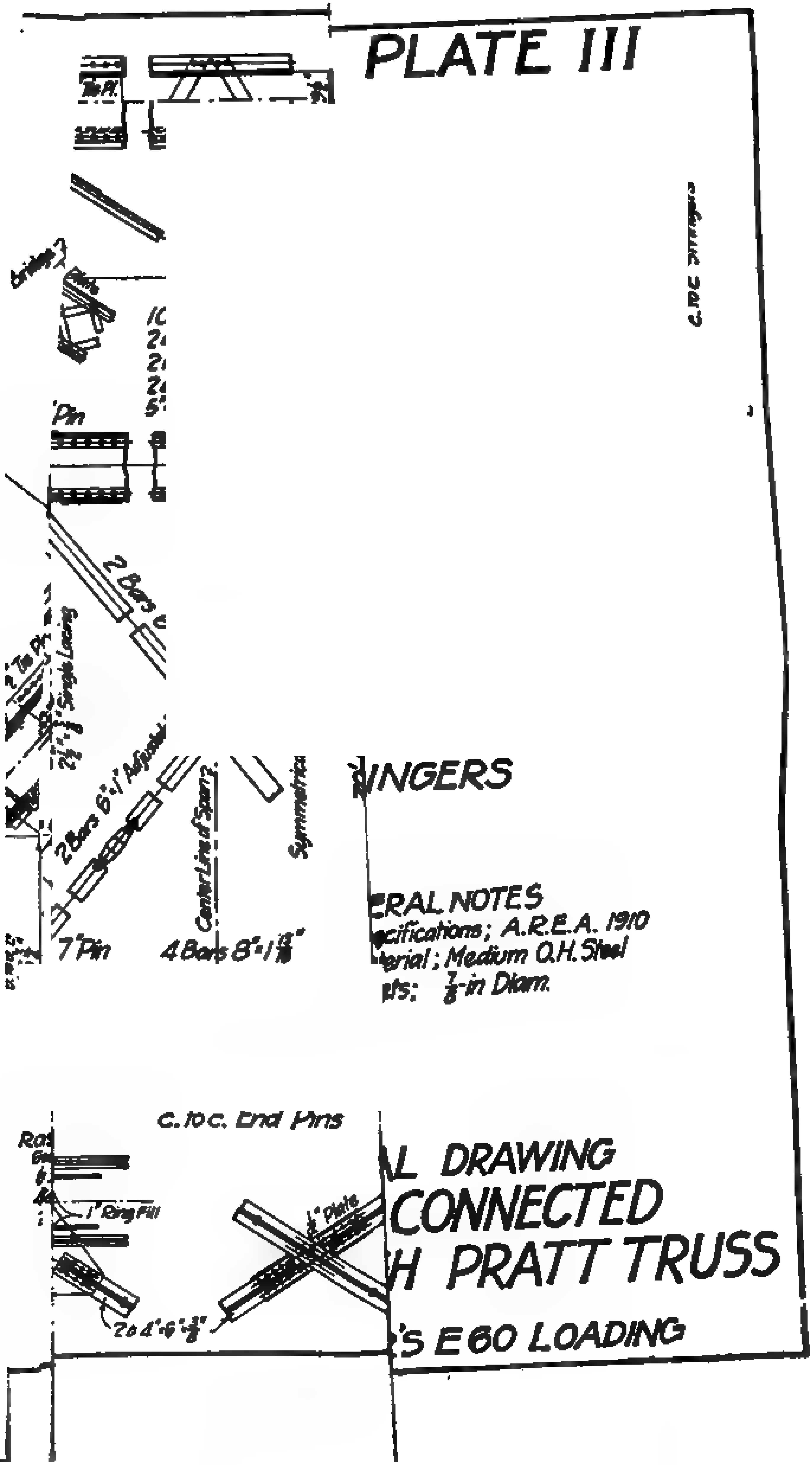


PLATE III
INGERS
ERAL NOTES
ecifications; A.R.E.A. 1910
erial; Medium O.H. Steel
lts; 7/8-in Diam.
L DRAWING
CONNECTED
H PRATT TRUSS
'S E 60 LOADING
Pin
2 Bars 6"x1" Adjustable
7"Pin
4 Bars 8"x1 13/16"
Center Line of Span
Symmetrical
Single Lacing
C. to C. End Pins
1" Ring Fill
1" Plate

ways. The first method of providing camber is an exact method. It requires the calculation of the exact elongation or shortening of the several members under full load. The lengths of the members as manufactured are then modified by these amounts, the compression members being made longer and the tension members shorter than the lengths calculated for the undeformed truss.

The other method, which is the one in general use, is to increase the lengths of only the top chord members. This increase in length is determined by a rule, which is, that the top chord panel lengths shall be increased $\frac{1}{8}$ in. for each 10 ft. of length. For the truss under consideration, the top chord panels will then be increased by $25/10 \times \frac{1}{8} = \frac{5}{16}$-in., making the manufactured length of these members 25 ft. – $\frac{5}{16}$ in., as shown on the general drawing. All other members are made up as calculated for panels of 25 ft. and a truss height of 30 ft.

CHAPTER IX

DESIGN OF RIVETED TRUSSES

THE DESIGN OF A RIVETED WARREN GIRDER

238. General Data.—The deck Warren girder is often used in place of a deck plate girder for spans of from 100 to 125 ft. This span length requires a depth of girder of from 12 to 15 ft. Plate girder webs of this depth cannot be obtained in single pieces, as the maximum width of roller plates is about 11 ft. It is then necessary to provide horizontal web splices, or reduce the depth of girder to that for which rolled plates can be obtained. This increases the flange stress and results in very heavy girders.

Where the head room is not restricted, the required depth can readily be obtained with a Warren girder. The substitution of truss members for a web plate usually results in a saving of material. The ties are placed directly on the top chord, as in the case of the deck plate girder designed in Chap. VI, so that the chord members must carry bending moment in addition to direct stress. This results in deep and heavy chord sections. The extra material required in the chords will about balance the saving of material in the webs, thus resulting in a structure of nearly the same weight as the plate girder of the same span. Since the weights of the Warren and plate girders are practically the same, the choice between the two types of structure will depend upon other conditions. The principal advantage of the Warren girder over the plate girder is that the former can be broken up into small parts for shipment, while the latter is generally shipped in one piece.

Where very heavy loads are to be carried, the top chord section becomes so large that the secondary stresses due to continuous girder action, and truss deflection, discussed in Chapter IV, cannot be kept within the specified limits without an uneconomical use of material. In such cases the floor should be carried by a stringer and floor-beam

system, as in the through plate girder designed in Chapter VI. For a live load not exceeding Cooper's E–50, a substantial and economical structure is provided by the deck Warren girder, ties placed directly on the top chord.

In order to bring out the principles involved in the design of this type of structure, a 125-ft. deck Warren girder will be designed. The trusses will be made 15 ft. deep, and they will be spaced 10 ft. centre to centre. In order to avoid large bending stresses in the top chord members, a relative short panel length will be used. Fig. 1 shows the adopted arrangement. The live load will be taken as Cooper's E–40, and the main features of the design will be governed by the Specifications of the American Railway Engineering Association, as given in Appendix A.

239. Design of the Floor.—The methods of design and the requirements of the Specifications are the same as given in Art. 137

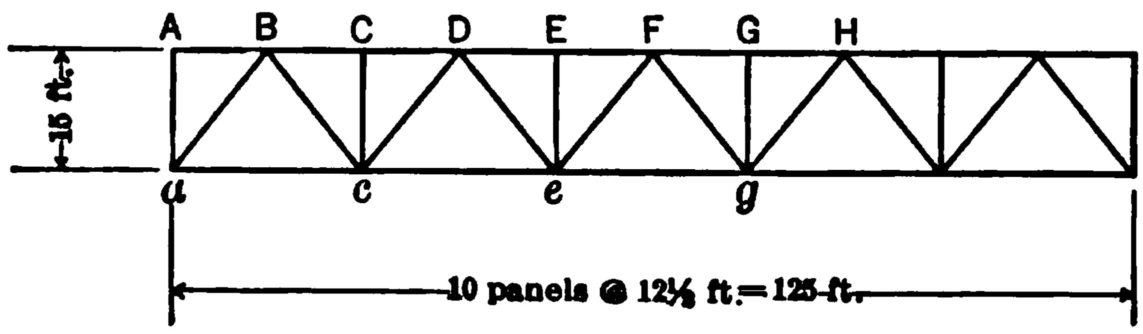


FIG. 1.

for the deck plate girder design. Fig. 2 shows the loading conditions for the case under consideration. The live-load bending moment for one tie, including 100 per cent impact, is

$$M = \tfrac{1}{3} \times 50{,}000 \times 2.5 \times 12 = 500{,}000 \text{ in.-lbs.}$$

This requires a section modulus of 500,000/2,000 = 250. A 10 × 12-in. tie has a section modulus of 240, but it will be adopted. The ties will be 14 ft. long. They will be held in place by two 6 × 8 in. guard rails.

240. Stresses in Members.—The dead, live, and impact stresses, calculated by the methods given in Part I, are tabulated in Table A. It was found that the wind load chord stresses can be neglected, as they are less than 25 per cent of those for vertical loading.

The dead weight of the structure was estimated by the formula

used in Art. 176 for the pin-connected span, as a rough estimate, showed that this formula gives approximately correct results for the case under consideration. With $l = 125$, the formula gives $w = \frac{7}{8}(8 \times 125 + 700) = 1{,}488$ lbs. per ft. of bridge. A 10 × 12-in. tie 14 ft. long weighs 630 lbs., or $\frac{12}{16} \times 630 = 472$ lbs. per ft. of bridge, and two 6 × 8-in. guard rails weigh 36 lbs. per ft. of bridge. With rails and fastenings at 150 lbs. per ft., the total weight of the floor is 658 lbs. per ft. of bridge. The total dead load is $658 + 1{,}488 = 2{,}146$ lbs. per ft. Considering all the dead load to be applied at the top chord joints, the dead panel load is $2{,}146 \times 12.5 = 26{,}825$ lbs., or 13,400 lbs. per panel per truss. Table A gives the resulting dead-load stresses. The member notation is as shown in Fig. 1.

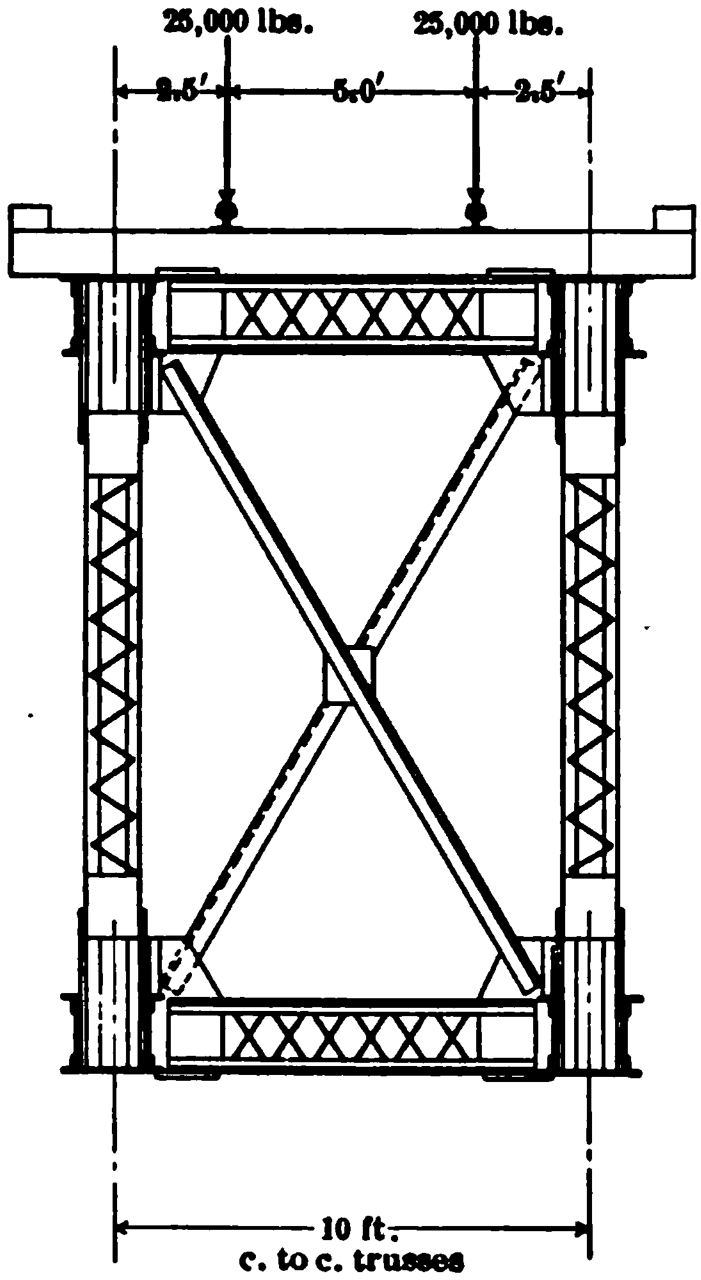


FIG. 2.

It will be noted from Table A that members *De* and *eF* are subjected to a reversal of stress during the passage of one train. The maximum stresses in these members are governed by the following articles from the Specifications:

(22) Members subject to alternate stresses of tension and compression shall be proportioned for the stresses giving the largest section. If the alternate stresses occur during the passage of one train, as in stiff counters, each stress shall be increased by 50 per cent of the smaller. The connections shall in all cases be proportioned for the sum of the stresses.

(23) Wherever the live- and dead-load stresses are of opposite character, only two-thirds of the dead-load stresses shall be considered as effective in counteracting the live-load stress.

The maximum stress to be used in the design of these members is then $191{,}300 + \frac{1}{2} \times 28{,}650 = 205{,}625$ lbs. for *De* and $125{,}410 + \frac{1}{2} \times 73{,}300 = 162{,}060$ lbs. for *eF*. In the tabulation given in Table A for *De* and *eF* the dead-load stresses given in the second line

for each member are only ⅔ of the true dead-load stress in order to comply with Art. 23, Specifications.

TABLE A
STRESSES IN NUMBERS

Member	Dead Load	Live Load	Impact	Total Stress
AB	0	0	0	0
BCD	− 89,500	−221,000	−156,000	−466,500
DEF	−134,000	−325,000	−229,500	−688,500
ac	+ 50,300	+127,000	+ 89,600	+266,900
ce	+117,100	+285,500	+201,500	+604,100
eg	+139,500	+332,000	+234,000	+705,500
aB	− 78,500	−198,100	−140,000	−416,600
Bc	+ 61,000	+159,000	+116,900	+336,900
cD	− 43,600	−123,900	− 94,000	−261,500
	− 29,000	+ 10,500	+ 9,750	....
De	+ 26,100	+ 92,100	+ 73,100	+191,300
	+ 17,400	− 24,650	− 21,400	− 28,650
eF	− 8,710	− 64,500	− 52,200	−125,410
	− 5,800	+ 43,000	+ 36,100	+ 73,300
Cc, Ee, Gg	− 13,400	− 48,000	− 44,300	−105,700
Aa	− 6,700	− 36,000	− 34,600	− 77,300

+ Denotes tension. — Denotes compression.

The stresses in the members of the lateral system are determined by the same methods as used in Art. 150 for the deck-plate girder. This will be left as a problem for the student.

241. Design of Members.—As the joints are to be riveted, all members will be made up of rolled shapes and plates. The details are as shown on the general drawing, Plate IV, and Table B gives all necessary data for the design of the members.

All web members are made of rolled channels placed with the flanges turned inward. The discussion given in Art. 164, Chap. VII, shows that this is the best section for such compression members. For appearance the tension members will be made of the same form as the compression members.

The lower chord consists of members built up of angles and plates. In determining net areas, one rivet hole is deducted from each angle, four from each web plate and cover plate, and two from each side plate. The angles are placed with the flanges turned outward. This allows the verticals to be run down between the web plates to form a diaphragm. It also simplifies the connection of the cross-frames to the truss.

Members *ce* and *eg* have angles, web plates, and side plates of the same size. Additional area is provided for member *eg* by cover plates. Member *ac* is made of angles and plates of minimum thickness corresponding to the sizes used for the other members.

TABLE B

Design of Members

Member	Stress (Lbs.)	Length (Ins.)	Radius of Gyration (Ins.)	Unit Stress (Lbs./In.²)	Area Required (Sq. Ins.)	Section	Area Provided (Sq. Ins.) Gross	Net
Top Chord	....	....	.....	......	.....	Special Design.	.....	.....
ac.....	+266,900	....	.....	16,000	16.7	4 angles $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$ ins. 2 plates $16\times\frac{3}{8}$ ins......	21.96	17.46
ce......	+604,100	....	.....	16,000	37.8	4 angles $3\frac{1}{2}\times3\frac{1}{2}\times\frac{9}{16}$ ins. 2 web plates $16\times\frac{3}{4}$ ins.. 2 side plates $9\times\frac{9}{16}$ ins...	48.65	38.15
eg.....	+705,500	....	.....	16,000	44.1	4 angles $3\frac{1}{2}\times3\frac{1}{2}\times\frac{9}{16}$ ins. 2 web plates $16\times\frac{3}{4}$ ins.. 2 side plates $9\times\frac{9}{16}$ ins... 2 cover plates $15\times\frac{3}{8}$ ins.	59.90	46.40
aB.....	−416,600	234.5	5.16	12,820	32.3	2 15-in. channels @ 55 lbs.	32.36	.....
Bc.....	+336,900	....	.....	16,000	21.1	2 15-in. channels @ 45 lbs.	26.48	22.76
cD.....	−261,500	234.5	4.09	11,980	21.9	2 12-in. channels @ 40 lbs.	23.52	.....
De.....	+205,625	....	.....	16,000	12.8	2 10-in. channels @ 25 lbs.	14.70	12.60
eF.....	−162,060	234.5	3.52	11,340	14.3	2 10-in. channels @ 25 lbs.	14.70	.....
Cc, Ee, Gg.	−105,700	180.0	3.66	12,560	8.42	2 10-in. channels @ 20 lbs.	11.76	.....
Aa.....	− 77,300	180.0	3.66	12,560	6.14	2 10-in. channels @ 20 lbs.	11.76	.....

The top chord is riveted from end to end, and thus forms a continuous girder of ten equal panels when subjected to vertical loading. Chap. I, Part II, gives the methods of calculation for bending moments in this girder. The calculation of moments by continuous girder methods is laborious, and such refinement is hardly warranted by the conditions of the problem. It will be sufficiently accurate, and in accordance with the usual practice, to use an approximate method of design. This approximate method assumes that the positive moment at the centre of a panel and the negative moments at the supports are each equal to three-quarters of the bending moment at the centre of the panel when considered as a simple beam.

The top chord members will be designed for two combinations of bending moment and direct stress, as follows: Case A: Maximum bending moment and simultaneous chord stress. Case B: Maximum

chord stress and simultaneous bending moment. The chord section is to be made up for the combination which requires the greater area. Art. 70, Chap. IV, gives the methods of design.

The calculations will be followed through in detail for member *B C D*. Case A, as outlined above, requires the calculation of the

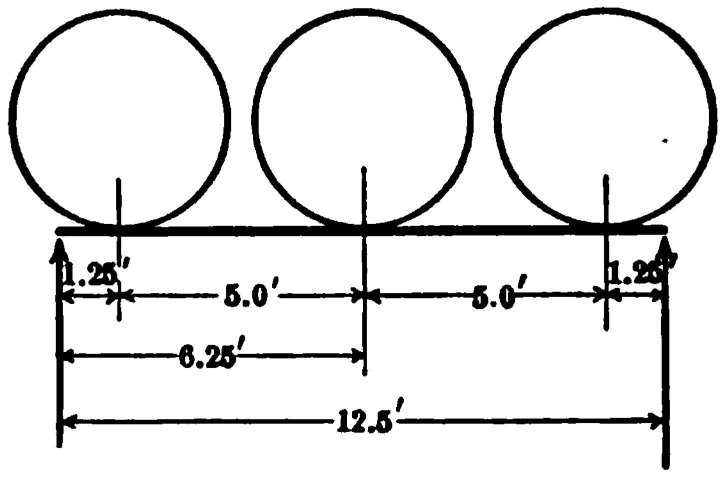


FIG. 3.

maximum bending moment in a panel 12½ ft. long. For live load this will be found to occur when three drive wheels are located in a panel. Fig. 3 shows the loads in position. The maximum moment occurs under the centre wheel, and for E–40 loading, the moment is 87,500 ft.-lbs. Considering the loaded length equal to a panel length, the allowance for impact is 0.96 × 87,500 = 84,000 ft.-lbs. (Art. 9, Specifications). The dead-load moment due to the floor and chord member must also be included. Based on a trial section, it was found that, adding 20 per cent for details, member *B C D* weighs 215 lbs. per ft. Adding the weight of the floor, which from Art. 240 is 330 lbs. per ft., the total dead load is 545 lbs. per ft. and the centre moment is 10,700 ft.-lbs. The total simple-beam moment is then 182,200 ft.-lbs. Reducing this to an equivalent continuous girder moment by multiplying by the coefficient three-quarters, the moment to be used in the design is 1,640,000 in.-lbs. positive at the centre of the panel, and negative at the supports.

For member *B C D* it will be found that the position of live load which will realize the conditions shown in Fig. 3 and give the maximum simultaneous chord stress is for wheel 4, placed 1¼ ft. to the left of panel point *C* of Fig. 1. The stress in the member for E–40 loading is 220,000 lbs. With a loaded length of 125 ft., the allowance for impact is 0.706 × 220,000 = 155,500 lbs. From Table A, Art.

240, the dead-load stress in BCD is 89,500 lbs. The total stress in BCD is then 465,000 lbs.

Case B requires the calculation of the maximum stress in BCD and the simultaneous bending moment. From Table A, the maximum stress in BCD is 466,500 lbs., which occurs when wheel 4 is placed at panel point C of Fig. 1. With wheel 4 at the right end of a 12½-ft. panel, the maximum bending moment in the panel occurs under wheel 3, and it is 80,000 ft.-lbs. Using the same impact coefficient and dead-load moment as for Case A, the equivalent continuous girder moment is 1,510,000 in.-lbs.

Applying similar methods of calculation to member DEF, it will be found that the corresponding values are as follows: Case A: Stress 686,000 lbs., moment 1,650,000 in.-lbs. Case B: Stress 688,500 lbs., moment 1,520,000 in.-lbs. It is to be noted that the bending moment at panel point g of Fig. 1 is greater than that at point e. The above values are therefore calculated for member FGH.

The chord sections are to be designed by the methods given in Art. 70. Conditions at the supports will determine the required area, for at these points the distance from the neutral axis to the extreme compression fibre is greatest. The value of f_c is the same at the centre and ends of the member, due to the large radius of gyration and short length of member, being 14,000 lbs. per sq. in., the maximum allowed by Art. 16, Specifications.

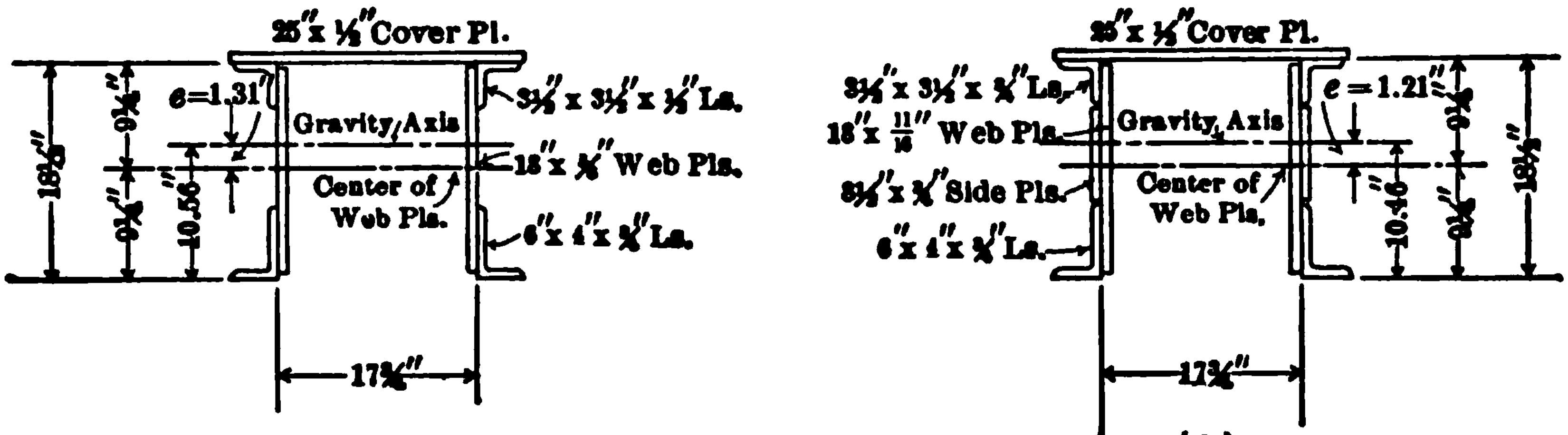
Fig. 4(a) shows the section assumed for member BCD. The gross area of this section is 55.38 sq. ins. By the methods used in Art. 189, Chap. VIII, it will be found that the gravity axis is located 1.31 ins. above the centre of web plate. For this axis the moment of inertia is 2,847 ins.,[4] the radius of gyration is 7.16 ins.; and the distance to the extreme compression fibre for negative moment is 10.56 ins. From Eq. (21), Art 70, the area required for Cases A and B are as follows:

$$\text{Case A.—}A = 465{,}000/14{,}000 + (1{,}640{,}000 \times 10.56)/(16{,}000 \times 7.16^2) = 54.2 \text{ sq. ins.}$$

$$\text{Case B.—}A = 466{,}500/14{,}000 + (1{,}510{,}000 \times 10.56)/(16{,}000 \times 7.16^2) = 52.7 \text{ sq. ins.}$$

The assumed section provides the required area and will be adopted.

Fig. 4(*b*) shows the section assumed for member *D E F*. From the values of moments and stresses given for members *B C D* and *D E F*, it will be noted that the moments for these two members are nearly equal, but that the direct stress for *D E F* is considerably greater than that for *B C D*. In making up the section for member



(*a*) (*b*)

FIG. 4.

D E F the same general dimensions were used as for *B C D*, but the additional area was placed in the webs and angles, for in this position it has its greatest value as compressive area. As the gusset plates are preferably spaced with the same clear distance between inside faces throughout the truss, it is best not to make any great change in web plate thickness. Most of the additional required area will be placed in the top angles and in a side plate which fits between the angle legs, as shown in Fig. 4(*b*). Proceeding as in the design for member *B C D*, it will be found that the assumed section provides the required area.

Member *A B* has no direct stress. It acts as a beam to carry the loads from points *B* to *A*. A section of the same area as *B C D* will be used. This section provides some excess area, but as it will permit the elimination of a splice at joint *B*, it will be adopted.

Minor details, such as tie plates, lacing, and diaphragms are designed by the principles outlined in Art. 234, Chap. VIII. All details of members are as shown on the general drawing, Plate IV.

The lateral bracing, sway bracing, and end cross-frames are arranged as shown on Plate IV. As the adopted arrangement is governed by the same general requirements as for the deck plate girder designed in Chap. VI, the student can easily check the design as given.

242. Design of Joints.—The general principles governing riveted

joint design are given in Chap. VII. Plate IV shows the details of the joints as designed. It can be seen that the arrangement of joint details is such that the rivets connecting the members to the gusset plates are in single shear. As the truss is to be broken up into small parts for shipment, field connections are to be provided at nearly all joints.

Since the members are connected by field rivets in single shear, the number required in any member is equal to the maximum stress given in Table A, Art. 240, divided by 6010, the value of a rivet. One-half the number required are shown on the general drawing in each half of the member.

Members *De* and *eF* are subject to a reversal of stress during the passage of one train. Art. 22, Specifications, states that the end connections for such members are to be designed for the sum of the alternate stresses. For *De*, $(191{,}300 + 28{,}650)/6{,}010 = 37$ rivets are required in each end of the member. The required number are shown in position on Plate IV.

The design of gusset plates has been discussed in Art. 170, Chap. VII. Gusset plates must provide an area on a section cut through the last row of rivets at least equal to the area of the member connected. On Plate IV such a section is shown for member *De* at joint *e* by the curved line *x-y*. This section has a length of about 3.5 ft. The area to be provided by each gusset plate is one-half the net area of *De*, which from Table *B*, Art. 241, is 6.3 sq. ins. A thickness of $6.3/42 = 0.15$ ins. is required. Similar calculations for other joints show that in no case is the required thickness of gusset plates greater than 3/8 in., which is the minimum allowable thickness of material.

The adopted thickness of gusset plates will be determined by conditions fixed by the distance between inside faces of web plates on the chord members as mentioned in the preceding article. For bottom chord members this distance can be taken as desired, as the member is composed of two separate parts. The top chord has a cover plate whose width determines the distance between inside faces of web plates. Since member *D E F* has the thickest web plates, the distance between inside faces of web plates is least for this member, and is 16 3/8 ins. as shown by Fig. 4 (*b*). At top chord joints *D*, *E*, and *F*, gusset plates of minimum thickness, 3/8 in., will be used. The

web plates on the members from joints A to D are $^1/_{16}$ in. thinner than those for $D\,E\,F$. If the same clear distance between inside faces of web plates is to be maintained, which is desirable, the gusset plates at all other top chord joints must be made $^7/_{16}$ in. thick. The lower chord gusset plates will also be made $^7/_{16}$ in. thick. All details are as shown on Plate IV.

The lower chord will be spliced at each panel point. These splices must develop the full net strength of the members. At joint e, the splice will be placed just to the left of the joint. This requires a splice for the stress in member ec. From Table B, Art. 241, the area of ec is 38.15 sq. ins. The load to be carried to each gusset plate is then $19.08 \times 16{,}000 = 306{,}000$ lbs. By extending the cover plate of member eg over the splice, the rivets through member ec are placed in double shear. Field rivets in double shear have a value of 12,020 lbs. per rivet, and $306{,}000/12{,}020 = 26$ rivets are required.

At joint c the splice is also located to the left of the joint. The load to be carried to each gusset plate is $\frac{1}{2} \times 17.46 \times 16{,}000 = 139{,}500$ lbs. By extending the side plate of ec over the splice, part of the rivets in the web of ac are placed in bearing. Plate IV shows 17 rivets in the single shear and 11 rivets in bearing on the $\frac{3}{8}$ in. web plate. The value of these rivets is 174,330 lbs. In making this splice, sufficient rivets must be placed in each part to carry its stress to the gusset plate. As each angle has a net area of 2.115 sq. ins., its proportion of the total load is $2.115 \times 16{,}000 = 33{,}800$ lbs. This requires $33{,}800/6{,}010 = 6$ rivets in each angle, the number shown on Plate IV.

The top chord will be made in three pieces. A splice will be located just to the left of joint D, and another splice at the corresponding point on the right hand side of the span. As the top chord acts as a beam, any bending moment which exists at the point of splicing must be provided for. By placing the splice near the quarter point of the panel, where the bending moment is practically zero, and the shear is not large, a direct compression splice can be provided. The splice shown on Plate IV will develop the full compressive strength of the section.

At joint a, the connection between the truss and the shoe is provided by a 6-in. pin. The design of the pin and the necessary bearing

plates is similar to the design of joints for the pin-connected truss given in Chap. VIII. Plate IV shows the adopted arrangement.

The Design of a Riveted Pratt Truss

243. **General Data.**—Riveted Pratt trusses are now in general use for single- and double-track spans up to about 175 ft., and occasionally for spans over 200 ft. long. For spans shorter than these, the riveted structure is more rigid than the pin-connected structure. In longer spans, the greater ease of erection gives the pin-connected span the advantage over the riveted span.

In general, it will be found that the span with an even number of panels is best adapted to the riveted structure, while the span with an odd number of panels is best for the pin-connected structure. The even number of panels permits symmetrical joint details and avoids the use of a double set of rigid diagonals in a centre panel. In pin-connected spans, an odd number of panels simplifies the lower chord-bar packing near the centre of the span.

A brief discussion of the most important points in the design of a riveted Pratt truss will be given in the following articles. The general dimensions and loading for the structure to be designed will be the same as for the pin-connected span designed in Chap. VIII. This structure has an odd number of panels, but since all compression members, as designed for the pin-connected span, can be used for the riveted span and since the stresses for all tension members are as given in Table A, Art. 180, the same data will be assumed. Fig. 3, Art. 180, shows the general dimensions of the structure under consideration.

244. **Design of Members.**—As stated in the preceding article, all compression members will be taken the same as given in Table C, Art. 186, for the pin-connected span. Since all joints are to be riveted, the tension members will be built up of plates and rolled shapes. Riveted tension members were used in the pin-connected span for members *Bb* and *abc*, as shown on Plate III. Similar sections will be used for these members in the riveted span. The web plate of *Bb* will be made wider than before, in order to make it fit in between the gusset plates at joint B. Plate V shows that a plate 16 ins. wide

is required. For member *abc*, the same plates and angles will be used as in Table C, Art. 186, but the horizontal legs of the angles will be turned in, as shown in the cross-section on Plate V. All other tension members are designed by the same methods as used in Art. 186 for member *abc*. The make-up of the sections as designed are given in the following table. The stresses used in designing are taken from Table A, Art. 180.

TABLE C

SECTIONS OF TENSION MEMBERS FOR RIVETED PRATT TRUSS

Member	Stress (Lbs.)	Unit Stress (Lbs./Ins.²)	Area Required (Sq. Ins.)	Section	Area Provided (Sq. Ins.)	
					Gross	Net
abc......	477,400	16,000	29.85	4 angles $4 \times 4 \times \frac{1}{2}$ ins..... 2 plates $20 \times \frac{9}{16}$ ins.......	37.5	31.0
cd.......	772,000	16,000	48.30	4 angles $4 \times 4 \times \frac{1}{2}$ ins..... 4 plates $20 \times \frac{9}{16}$ ins.......	60.0	49.0
de.......	918,200	16,000	57.40	4 angles $4 \times 4 \times \frac{1}{2}$ ins..... 4 plates $20 \times \frac{9}{16}$ ins....... 2 plates $12 \times \frac{1}{2}$ ins.......	72.0	59.0
Bc......	540,900	16,000	33.80	4 angles $4 \times 4 \times \frac{1}{2}$ ins..... 2 plates $18 \times \frac{3}{4}$ ins.......	42.0	34.0
Cd......	381,600	16,000	23.85	4 angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ ins. 2 plates $16 \times \frac{9}{16}$ ins.......	29.0	24.5

Member *Cd* is subject to a reversal of stress during the passage of one train. Its maximum stress, as given in the above table, is made up to conform to the requirements of Art. 22, Specifications. The required stress can be obtained from Table C, Art. 186, by adding to the stress for *Cd*, one-half the stress in *Dc*, the counter in the same panel.

The design of the floor system is exactly the same as that given in Chap. VIII for the pin-connected truss. Plate V shows the details of an intermediate floor-beam. On comparing this design with the one shown in Fig. 20, Art. 197, for the pin-connected span, it will be noted that the floor-beam flange and the web sections are the same, but that the end details for the riveted structure are much simpler than those for the pin-connected structure. This is largely due to the fact that the lower chord angles are turned in, as stated in the preceding article. If these angles were turned out, the end of the

floor-beam would have to be notched out, as in the pin-connected span. The connection between floor-beam and truss shown on Plate V is similar to that shown in Fig. 16, Art. 194, and the methods of design are the same in both cases.

The design and details of the end floor-beam will be taken the same as given in Art. 201 for the pin-connected span. An end shoe and bearings similar to those shown in Fig. 55, Art. 233, will be used. All details are similar to those shown on Plate IV. The stresses in the laterals and portals are the same as for the pin-connected span. All details are the same as shown for the pin-connected span.

245. Design of Joints.—The principles of riveted joint design are given in Chap. V and in Art. 170, Chap. VII. In the truss under consideration, the top chord angles are turned out, and the bottom chord and web member angles and channels are turned in, as shown on Plate V, for a portion of the truss. The gusset plates will be placed inside the web plates of the top chord members, and outside the web plates of the lower chord members. All web members will be placed inside the gusset plates.

In some cases, the web members are placed outside the gusset plates. The floor-beam connections are somewhat simplified by placing the channels of the vertical posts inside the gusset plates, and the size of tie plates and lacing on all members are reduced when placed inside the gusset plates.

As all web members are placed inside the gusset plates, the connecting rivets will in all cases be in single shear. The number required in each end of a member is equal to the maximum stress divided by the value of a field rivet in single shear. as field connections must be provided at all joints.

Several members near the centre of the span are subject to a reversal of stress during the passage of one train. From Art. 22, Specifications, the connections must be designed for the sum of the alternate stresses. Thus for member *Cd*, it will be found from values given in Table A, Art. 180, that the stress are + 352,400 lbs., and − 58,400 lbs. (The compressive stress in *Cd* is the same as the tensile stress in *Dc*, the counter in panel *cd*.) This requires (352,400 + 58,400)/6,010 = 69 field rivets in each end of the member.

The top and bottom chord members are made continuous over

the joints. This requires that riveting between gusset plates and chords be sufficient to take up the difference in stress of the chord members entering the joint. Usually an excess of rivets is provided in such connections, the number being determined by the maximum allowable rivet spacing, and not by stress conditions.

Chord splices are placed near panel points. Top chord splices can be made the same as for the pin-connected span (Art. 236). The lower chord splices must develop the full net strength of the member. Plate V shows the splice at joint *c*. From Table *C*, Art. 244, the net area of *abc* is 31 sq. ins. The load to be carried by the splice is then 31 × 16,000 = 496,000 lbs. In arranging the connection between members *abc* and *cd* splice plates will be placed on the four sides of the members. On the webs, two 20 × ½ in. plates will be placed outside, and two 12 × ½ in. plates will be placed inside the member. The horizontal legs of the upper and lower angles will be spliced by means of 16 × ½ in. plates. Plate. V shows the arrangement of plates. These plates provide a net area of 40 sq. ins., which is somewhat in excess of that required. The difference in make-up of the members spliced requires that a 9/16-in. fill be placed on the inside of member *abc*.

The rivets are in single shear in the upper and lower angles and splice plates of the splice as arranged. Those passing through both the inside and outside web splice plates are in double shear. The splice shown on Plate V provides 40 rivets in single shear and 24 rivets in double shear on each side of the splice, giving a total strength of 528,900 lbs. (field rivets).

The proper distribution of rivets between the angles and web plate must be investigated. Of the total load carried to the splice, 13 × 16,000 = 208,000 lbs. is carried by the angles, and 18 × 16,000 = 288,000 lbs. is carried by the web, assuming the load to be divided in proportion to the net area of angles and plates. To transmit the load carried by the angles requires 208,000/6,010 = 35 field rivets on each side of the splice. Plate V shows 9 rivets in each angle, 5 in the vertical and 4 in the horizontal leg. For the web stress, 288,000/6,010 = 48 rivets in single shear are required. Plate V shows 24 rivets in double shear and 4 in single shear, not counting the rivets in the angles. As the rivets provided in the web are equivalent

to 52 rivets in single shear, the splice is sufficient. Splices at other points are designed by similar methods.

The gusset plates are designed by the methods outlined in Art. 170, Chap. VII. As an example of gusset-plate design, an investigation will be made of the conditions at joint *B*. At this joint large tensile stresses exist in members *Bc* and *Bb*. Fig. 36, Art. 216, shows the stress conditions used in the design of the pin at joint *B* for the pin-connected span. The resultant of the stresses shown for *Bc* and *Bb* is about 660,000 lbs. This is probably a reasonable estimate of the maximum total load brought to joint *B* by these members. In determining the required gusset-plate thickness, it will be assumed that this total load is uniformly distributed over the area of the gusset plates at the point where they are connected to the top chord members. From Plate V, the length of this section is about 7 ft. for each plate. The thickness of plate required is then $660{,}000 / (2 \times 84 \times 16{,}000) = 0.25$ ins. for each plate. Similar calculations for other joints show that plates of minimum thickness, 3/8 in., will answer at all joints.

The adopted thickness of gusset plates will be determined by other than the stress conditions. As the distance between inside faces of gusset plates is preferably kept the same throughout the truss, this condition will determine the gusset plate thickness. At the centre top chord joints, a 7/16 in. gusset plate will be used. The web plate of members *CDE* is 3/4 in. thick, and the web plates of *aB* and *BC* are 9/16 in. thick. To maintain a uniform spacing between inside faces of gusset plates, those at joint *B* must be 5/8 in. thick. The gusset plates at joint *a* will also be taken as 5/8 in. thick.

At lower chord joints *b*, *c*, *d*, etc., the floor-beam and vertical posts are attached to the same plates. In calculating the number of rivets used in these connections the rivets were assumed to be in single shear. In order to realize the assumed conditions, a gusset plate 11/16 in. thick must be used, so that the double shearing strength of a rivet is available. If a thinner plate is used, the rivets common to the floor-beam and post connection will be in bearing on the gusset plate. This would require more rivets and larger gusset plates. Plate V shows 11/16 in. plates at lower chord joints.

Joint *a* is a combination riveted and pin-connected joint. The

truss members entering the joint are riveted to gusset plates, and the truss and shoe are connected by a pin joint. This pin joint is used in order to obtain a uniform distribution of load to the rollers and masonry, as discussed in Art. 167, Chap. VII.

In designing joint *a*, the first step is to provide sufficient rivets to connect the main members to the gusset plates. The number of rivets required is determined by the methods used for the other joints. After this has been done, the connection between the shoe and the truss is to be designed. The size of pin and thickness and attachment of bearing plates are determined by the methods used in Chap. VIII for the pin-connected truss. Finally, the joint as a whole is to be investigated, in order to make certain that the loads are transmitted from the truss members to the shoe without overstraining the rivets or plates.

Member *aB* is connected to the gusset plates at joint *a* by means of shop rivets in single shear, and *abc* is connected by field rivets, also in single shear. The bearing plates and pin size are determined from the maximum end reaction, which from Art. 233 is 575,000 lbs. Assuming a 7-in. pin, which subsequent calculation shows to be satisfactory, the thickness of bearing plates required for each half of the member is 1.72 ins. As the web plate of *aB* is $\frac{9}{16}$ in. thick, and the gusset plate $\frac{5}{8}$ in. thick, the required bearing is provided by placing a $\frac{5}{8}$-in. pin plate inside the gusset plate, as shown in Plate V. The stress in this pin plate is transferred to the gusset plate by rivets in single shear. Assuming the total vertical load to be transmitted to the bearing plates in proportion to their thickness, the load on the pin plate is $287{,}500 \times 0.625 / 1.81 = 99{,}200$ lbs. This requires $99{,}200/7{,}220 = 14$ rivets. The position of these rivets must be such that they will transfer the vertical load to the gusset plate. This calls for a symmetrical arrangement of rivets with respect to a vertical line through the pin. Plate V shows a large excess of rivets provided, but this is necessary, since many of the rivets have already been considered as part of the connection between member *aB* and the gusset plate. Upon investigating the joint by methods similar to those in Art. 226, Chap. VIII, for pin-plate attachment, it will be found that the arrangement shown on Plate V is adequate.

In many designs, the bearing on the pin is provided by means of

plates placed on the web of member *aB*, forming a detail similar to that shown on Plate III for this member. Since the load on the joint due to the shoe connection is vertical, it is evident that this detail is not efficient, for only the rivets symmetrically placed with respect to the pin are in position to carry the applied loads. The rivets beyond this region cause the connection to be eccentric with respect to the applied load. It can be shown by the methods given in Art. 91, Chap. V, that these extra rivets weaken instead of strengthen the joint.

Plate IV shows another arrangement at joint *a* by means of which a symmetrical arrangement of bearing-plate rivets can be obtained. In this detail member *abc* is carried over joint *a* and the end post is cut away above the joint. Bearing plates are then placed on the web of *abc*.

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CHAPTER X

DESIGN OF A RIVETED HIGHWAY BRIDGE

246. General Data.—The principles of riveted highway truss bridge design will be illustrated by the design of a pony truss of the dimensions shown in Fig. 1. A 16 ft. roadway will be provided, and the trusses will be spaced 18 ft. centres. The bridge floor will consist of oak plank laid on steel stringers. The material will be

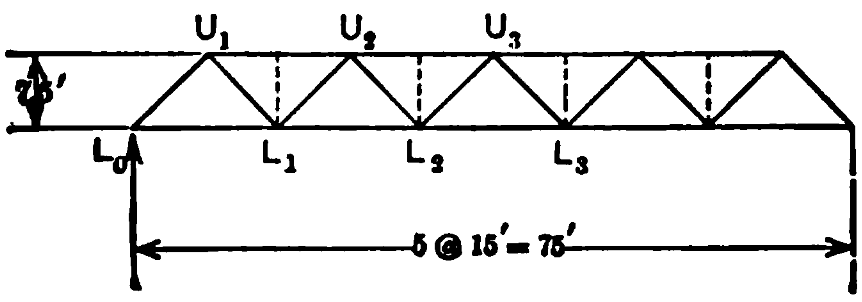


FIG. 1.

taken as structural steel, with a minimum thickness of ¼ in. Rivets ¾ in. in diameter will be used. As a great variety of loadings exist for highway bridges, the problem will be simplified by assuming that the bridge is to be designed for light country service. The assumed live load will consist of a uniform load of 100 lbs. per sq. ft. of floor, or a 6-ton truck carried on two axles 10 ft. centres with wheels 5 ft. centres, the total load to be considered as equally divided among the four wheels. Only one set of loads will be considered on this bridge at any one time. Provision for impact will be made by increasing all live-load stresses by 25 per cent. Many rules and formulas are in use for determination of impact in highway bridges. The impact allowance assumed above seems reasonable, and is readily applied.

247. Design of the Floor System.—*The Plank Floor.*—The bridge floor will be taken as 3-in. oak plank resting on steel stringers spaced 2½ ft. centres. A common rule is that the spacing of stringers in feet must not exceed the thickness of floor plank in inches. Fig. 2 shows the maximum load to be carried by the portion of the floor

between stringers. This load is caused by one wheel of the truck placed half way between stringers. Assuming the floor plank to act as a simple beam between stringers, the bending moment to be carried is $1500 \times 15 = 22,500$ in.-lbs. Increasing this moment by 25 per cent for impact, the total moment to be carried is 28,125 in.-lbs.

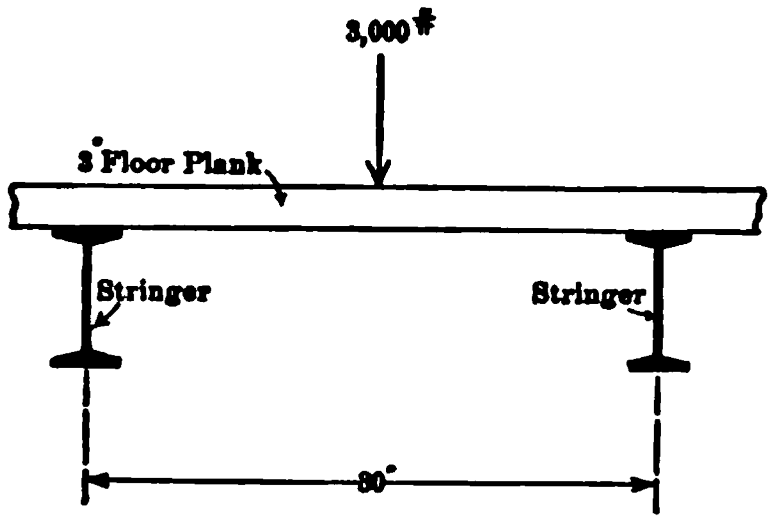


FIG. 2.

The moment due to the weight of the floor has been neglected, as it is very small compared to the live-load moment.

Assuming the above moment to be carried by a single plank, which will be taken as 3×12 in., the section modulus is $\frac{1}{6}\, b\, d^2 = \frac{1}{6} \times 12 \times 9 = 18$, and the fibre stress is $28,125/18 = 1560$ lbs. per sq. in. The usual allowable fibre stress for oak is 1500 lbs. per sq. in. Since the floor plank is continuous over several stringers, the actual bending moment is probably less than given above. The 3×12-in. plank will therefore be adopted.

248. *Design of the Stringers.*—The stringers are to be designed for either the uniform load of 100 lbs. per sq. ft. of floor, or for the 6-ton truck, in addition to the dead weight of the floor and stringer. As the stringers are spaced $2\frac{1}{2}$ ft. centres, the uniform load per foot amounts to 250 lbs. per stringer and the resulting bending moment is $\frac{1}{8}\, w\, l^2 = \frac{1}{8} \times 250 \times 15^2 \times 12 = 84,375$ in.-lbs. For the truck, the maximum moment occurs when one wheel is placed at the centre of the stringer. This position of loads produces a moment of $1500 \times 7.5 \times 12 = 135,000$ in.-lbs.

It is usually assumed that concentrated loads on a bridge floor are distributed over more than one stringer due to the rigidity of the floor plank. A common rule is that for stringers spaced not to exceed 2 ft. apart, each stringer takes half the load. When the

stringers are spaced more than 2 ft. and less than 3 ft. apart, the stringer under the load is assumed to take $\frac{2}{3}$ of the load, and when stringers are spaced more than 3 ft. apart, one stringer is assumed to take the whole load.

Applying the above rule to the case under consideration the moment carried by a stringer is $\frac{2}{3} \times 135{,}000 = 90{,}000$ in.-lbs. As this moment is greater than for the uniform load, the truck loading governs the design of the stringers. Increasing the moment due to the truck by 25 per cent to provide for impact, the maximum live load and impact moment is 112,500 in.-lbs. The dead load consists of the weight of the floor and the stringer. At $4\frac{1}{2}$ lbs. per ft. board measure, the usual assumed weight of timber in bridge floors, the plank weighs $3 \times 4\frac{1}{2} \times 2\frac{1}{2} = 33.75$ lbs. per lin. ft. of stringer. Assuming a stringer section consisting of a 7-in., 15-lb. I-beam, the dead load is $33.75 + 15.0 = 48.75$ lbs. A nailing strip is usually bolted to the top of the I-beam stringer, and the floor planks are nailed to this strip. Using a 3×6-in. strip, whose weight is 6.75 lbs. per foot, the total dead load becomes 55.5 lbs. per ft. of stringer. The resulting bending moment is $\frac{1}{8} \times 55.5 \times 15^2 \times 12 = 18{,}750$ in.-lbs. Adding this moment to the live-load moment given above, the total moment is found to be $112{,}500 + 18{,}750 = 131{,}250$ in.-lbs.

With an allowable fibre stress in bending of 16,000 lbs. per sq. in., the assumed stringer section must provide a section modulus of $131{,}250/16{,}000 = 8.2$ ins.^{3} A 7-in., 15-lb. I-beam has a section modulus of 10.4 ins.,^{3} and is therefore ample. In order to avoid excessive deflection of floor stringers, it is generally specified that the depth of stringer shall not be less than $\frac{1}{30}$ of the span, or 6 ins. in this case. The assumed I-beam answers the depth requirements, and as the web is $\frac{1}{4}$ in. thick, the requirement regarding minimum thickness of material is also answered. The outside stringers are sometimes made of channels or lighter I-beams, as the load to be carried is less than for the other beams. In this design the same size beams will be used for all stringers.

Standard I-beam end connections, as given in the rolling mill handbooks, will be used to connect the stringer to the floor-beam. This connection for a 7-in. I-beam consists of two $6 \times 4 \times \frac{7}{16}$-in. angles which are fastened to the web of the beam by 4 shop rivets.

The connection to the floor-beam is made by means of 4 field rivets in single shear. It will be found that the maximum end shear for the stringer occurs when one wheel of the truck is at the end of the beam. Assuming the same distribution of live load as before, and adding impact and dead load, the total end shear is $\frac{2}{3} \times 4{,}000 \times 1.25 + 7.5 \times 55.5 = 3{,}750$ lbs. The standard connection, therefore, provides excess strength for rivet values given in Art. 254.

249. *Design of the Floor-Beams.*—As shown on the General Drawing, Plate VI, the stringer loads are brought to the floor-beam by means of 7 stringers spaced $2\frac{1}{2}$ ft. centres. In calculating dead-load and uniform live-load moments and shears it can be assumed that the loads are uniformly distributed over the floor-beam. Loads brought to the floor-beam, due to the truck loading, will be considered as concentrated loads.

Fig. 3 shows the truck in position for maximum moment in the floor-beam. As shown in Fig. 3 (*a*), one axle is placed over the floor-beam and the other on an adjacent stringer. The reaction at the middle floor-beam is then 8,000 lbs. Fig. 3 (*b*) shows the wheels placed in position for absolute maximum moment (Art. 125, Part I). For this position of loads, the maximum moment occurs under the wheel to the left of the beam centre, and the moment is $8{,}000 \times 7.75^2 \times 12/18 = 320{,}000$ in.-lbs.

The uniform load of 100 lbs. per sq. ft. brings a load of $16 \times 100 \times 15 = 24{,}000$ lbs. to each floor-beam. Fig. 3(*c*) shows the loading conditions when this load is considered as uniformly distributed over the 16 ft. roadway. The bending moment at the beam centre is $(12{,}000 \times 9 - 12{,}000 \times 4)12 = 720{,}000$ in.-lbs. As this moment is greater than that found above for the truck, the floor-beam design is governed by uniform load conditions. Increasing this moment by 25 per cent for impact, the live-load and impact moment is 900,000 in.-lbs.

The dead load consists of the weight of the floor and stringers, and the floor-beam. Expressed in lbs. per ft. of bridge, the floor weight is made up of the following items: Stringers, 7 at 15 lbs. = 105 lbs.; nailing strips, 7 at 6.75 = 47.3 lbs.; floor plank, $16 \times 3 \times 4\frac{1}{2} = 216$ lbs.; guard rails, two 4×6 in. pieces: 18 lbs. = a total of 386.3 lbs. As each floor-beam carries 15 ft. of floor, the dead load is $386.3 \times 15 = 5{,}795$ lbs. Considering this load as distributed as

shown in Fig. 3(c), the bending moment is $\frac{1}{2} \times 5{,}795 \times 5 \times 12 = 173{,}850$ in.-lbs. Assuming the floor-beam section to be an 18-in., 55-lb. I-beam, the bending moment due to its weight is $\frac{1}{8} \times 55 \times 18^2 \times 12 = 26{,}730$ in.-lbs. The total dead, live, and impact moment is then $900{,}000 + 173{,}850 + 26{,}730 = 1{,}100{,}580$ in.-lbs., which re-

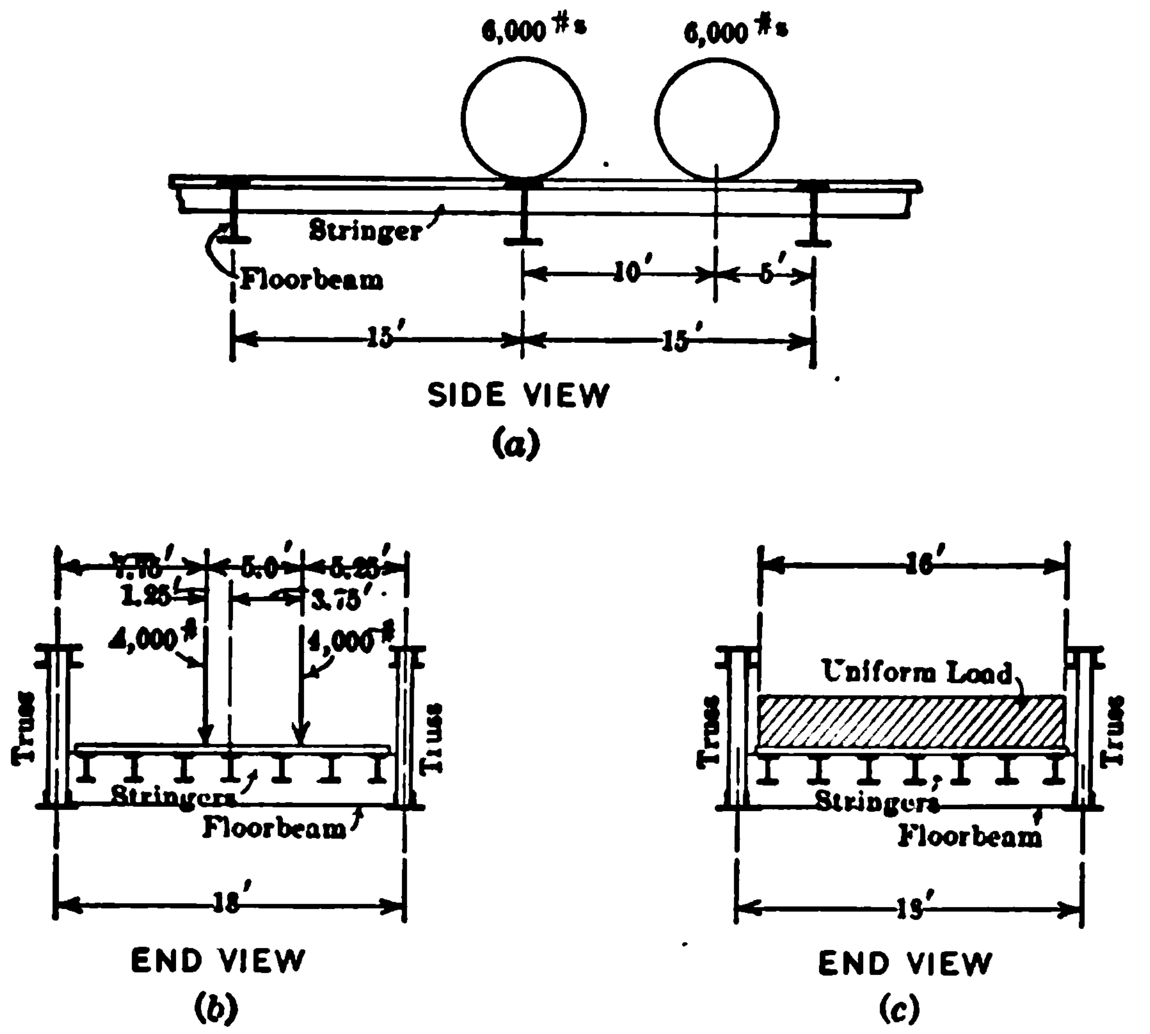


FIG. 3.

quires a beam with a section modulus of $1{,}100{,}580/16{,}000 = 68.9$ ins.[3] As the assumed beam has a section modulus of 88.4 ins.,[3] it will be used. It will be found that a 15-in., 55-lb. I-beam also furnishes the required section modulus. The 18-in. beam is selected because its greater depth provides a more rigid floor-beam, and also makes possible a more rigid connection between floor-beam and truss.

A standard I-beam connection, as given in the rolling mill handbooks, will be used to connect the floor-beam to the truss. This connection consists of two $4 \times 4 \times \frac{7}{16}$-in. angles, which are connected to the I-beam by 5 shop rivets in bearing on the web. The connection between the beam and truss is made by 10 field rivets in single shear. As the end reaction due to dead and live load plus impact is $\frac{1}{2} \times 5{,}795 + \frac{1}{2} \times 18 \times 55 + 1.25 \times 12{,}000 = 18{,}392$ lbs., the standard connection is adequate.

The end floor-beam carries a floor area which is one-half that carried by the intermediate beams. It will be found that the uniform live load governs the design of this beam, and that the live-load and impact moment is 450,000 in.-lbs., or just half that given above. In the same way the moment due to the floor load is half that for an intermediate beam, or 86,925 in.-lbs.

A 12-inch, 31.5-lb. I-beam will be assumed. The bending moment due to its own weight is 15,300 in.-lbs., giving a total moment of 552,225 in.-lbs. This requires a section modulus of 34.4 ins.^{3} As the assumed beam has a section modulus of 36.0 ins.,^{3} it will be adopted.

The details of the intermediate and end floor-beams are shown on the general drawing, Plate VI.

250. Determination of Stresses.—*Dead-Load Stresses.*—The dead load brought to each floor-beam due to the floor load is given in the preceding article as 5,795 lbs. Each truss takes half this load, or say, 2,900 lbs. Art. 66, Part I, gives the formula $w = 2l + 50$ for dead weight of truss and floor-beam, where w = weight per foot of bridge, and l = span. With $l = 75$, we find $w = 100$ lbs. per foot per truss, which gives a panel load of 1,500 lbs. The total panel load is $2,900 + 1,500 = 4,400$ lbs.

A preliminary design was made using this panel load. Upon making an estimate of the weight of the structure as designed, it was found that the estimated weight was too small. The true panel load was found to be 2,300 lbs. per truss. This gives a total panel load of $2,900 + 2,300 = 5,200$ lbs. Table A gives the dead-load stresses calculated for this panel load.

251. *Live-Load Stresses.*—It will be found that the live-load stresses in the truss members are governed by the uniform load of 100 lbs. per sq. ft. This load gives a panel load of $\frac{1}{2} \times 16 \times 100 \times 15 = 12,000$ lbs. per truss. Table A gives the resulting live-load stresses, calculated by the conventional method of loading given in Chap. IV, Part I. The impact stresses are in all cases taken as 25 per cent of the live-load stresses.

252. Maximum Stresses.—Table A gives the maximum stresses due to dead load, live load, and impact. The member notation is as shown in Fig. 1.

TABLE A

STRESSES IN MEMBERS

Member	Dead-Load Stress	Live-Load Stress	Impact Stress	Maximum Stress
$U_1 U_2$	−20,800	−48,000	−12,000	− 80,800
$U_2 U_3$	−31,200	−72,000	−18,000	−121,200
$L_0 L_1$	+10,400	+24,000	+ 6,000	+ 40,400
$L_1 L_2$	+26,000	+60,000	+15,000	+101,000
$L_2 L_3$	+31,200	+72,000	+18,000	+121,200
$L_0 U_1$	−14,700	−34,000	− 8,500	− 57,200
$U_1 L_1$	+14,700	+34,000	+ 8,500	+ 57,200
$L_1 U_2$	− 7,350	−20,400	− 5,100	− 32,850
$U_2 L_2$	+ 7,350	+20,400	+ 5,100	+ 32,850
$L_2 U_3$	± 0	±10,200	± 2,550	± 12,750

+ Denotes tension, − denotes compression.

253. Design of Members.—The allowable working stress in tension will be taken as 16,000 lbs. per sq. in. Since the rivets are ¾ in. in diameter, rivet holes will be considered as ⅞ in. in diameter in determining net section. Working stresses for compression members will be given by the formula $p = 16{,}000 - 70\,l/r$, with l/r limited to 125, and p not to exceed 14,000 lbs. per sq. in. As stated in Art. 246, the minimum thickness of material will be taken as ¼ in. Table B gives the necessary data for the design of members.

TABLE B

DESIGN OF MEMBERS

Member	Stress (Lbs.)	L'nth (Ins.)	r (Ins.)	l/r	Unit Stress (Lb. per Sq. In.)	Area Req'd (Sq. Ins.)	Section	Area Used (Sq. Ins.) Gross	Net
U_1U_2...	− 80,800	90	2.77	32.5	13,730	5.88	2[s 7″ at 9¾ lbs.	9.2	....
U_2U_3...	−121,200	90	2.77	32.5	13,730	8.83	1 *Pl.* 14 × ¼ in.	9.2	....
L_0L_1....	+ 40,400	...	....	....	16,000	2.53	2 ∠s 3½×3½×5/16 in	4.18	3.09
L_1L_2....	+101,000	...	....	....	16,000	6.32	2 ∠s 6×4×7/16 in.	8.38	6.85
L_2L_3....	+121,200	...	....	....	16,000	7.58	2 ∠s 6×4×½ in.	9.50	7.75
L_0U_1...	− 57,200	127	2.77	45.8	12,790	4.47	2[s 7″ at 9¾ lbs. 1 *Pl.* 14×¼ in.	9.2	....
U_1L_1...	+ 57,200	...	....	....	16,000	3.58	2 ∠s 5×3×5/16 in.	4.82	3.73
L_1U_2...	− 32,850	127	1.27	100	9,000	3.65	2 ∠s 4×3×5/16 in.	4.18	....
U_2L_2...	+ 32,850	...	....	....	16,000	2.06	2 ∠s 3½×2½×¼ in	2.88	2.44
L_2U_3...	± 12,750	127	1.12	113	16,000 / 8,050	1.19 / 2.38	2 ∠s 3½×2½×¼ in	2.88	2.44
Struts..	Nominal	...	....	....	.....	....	2 ∠s 4×3×5/16 in.	....	....

As shown in Table B and on the General Drawing, Plate VI, the top chord section is made uniform for its entire length. Its unsup-

ported length is taken as 7½ ft. in a horizontal plane. Since the top chord is supported laterally only by the web members, it is necessary that the chord and web members be made up of wide members. In many cases highway bridge failures in trusses of this type have been traced directly to a narrow, inadequately supported top chord.

No general rules or specifications are given regarding the width of top chord required for the necessary rigidity. From examples of trusses in general use which have been found to possess the required rigidity, it seems that the l/r for an axis parallel to the plane of the truss should be at least 200, where l is the total span length. In the truss under consideration, r for a vertical axis is 4.88 ins. and $l = 75$ ft., so that $l/r = 184$. The end post as made up provides considerable excess area, but the section is used for the sake of appearance and also because it provides a rigid connection between the top chord and the end shoe.

The design of member $U_3\ L_2$ requires special consideration. This member is subjected to a reversal of stress during the passage of the live load. As shown in Table A, the stress changes from 12,750 lbs. compression to 12,750 lbs. tension. The usual practice in such cases is to design the member for a stress equal to the greater stress plus one-half the lesser stress. In this case a stress of 19,125 lbs. compression or tension is used, and the areas given in Table B are made up for this stress.

The details of the tie plates used in this design are shown on the general drawing. These details conform to the standard practice for this type of structure. The general practice is to omit lacing on members of small trusses, and to use tie plates spaced 2 or 3 ft. apart in place of lacing. Tie plates consisting of $\frac{5}{16}$ in. material will be used on the top chord and ¼ in. material for all other members.

254. Design of the Joints.—The arrangement of members with respect to the centre line of the truss is shown on the general drawing. Members consisting of angles are placed with the gauge line on the centre line of truss. Where an angle has two gauge lines, the one nearer the backs of the angles is used. The top chord has been placed with its centre of gravity on the centre line of the truss, which

requires the centre of the channel to be placed 1⅜ ins. below the centre line of truss. A similar arrangement is adopted for the end post. Gusset plates ⅜ in. thick are used at all points. These plates are placed inside the top and bottom chord members. The web members are arranged to fit inside the gusset plates.

For members arranged as described above all rivets are in single shear, except where the material connected is less than $\frac{5}{16}$ in. thick. In such cases bearing governs the rivet strength. Working values for shop rivets will be taken as 12,000 lbs. per sq. in. in shear, and 24,000 lbs. per sq. in. in bearing. Corresponding values for field rivets are 10,000 and 20,000 lbs. per sq. in. respectively. Rivet values for these working stresses are given in the Tables of Appendix B.

Member $L_0\ U_1$ has a stress of 57,200 lbs. As the web of the channel is 0.25 in. thick, bearing governs, and the rivets have a value of 4,500 lbs. each. The total number required is 57,200/4,500 = 13 rivets, or 7 rivets on each side of the member. As shown in the general drawing, 8 rivets are used in each channel at joint L_0. A short piece of 3 × 3 in. angle is riveted inside the channels in order to form a connection between the gusset plates and the cover-plate. In order to square off the gusset plate at joint U_1, 8 rivets have been placed in each channel. All other members are designed in the same manner. Member $U_3\ L_2$ and the splices in the top and bottom chord require special consideration.

Member $U_3\ L_2$ is subjected to a reversal of stress, which changes from 12,750 lbs. compression to an equal amount in tension. The usual practice in such cases is to design the connection for a stress equal to the sum of the compression and tension, or 25,500 lbs. in this case. As the angles are ¼ in. thick bearing governs and 25,500/4,500 = 6 rivets are required, or 3 rivets in each angle, as shown on the general drawing, Plate VI.

A field splice is made at joint U_2 of the top chord, as shown on Plate VI. If the ends of the channels are milled to form an even bearing, only enough rivets are required to hold the members firmly together. Two rows of rivets each side of the splice are sufficient. In this case extra rivets are provided in order to fill out the gusset plate.

The lower chord is spliced at two points. At joint L_1 the splice is made on the gusset plate. Part of the load is carried across the

splice by means of the lateral plate, and the balance is carried into the gusset plate by the vertical legs of the angles. The number of rivets which can be placed in the lateral plate is limited by the carrying capacity of the horizontal legs of the smaller angles to be connected. From Table B, Art 253, the 3½-in. angles of member $L_0\ L_1$ have a total net area of 3.09 sq. ins., or 0.77 sq. ins. for each leg. At full working strength of 16,000 lbs. per sq. in. the horizontal leg of an angle can then carry $0.77 \times 16{,}000 = 12{,}300$ lbs. As the rivets in the lateral plate are in single shear, their value is 5,300 lbs. per rivet, and 3 rivets will transfer to the angle a load slightly in excess of the capacity of the angle leg. Using 6 rivets in the splice plate, there is left a stress of $101{,}000 - 6 \times 5{,}300 = 69{,}200$ lbs. to be transferred from member $L_1\ L_2$ to the gusset plate, which requires $69{,}200/5{,}300 = 13$ rivets, or 7 in each angle, the number which has been provided. In the same way, the stress to be transferred from member $L_0\ L_1$ to the gusset plate is $40{,}400 - 6 \times 5{,}300 = 8{,}600$ lbs., which requires $8{,}600/5{,}300 = 2$ rivets. The general drawing shows 3 rivets in place in each angle.

At joint L_2, the splice has been made a short distance to the left of the joint. If a splice is used similar to that at joint L_1, the heavy stress in $L_2\ L_3$, and the fact that a field splice is to be provided for member $L_1\ L_2$ would require a very large gusset plate at this joint.

The splice shown on the general drawing, Plate VI, consists of a tie-plate connecting the horizontal legs of the angles, and two side plates connecting the vertical legs of each angle. Using 8 field rivets in the tie-plate on each side of the splice leaves $101{,}000 - 8 \times 4420 = 65{,}640$ lbs. to be carried by the vertical legs of the angles. As the rivets through the vertical legs of the angles on the left-hand side of the splice are field rivets in bearing on a $\frac{7}{16}$-in. angle, they have a value of 6,560 lbs. per rivet. The number required is $65{,}640/6{,}560 = 10$ rivets, or 5 in each angle. An equal number will be used on the right-hand side of the splice. On the vertical legs of the angles side plates will be used which have a net area equal to that of these legs. From Table B, Art. 253, the net area of the two $6 \times 4 \times \frac{7}{16}$-in. angles of $L_1 \cdot L_2$ is 6.85 sq. ins. Of this area, $\frac{6}{10}$, or 2.06 sq. ins. per angle, is provided by the vertical legs. Two $5 \times \frac{3}{8}$-in. side plates will be used on each angle. This arrangement brings

to the vertical and horizontal legs of the chords angles loads which will not exceed the net carrying capacity of the angles legs.

The number of rivets required at joint L_2 between the lower chord angles and the gusset plate must be sufficient to transfer to the gusset plate the maximum difference in stress between members $L_1 L_2$ and $L_2 L_3$. This will be found to occur when the diagonal $U_2 L_2$ has its maximum stress. It was found that this difference in stress is 26,200 lbs. As the rivets are in single shear, only 5 are required to carry this stress, but in order to make a firm connection a greater number has been used.

At joints L_1 and L_2 the floor-beam is directly connected to the inside gusset plate. In order to transfer part of the load to the outside gusset plate, a diaphragm consisting of four $4 \times 3 \times 5/16$ in. angles and a $5/16$-in. plate will be placed between the gusset plates. This diaphragm extends over the portion of the gusset plates covered by the floor-beam connection. Two of the angles forming the diaphragm are extended upward and riveted to the top chord, thus providing lateral support for the chord. The sub-strut thus formed is stiffened by tie plates, as shown in the general drawing.

A similar diaphragm is used at the end floor-beam connection. At this point one of the angles is extended upward to be used as a support for the hand-rail.

The end shoe is formed by riveting a base plate to an extension of the gusset plates at joint L_0. From the panel loads given in Art. 250, the maximum end reaction for dead load, live load, and impact is $5/2\ (5{,}200 + 1.25 \times 12{,}000) = 50{,}500$ lbs. per truss. With an allowable bearing on the masonry of 300 lbs. per sq. in., a base area of $50{,}500/300 = 170$ sq. ins. is required. As shown on the general drawing, a $12 \times 19 \times 3/4$ in. base plate is used which provides an area of 228 sq. ins. The base plate is connected to the gusset plates by four $4 \times 4 \times 1/2$ in. angles. As the rivets are in bearing on the $3/8$-in gusset plates, the number required is $50{,}500/6{,}750 = 8$ rivets, or 4 in each pair of angles. Anchor bolts $3/4$ in. in diameter placed in $7/8 \times 1\,5/8$ in. slotted holes provide the required anchorage, and allow for a 100-degree temperature change.

255. Design of the Lateral System.—The lateral system of pony highway bridges is usually designed to withstand a horizontal force

due to a moving uniform load of 300 lbs. per lin. ft. applied at the lower chord joints. For this load, the panel load for the truss in question is $300 \times 15 = 4{,}500$ lbs. per panel, which causes a maximum shear of 9,000 lbs. in the end panel. As the shear is small and the members are long, it will be assumed that the lateral members take tension only. The stress on a diagonal of the end panel is then $9{,}000 \times 1.3 = 11{,}700$ lbs., which requires a net area of $11{,}700/16{,}000 = 0.73$ sq. ins. A single $3 \times 3 \times \frac{1}{4}$ in. angle with one rivet hole deducted has a net area of 1.22 sq. ins. The rivets connecting this angle to the lateral plate are field rivets in bearing on the $\frac{1}{4}$-in. angle, and $11{,}700/3{,}750 = 4$ rivets are required.

All of the other lateral members have stresses which are less than those in the end panel. Since smaller angles will be deficient in rigidity, all members will be made the same as for the end panel. Lateral plates $\frac{5}{16}$ in. thick will be used throughout.

As the bottom chords of the main truss also act as chords for the lateral truss, it will be necessary to determine the lateral chord stresses. If these stresses are less than 25 per cent of those due to vertical loading, they are neglected. Assuming tension diagonals for the lateral system, the chord stresses for 4,500 lb. panel loads are found to be 7,500 lbs. for $b\,c$ and 11,250 lbs. for $c\,d$ of the leeward truss. As these stresses are 7.4 and 9.3 per cent respectively of those due to vertical loading, they can be neglected.

PLATE VI

2L
1P

Tie Pls

iaphragm
– 4Ls 4"×3"×5/16"
1 Pl. 8"×5/16"

L DRAWING
HIGHWAY BRIDGE

CHAPTER XI

DESIGN OF STEEL ROOF TRUSSES

256. General Data.—The principles of steel roof design will be illustrated by the complete design of a Fink truss of the general dimensions shown in Fig. 1. A span 60 ft. centre to centre of bearings will be taken, and the rise will be made 15 ft., that is, the roof has a *pitch* of *one-quarter*. At the ends the truss will be assumed to be supported on walls of brick or masonry. The distance between adjacent trusses will be taken as 15 ft. From practice it has been found that the economical spacing of trusses is about 1/3 of the span for trusses 30 to 40 ft. long, and about 1/5 of the span for 100-ft. trusses. The adopted spacing of 1/4 of the span length is, therefore, probably about right for the span in question.

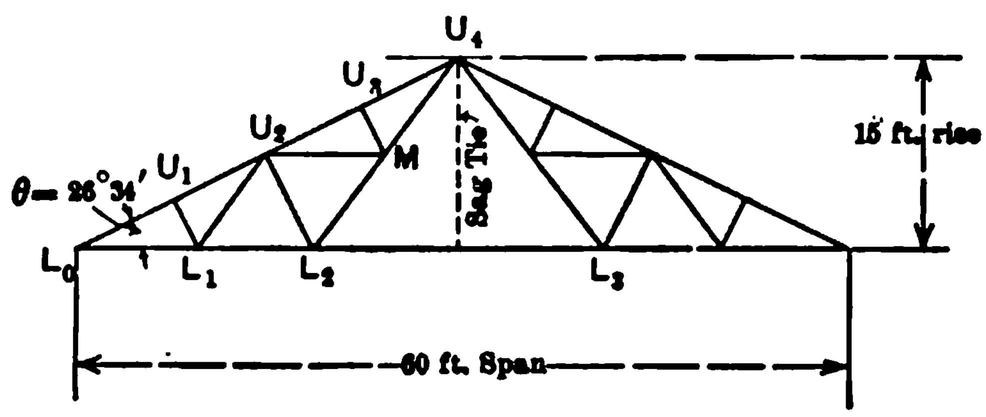


FIG. 1.

The roof covering will be taken as corrugated steel, supported directly on purlins. The distances between purlins allowed by good practice for different thicknesses of corrugated steel, and the weight of the same, are given in Table A.

TABLE A

Corrugated Steel	Allowable Span	Weight per Sq. Ft.
26 gage	2 feet—6 inches	1.0 pounds
24 "	3 " —0 "	1.3 "
22 "	4 " —0 "	1.6 "
20 "	4 " —6 "	1.9 "

Buildings used for shop or storage purposes, where drippings from condensation would cause damage to the contents of the building, are usually provided with a roof lining placed under the corrugated steel. This lining is made up in various ways. An effective lining consists of two layers of asbestos paper $\frac{1}{16}$-in. thick, and two layers of tar paper; all held in place by wire netting stretched over the purlins. Such a lining will weigh about 30 lbs. per square for each layer of paper, and 10 lbs. per square for the netting, or a total of 1.3 lb. per sq. ft. of roof.

The dead load, snow, and wind loads will be taken from the tables given in Chap. III, Part I. On pages 80 and 81 of this Part are given a number of formulas for weights of roof trusses. For trusses of the type and span under consideration, the formula of eq. (1) seems to fit the conditions better than the others. The dead weight of the truss will, therefore, be calculated from the formula

$$w = \frac{l}{25} + \frac{l^2}{6{,}000}$$

where w = weight per sq. ft. of horizontal covered area, and l = span of truss in feet.

Snow loads for different sections of the country are given in Table IV on page 85, Part I. Assuming the truss under consideration to be located in the Northwest States, the snow load for a metal roof of $\frac{1}{4}$ pitch is found to be 25 lbs. per sq. ft. of roof surface.

Wind loads are given in Table III, page 84, Part I. For $P = 30$ lbs. per sq. ft., the normal wind pressure on a roof of $\frac{1}{4}$ pitch is given as 22.4 lbs. per sq. ft. of roof surface.

In addition to the above loads, it is usually specified that no roof shall be designed for a load which is less than a certain minimum load per sq. ft. of horizontal covered area. The object of this specification is to make certain that a reasonably rigid structure will be obtained. For roof trusses of the size under consideration, this minimum load is taken as 30 lbs. per sq. ft. of horizontal area. This load will be used in the following design.

Unit stresses and details of construction will be used which conform to standard practice. These points will be discussed as they come up in the course of the design.

257. Design of Purlins.—It is usual in roof truss design to place the purlins at panel points of the top chord. Where corrugated steel is used as roofing, the allowable unsupported span of the roofing may determine the purlin spacing. If the top chord panel length of the assumed form of truss is greater than the allowable span of the roof covering, it may be necessary to place purlins at points between joints. The top chord must then act as a beam as well as a compression member. If this arrangement is considered undesirable, the form of truss shown in Fig. 1 can be altered so as to provide additional panel points. Two possible alterations are shown in Fig. 2. In the design under consideration, the form of truss shown in Fig. 1

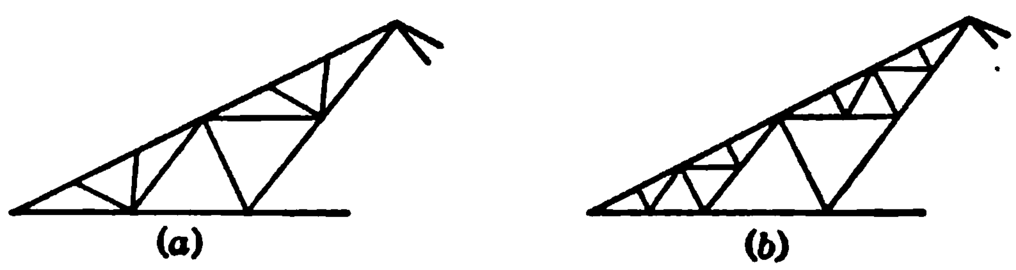


FIG. 2.

will be retained, and purlins spaced between panel points, if found necessary, in order to illustrate the principles of design for such cases.

From the dimensions given in Fig. 1, the length of a top chord panel is found to be $\frac{1}{4}(15^2 + 30^2)^{\frac{1}{2}} = 8.4$ ft. An examination of the allowable spans for corrugated steel given in Table A, Art. 256, shows that two arrangements of purlins can be made. By using two purlins between panel points, placed at the third points, with a spacing of $8.4/3 = 2.8$ ft. between purlins, 24-gage corrugated steel with an allowable span of 3 ft. can be used. Again, by placing a purlin at the centre of the top chord panel, or with a spacing of $8.4/2 = 4.2$ ft. 20-gage corrugated steel with an allowable span of 4.5 ft. can be used. The first arrangement will be called design A, and the second will be called design B. That arrangement will be adopted as final which will give the least total weight for corrugated steel and purlins.

The purlins must be designed for several combinations of dead, snow, and wind load. These combinations of loading are as follows:

(1) Dead load and snow load.

(2) Dead load and wind load.

(3) Dead load, wind load, and one-half snow load.

The dead load to be considered is that of the roofing, roof lining,

and the weight of the purlins. The purlin weight is not known when the design is begun, but must be assumed and later corrected when the purlin section is finally decided upon. Under Case III, only half the snow load is considered. This is due to the fact that maximum snow and maximum wind load can hardly occur at the same time. If a high wind is blowing at the time snow is falling, most of the snow will be blown off the roof as fast as it falls. In the case of a wet snow or sleet storm, part of the snow or sleet is likely to stay on the roof in spite of the wind. Therefore an allowance of one-half the maximum snow load seems to be reasonable for this condition.

Proceeding with the design of the purlins for design A, the weight of 24-gage corrugated steel is given in Table A, as 1.3 lb. per sq. ft. of roof. On page 398 the weight of the roof lining is given as 1.3 lb. per sq. ft. of roof. The purlin weight for the adopted section was found to be 4.4 lbs. per sq. ft. of roof, and this value appears in the following calculations. In the preliminary calculations a weight of 4 lbs. per sq. ft. was assumed, which was afterward corrected as stated above. The total load for design A is then $1.3 + 1.3 + 4.4 = 7.0$ lbs. per sq. ft. of roof.

The roof area carried by a purlin is found to be $2.8 \times 15 = 42$ sq. ft. In calculating bending moments the purlins will be assumed as simple beams between the trusses, which are 15 feet apart. Then the moments are given by the formula $M = W\,L/8$, where W is the total load to be carried and L is the length of the purlin. The moments to be carried for the cases outlined above are then as follows:

Case 1. *Dead Load and Snow Load.*—As given above, the dead load for roofing is 7.0 lbs. per sq. ft. of roof. From the data given in Art. 256, the snow load is a vertical load of 25 lbs. per sq. ft. of roof. The total load is then 32.0 lbs. per sq. ft. of roof. Each purlin carries a roof area of 42 sq. ft. The total load carried by a purlin is then $32.0 \times 42 = 1{,}344$ lbs., and the bending moment is $M = \frac{1}{8} \times 1{,}344 \times 15 \times 12 = 30{,}240$ in.-lbs.

Case 2. *Dead Load and Wind Load.*—The dead load for roofing is again 7.0 lbs. per sq. ft., a vertical load. As given in Art. 256, the wind load is 22.4 lbs. per sq. ft., acting normal to the roof. The resultant of these loads, as calculated graphically in Fig. 3(*a*), is found

to be 28.8 lbs. acting in the direction shown. Each purlin then carries a load of $28.8 \times 42 = 1{,}210$ lbs., and the resulting bending moment is $M = \frac{1}{8} \times 1{,}210 \times 15 \times 12 = 27{,}250$ in.-lbs.

Case 3. Dead Load, Wind Load, and One-half Snow Load.—The dead weight of roofing is 7.0 lbs. and one-half the snow load is 12.5 lbs., giving a total vertical load of 19.5 lbs. per sq. ft. of roof. As in Case 2, the wind load is a normal load of 22.4 lbs. per sq. ft. of roof. Fig. 3(*b*) shows the resultant pressure in amount and direction. The total load carried by the purlin is then $40.8 \times 42 = 1{,}715$ lbs., and

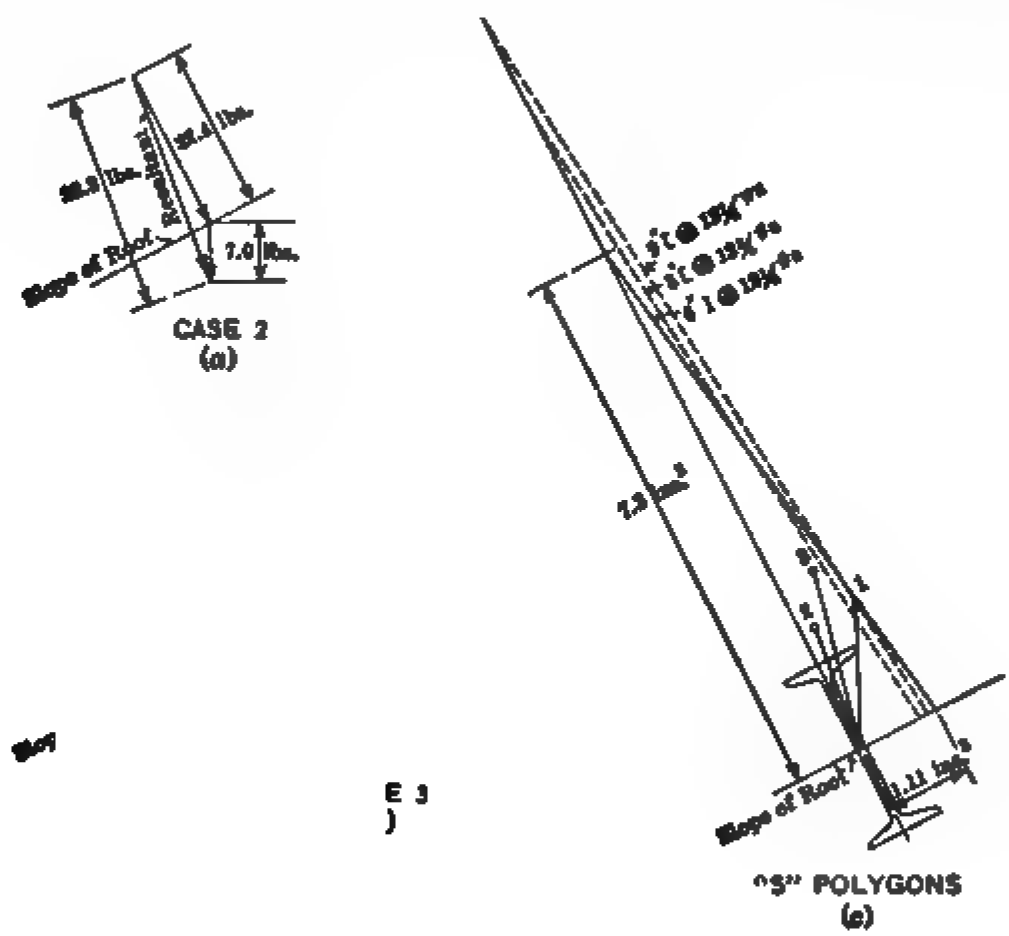


Fig. 3.

the bending moment is $M = \frac{1}{8} \times 1{,}715 \times 15 \times 12 = 38{,}600$ in.-lbs.

In general, purlins for trusses of the size considered in this design are made of single rolled channels or I-beams. The web of the purlin section is usually placed perpendicular to the top chord of the truss, as shown in Fig. 7. Therefore the plane of bending moments does not in any case correspond with a principal axis of the purlin section, as shown in Fig. 3(*c*). In order to keep the bending stresses on extreme fibres of the section within the limits allowed by good

practice, which is usually 16,000 lbs. per sq. in., the methods of design set forth in Appendix C must be used. This requires the calculation of the section modulus required for the several planes of bending. From the discussion given in Art. 2, Appendix C,

$$S = I/c = M/f = M/16{,}000.$$

The several values of S will be given a subscript similar to the corresponding case number. The required values of S are as follows:

Case 1. $S_1 = 30{,}240/16{,}000 = 1.89$ ins.^{3}; direction of bending vertical.

Case 2. $S_2 = 27{,}250/16{,}000 = 1.71$ ins.^{3}; direction of bending shown by resultant in Fig. 3(*a*).

Case 3. $S_3 = 38{,}600/16{,}000 = 2.41$ ins.^{3}; direction of bending shown by resultant in Fig. 3(*b*).

Applying the graphical method of design given in Art. 7, Appendix C, we have the construction shown in Fig. 3(*c*). This construction shows that the 6-in. 12¼-lb. I-beam is almost an exact fit and will therefore be adopted for design A. The S lines for several other sections are also shown in Fig. 3(*c*).

The adopted section is the smallest that can be used according to good practice, for the material of the web is 0.23 in. thick, which is slightly less than the minimum of ¼ in. allowed. Also, most specifications limit the depths of the purlins to $1/30$ of the span, or in this case $15/30 = 0.5$ ft. or 6 inches, which is the depth used. The true weight of the purlins per sq. ft. of roof surface must now be determined in order to check up the value used in the above calculations. As the adopted purlin section weighs 12.25 lbs. per ft., and as the purlins are spaced 2.8 ft. apart, the weight per sq. ft. of roof is $12.25/2.8 = 4.4$ lbs. Since this checks the assumed weight, no revision of the calculations is necessary.

In order to make a comparative estimate for design A and design B, the total weight of roofing and purlins per panel will now be calculated. For design A, each panel must include the weight of three purlins. As the adopted purlin weighs 12¼ lbs. per ft., the purlin weight is $3 \times 15 \times 12.25 = 552$ lbs. The 24-gage corrugated steel weighs 1.3 lb. per sq. ft., and the roof lining also weighs 1.3 lb. per sq. ft., a total for roofing of 2.6 lb. per sq. ft. As the area of a roof

panel is $8.4 \times 15 = 126$ ft., the roofing weighs $126 \times 2.6 = 328$ lbs. The total weight per panel for design A is then $552 + 328 = 880$ lbs.

Proceeding in a like manner for design B, in which 20-gage corrugated steel, with purlins spaced 4.2 feet apart, is used, it will be found that an 8-in. 18-lb. I-beam is required. The total weight of roofing and purlins is 943 lbs. This is the weight of two purlins at 18 lbs. per ft., or $2 \times 15 \times 18 = 540$ lbs., and a roof covering of corrugated steel at 1.9 lb. per sq. ft. and roof lining at 1.3 lb. per sq. ft. or $3.2 \times 126 = 403$ lbs.

Since design A has the least total weight, it will be adopted as final.

It will generally be found that I-beams will give the purlin section of least weight for roofs with a top chord making an angle less than about 30° to the horizontal. For roofs with a top chord at a greater slope than 30°, channel sections will generally be found best.

In some cases, wooden sheathing with a shingle or tile covering is used in place of the corrugated steel adopted in the preceding design. If this sheathing is made of 2-in. material, laid close together, purlins need not be placed at points between panel points of a truss of the size shown in Fig. 1, Art. 256. When a roof covering of this type is used, it is usually assumed that the purlins take only the component of loads perpendicular to the roof. The component of loads parallel to the roof is assumed as taken by the sheathing acting as a beam.

As an example of the methods of calculation for such cases, let it be assumed that the roof covering for the truss of Fig. 1 consists of shingles laid on 2-in. sheathing, with purlins at top chord joints. This roof covering will weigh about 10 lbs. per sq. ft. of roof. From the force diagrams of Fig. 3, page 401, it can be seen that a combination of dead load, wind load, and one-half snow load gives the greatest component of forces normal to the roof. Assuming the same wind and snow loads as for Case 3, and with a dead load of 10 lbs. per sq. ft. of roof, it will be found by a diagram similar to Fig. 3(*b*) that the load normal to the roof is 42.5 lbs. per sq. ft. The roof area carried by each purlin is $8.4 \times 15 = 126$ sq. ft., and the normal roof load is $126 \times 42.5 = 5{,}360$ lbs. To this must be added the weight of the purlin section which will be assumed as a 7-in. 15-lb.

I-beam. The component of purlin weight normal to the roof is $15 \times 15 \cos\theta = 202$ lbs., giving a total normal load of 5,562 lbs. As this is a uniformly distributed load, the bending moment is $M = \frac{1}{8} \times 5{,}562 \times 15 \times 12 = 125{,}000$ in.-lbs. The required section modulus for the purlin section is $125{,}000/16{,}000 = 7.82$ in.^{3} As the assumed section has a section modulus of 10.4 in.^{3}, it will be adopted. An estimate of the weight of this roof per panel, using the assumed weight of sheathing and shingles given above, gives a total weight of 1,485 lbs. This is made up of $10 \times 126 = 1{,}260$ lbs. for sheathing and shingles and $15 \times 15 = 225$ lbs. for the purlin. The corrugated steel roof of design A weighs 880 lbs. per panel. Based on comparative weights, the corrugated steel roof covering is therefore preferable for the truss under consideration.

258. Determination of Stresses in Truss Members.—The stresses in the truss members are to be determined by the methods given in Chaps. II and III of Part I. Art. 56, Part I, outlines the general methods of procedure, and Art. 58 gives the complete analysis of a truss similar in form to Fig. 1, page 397.

The roof truss under consideration will be analyzed for the following combinations of loadings:

(1) Dead load and snow load.

(2) Dead load and wind load.

(3) Dead load, wind load, and one-half snow load.

(4) A uniform load of 30 lbs. per sq. ft. of horizontal covered area.

The first three cases of loading are similar to those already discussed in Art. 257 under purlin design. Case 4 is considered in order to meet the minimum load requirements mentioned in Art. 256.

259. *Dead-Load Stresses.*—The dead panel load is made up of the weight of the roofing, the purlins, and the estimated weight of the truss. From Art. 257 the weight of the adopted roof covering (corrugated steel and roof lining) is 2.6 lbs. per sq. ft. of roof. As the area of a roof panel is $8.4 \times 15 = 126$ sq. ft., the weight of roofing per panel is $126 \times 2.6 = 328$ lbs. Each panel carries the weight of three 6-in. 12.25-lb. I-beams, giving a purlin weight of $3 \times 12.25 \times 15 = 552$ lbs. per panel.

The weight of the truss is to be estimated by the formula of Art. 256, which is $w = l/25 + l^2/6{,}000$. With $l = 60$ ft. we have $w = 3.0$ lbs. per sq. ft. of horizontal covered area. Each panel has a horizontal covered area of $15 \times 8.4 \cos \theta = 112.5$ sq. ft. The weight of truss per panel is then $112.5 \times 3 = 337.5$ lbs. This gives a total dead panel load of $328 + 552 + 337.5 = 1217.5$ lbs. A load of 1,220

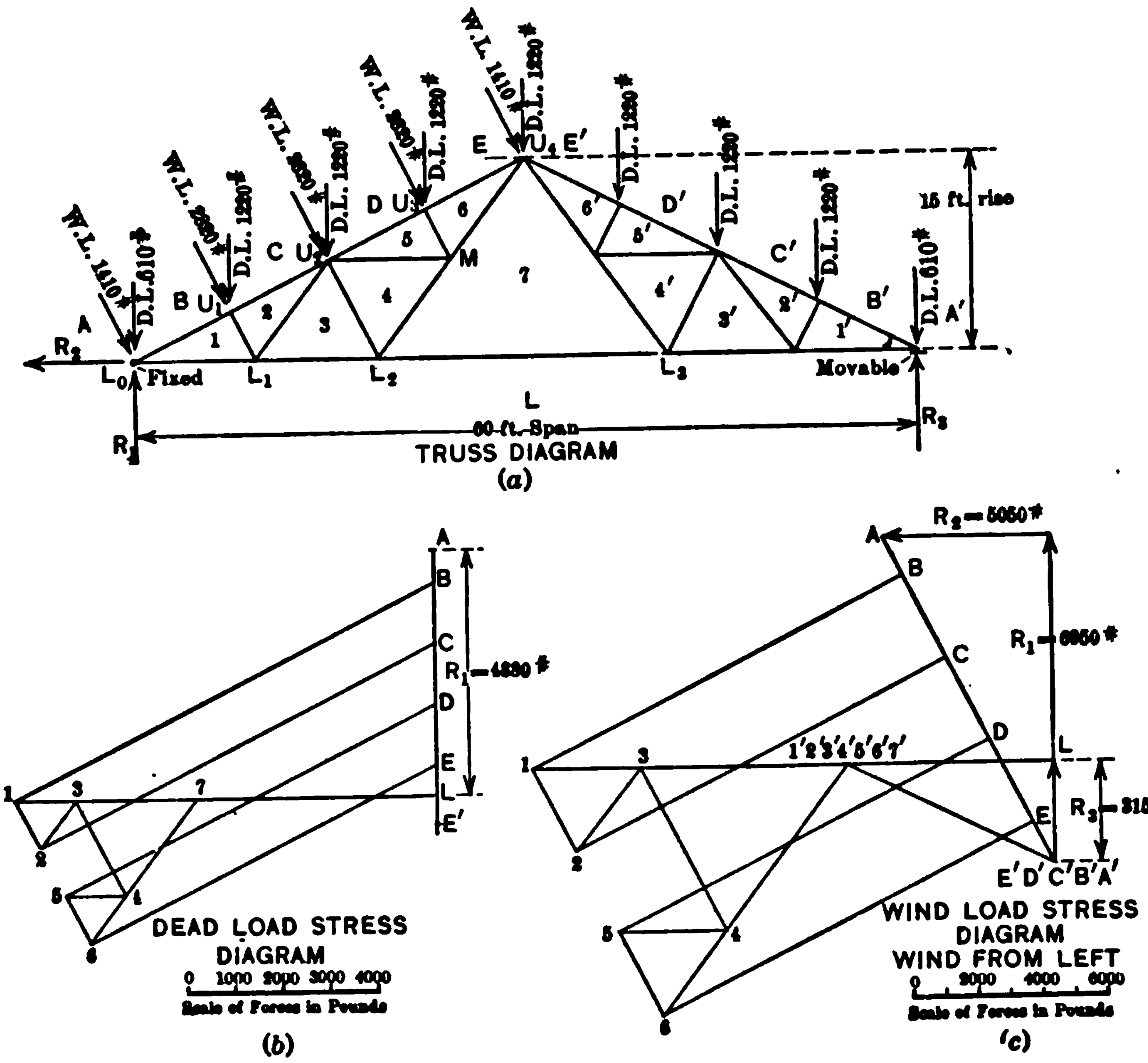


FIG. 4.

lbs. has been used in constructing the dead-load stress diagram, which is shown in Fig. 4(*b*). The resulting stresses are given in column 1 of Table B, Art. 263.

260. *Snow-Load Stresses.*—The snow load, per sq. ft. of roof, is given in Art. 256 as 25 lbs. As the area of a panel is 126 sq. ft., the snow panel load is $25 \times 126 = 3{,}150$ lbs. The snow-load stresses, as calculated by ratio from the dead-load stresses, are given in column 2 of Table B. Values for minimum snow load are given in column 3.

261. *Wind-Load Stresses.*—The normal wind pressure is given in Art. 256 as 22.4 lbs. per sq. ft. of roof. This gives a wind panel load of $22.4 \times 126 = 2{,}820$ lbs. Fig. 4(*c*) is the stress diagram drawn for the wind considered as blowing from the left. In drawing this diagram the reactions are determined on the assumption that the left end of the truss is fixed and that the right end is free to slide.

The end supports of trusses of the size considered in this design usually consist of two smooth flat plates. One plate forms the sole of the shoe and the other plate forms a masonry bed plate. Connection between the truss and wall is made by means of bolts which pass through slotted holes in the sole plates. The truss is then free to move on its bearings under temperature changes. If identical conditions could be maintained at each end of the truss, so that both ends could move at the same time, it would be possible to assume that the horizontal components of the wind reaction was equally divided between the two supports. It is probable, however, that due to movements of the truss under temperature changes, or due to unequal bedding of the anchor bolts, the end conditions will become such that all of the horizontal component of the wind reaction will be carried to one support. The end conditions will then become as assumed for Fig. 4(*c*). A similar assumption will also be made for wind blowing from the right. The same diagram can then be used for wind in each direction. Wind stresses for wind from the left are given in column 4 of Table B, and column 5 gives stresses for wind from the right.

262. *Stresses for Uniform Load of* 30 *Lbs. Per Sq. Ft.*—The minimum loading conditions are stated in Art. 256 to be 30 lbs. per sq. ft. of covered area. As each panel has a horizontal area of 112.5 sq. ft. the resulting panel load is 3,375 lbs. The resulting stresses, as calculated by ratio from the dead-load stresses, are given in column 6, Table B.

263. *Maximum Stresses.*—The maximum stresses are found by making up the combinations of stress called for in the four cases stated in Art. 258. The greatest of these stresses appear in column 7 of Table B. Under the column headed "Remarks" are given the numbers of the columns which were added to make the maximum.

264. Design of Members.—It is usual in roof trusses of this

type to make all main members of symmetrical sections, such as two angles placed back to back. The top chord, bottom chord, all tension diagonals, and the member $U_2 L_2$ (Fig. 4*a*) are usually

TABLE B

STRESSES IN MEMBERS

Member	1 Dead Load	2 Snow Load	3 One-Half Snow Load	4 Wind from Left	5 Wind from Right	6 Case 4	7 Maximum Stress	Remarks
L_0U_1	−9,550	−24,650	−12,325	−12,900	−7,200	−26,400	−34,775	1.3.4
U_1U_2	−9,000	−23,250	−11,625	−12,900	−7,200	−24,900	−33,525	1.3.4
U_2U_3	−8,450	−21,850	−10,925	−12,900	−7,200	−23,400	−32,275	1.3.4
U_3U_4	−7,900	−20,450	−10,225	−12,900	−7,200	−21,900	−31,025	1.3.4
L_0L_1	+8,540	+22,050	+11,025	+16,000	+6,400	+23,650	+35,565	1.3.4
L_1L_2	+7,320	+18,900	+ 9,450	+12,850	+6,400	+20,250	+29,620	1.3.4
L_2L_3	+4,880	+12,600	+ 6,300	+ 6,400	+6,400	+13,500	+17,580	1.3.4
U_1L_1	−1,090	− 2,820	− 1,410	− 2,820	0	− 3,020	− 5,320	1.3.4
U_2L_2	−2,180	− 5,640	− 2,820	− 5,640	0	− 6,040	−10,640	1.3.4
U_3M	−1,090	− 2,820	− 1,410	− 2,820	0	− 3,020	− 5,320	1.3.4
U_2L_1	+1,220	+ 3,150	+ 1,575	+ 3,200	0	+ 3,375	+ 5,995	1.3.4
U_2M	+1,220	+ 3,150	+ 1,575	+ 3,200	0	+ 3,375	+ 5,995	1.3.4
L_2M	+2,440	+ 6,300	+ 3,150	+ 6,400	0	+ 6,750	+11,990	1.3.4
MU_4	+3,660	+ 9,450	+ 4,725	+ 9,600	0	+10,125	+17,985	1.3.4

+ denotes tension; − denotes compression.

considered as main members. Members such as the sub-struts $U_1 L_1$ and $U_3 M$ and the sag tie at the centre of the truss are considered as secondary members. Their stresses are small, and the section is usually made of a single angle. All joints in such structures are usually riveted, although in some cases field splices are made with bolts.

The size of rivet usually adopted for small roof trusses is ⅝-in. in diameter. This determines the smallest angle which can be used. Structural rolling mill handbooks show that the smallest angle leg in which a ⅝-in. rivet can be driven is two inches wide. A 2 × 2-in. angle will therefore be adopted as the minimum angle. The minimum thickness of material for roof truss work is usually taken as ¼ in. In some cases, where the steel work is exposed to corrosive gases, the minimum is made 5/16 in.

The usual working stress in tension is 16,000 lbs. per sq. in. on the net section. In determining net section, rivet holes are taken ⅛ in. larger than the diameter of rivet, or ¾ in. in this case. The

working stress in compression on the gross area of the section is given by the formula $p = 16{,}000 - 70\, l/r$, where l is the length of the member and r its least radius of gyration. Compression members are limited by the condition that l/r must not exceed 125.

TABLE C

DESIGN OF MEMBERS

Member	Stress (Lbs.)	L'gth (Ins.)	r (Ins.)	$\frac{l}{r}$	Unit Stress (Lbs. per Sq. In.)	Area Reqd. (Sq. Ins.)	Section	Area Provided (Sq. Ins.) Gross	Area Provided (Sq. Ins.) Net
Top Chord	Special Design See Art. 268						2∠s 5x3x3/8 ins.	....	.....
L_0L_1....	+35,565	.....	....	.....	16,000	2.22	2∠s 2 1/2x2 1/2x5/16 ins.	2.94	2.48
L_1L_2....	+29,620	.....	....	.....	16,000	1.85	2∠s 2 1/2x2 1/2x5/16 ins.	2.94	2.48
L_2L_3....	+17,580	.....	....	.....	16,000	1.10	2∠s 2 1/2x2 1/2x 1/4 ins.	2.38	2.00
U_1L_1....	− 5,320	50.2	0.42	119.5	7,630	0.70	1∠ 2 1/2x2x 1/4 ins.	1.07	....
U_2L_2....	−10,640	100.4	0.95	105.8	8,600	1.24	2∠s 3x2 1/2x 1/4 ins.	2.64	....
U_3M....	− 5,320	50.2	0.42	119.5	7,630	0.70	1∠ 2 1/2x2x 1/4 ins.	1.07	....
U_2L_1....	+ 5,995	.....	....	.....	16,000	0.374	2∠s 2x2x 1/4 ins.	1.88	1.50
U_2M....	+ 5,995	.....	....	.....	16,000	0.374	2∠s 2x2x 1/4 ins.	1.88	1.50
L_2M....	+11,990	.....	....	.....	16,000	0.748	2∠s 2x2x 1/4 ins.	1.88	1.50
U_4M....	+17,985	.....	....	.....	16,000	1.12	2∠s 2x2x 1/4 ins.	1.88	1.50
Sag Tie .	Nominal	.....	....	.....	.....	.....	1∠ 2x2x 1/4 ins.	....	....

+ denotes tension; − denotes compression.

Table C gives in convenient form the data necessary for the design of members. The conditions governing the arrangement of a few of the more important members and the detail calculation involved in the design will now be given.

265. *Bottom Chord.*—The bottom chord member for panels $L_0\,L_1$ and $L_1\,L_2$ will be made in one piece. It will usually be found that it will cost more to place a splice at point L_1 than it will to use excess material in member $L_1\,L_2$, due to the increased size of gusset plates, the handling of smaller pieces, and the driving of extra rivets. At point L_2 a field splice is usually provided in order to break the truss up into small parts for shipment.

The maximum stress in the end section of the bottom chord is 35,565 lbs., which requires a net area of 35,565/16,000 = 2.22 sq. ins. Two 2½ × 2½ × 5/16-in. angles, with one rivet hole deducted from each angle, provide a net area of 2.48 sq. ins. The centre portion of the bottom chord will be made up of two 2½ × 2½ × ¼-in. angles. These angles provide some excess area, but as a rigid member is desirable at this point, the assumed angles will be used. As

member $L_2 L_3$ is quite long, a sag tie is provided in order to prevent the member from deflecting excessively under its own weight. A single angle of minimum size is sufficient. In trusses of ⅓ pitch, member $L_2 L_3$ is quite short, and a sag tie is not needed.

266. *Diagonal Tension Members.*—As shown in Table C, all diagonal tension members are made of two angles of minimum size. Considerable excess area is provided by this arrangement, but this cannot be avoided if symmetrical sections are used, as required by good practice. If single angles are used for these members, the stresses are brought to the gusset plate at the centre of gravity of the member, which is some distance from the plane of the centre of the plate. This tends to cause twisting and warping of the truss. Two angles, one placed on each side of the gusset plate, bring the stress to the plate at the plane of its centre, and thus avoid warping of the truss.

267. *Compression Diagonals.*—The compression diagonal $U_2 L_2$ is usually considered as a main member, and will therefore be made of two angles. This member has a maximum stress of 10,640 lbs., and a length of 100.4 ins. The section to be used for this member must have a radius of gyration not less than $r = l/125$, in order to answer the requirements for limiting length of compression members. For this case, minimum $r = 100.4/125 = 0.805$ ins. From the rolling mill handbooks, it will be found that the smallest angles which will answer this requirement are two $3 \times 2\frac{1}{2} \times \frac{1}{4}$-in. angles, placed with the 3-in. legs separated by a ¼-in. space to allow for the gusset plates at the ends of the members, as shown on the general drawing, Plate VII. These angles have a value of r of 0.95 in. for the gravity axis parallel to the 2½-in. legs, and a value of 1.09 in. for an axis parallel to the 3-in. legs. The allowable working stress is then $16{,}000 - 70 \times 100.4/0.95 = 8{,}600$ lbs. per sq. in., and the gross area required is $10{,}640/8{,}600 = 1.24$ sq. ins. The assumed angles provide an area of 2.64 sq. in. This is in excess of the area required, but the assumed angles must be used in order to meet the requirements for limiting length of compression members.

Members $U_1 L_1$ and $U_2 M$ have a stress of 5,320 lbs. compression and a length of 50.2 ins. As these members are sub-struts, a single angle section will be used. The angle used must have $r = 50.2/125 =$

0.402 ins. A $2\frac{1}{2} \times 2 \times \frac{1}{4}$-in. angle will be found to be the smallest usable angle.

268. *Top Chord.*—In trusses of the size under consideration, the top chord is usually made of the same section throughout. If the chord is spliced at each panel point, the added cost, due to increased size of gusset plates and driving of additional rivets at splices, will probably exceed the cost of the excess material in a chord of uniform section. For very long trusses it may, however, be of advantage to splice the top chord. Such splices would be located with reference to breaking the truss into small parts for shipment.

The top chord of this truss must carry bending moments due to the fact that purlins have been placed at points between panels. These moments must be taken care of in addition to the direct stresses in the members. Art. 70, Chap. IV, gives the method used in the design of such members. As the loads and moments to be carried are comparatively small, it can be shown that the effect of deflection of the member can be neglected. The formula to be used can then be written

$$A = \frac{P}{f_c} + \frac{Mc}{f_b r^2}$$

Where A = gross area of member; P = compression in chord member; M = bending moment to be carried; c = distance to extreme compression fibre; r = radius of gyration of section; f_b = allowable bending stress; and f_c = allowable compressive stress. The allowable compressive stress is to be determined by the formula $f_c = 16{,}000 - 70\, l/r$. If the point under consideration is at the end of the member, where column action does not enter, $f_c = 16{,}000$ lbs. per sq. in.

The bending moment is to be determined with respect to the assumed end conditions for the top chord sections considered as beams between panel points. As the loads are symmetrically placed, the top chord of uniform section, and the panels equal in length, it may be assumed that each top chord panel forms a beam fixed at the ends. Fig. 5 gives bending moment diagrams based on this assumption for several loading conditions. These moments are calculated by means of the formulas given on page 13, Part II.

The area of the chord section is to be determined for loading conditions which give the greatest compression in the member, together with the greatest simultaneous bending moments. From Table B and Fig. 3, page 401, it will be found that the combination of dead, wind, and one-half snow load is the required loading. Fig. 3(*b*) shows that the component of the purlin load perpendicular to the chord is 39.8 lbs. per sq. ft. of roof, and that parallel to the roof is 8.8 lbs. per sq. ft. Since two purlins are used in each panel, and since the roof area for each purlin is 42 sq. ft., the loads to be carried are $39.8 \times 42 = 1{,}672$ lbs. normal to the roof, and $8.8 \times 42 = 370$ lbs. parallel to the roof. Fig. 6(*a*) shows the loads in place. From the moment diagram of Fig. 5(*b*) the moment at the end of the beam is $M = -2/9 \times 1{,}672 \times 8.4 \times 12 = -37{,}440$ in.-lbs., and at the centre the moment is $M = 1/9 \times 1{,}672 \times 8.4 \times 12 = +18{,}720$ in.-lbs. In Table B, the maximum stress for dead, wind, and one-half snow load is 34,775 lbs. Fig. 6(*a*) shows that certain loads parallel to the member must also be taken care of. Assuming these loads to be carried to the lower end of the member, the total compression at L_0 is $34{,}775 + 2 \times 370 = 35{,}515$ lbs., and at the centre the total compression is $34{,}775 + 1 \times 370 = 35{,}145$ lbs.

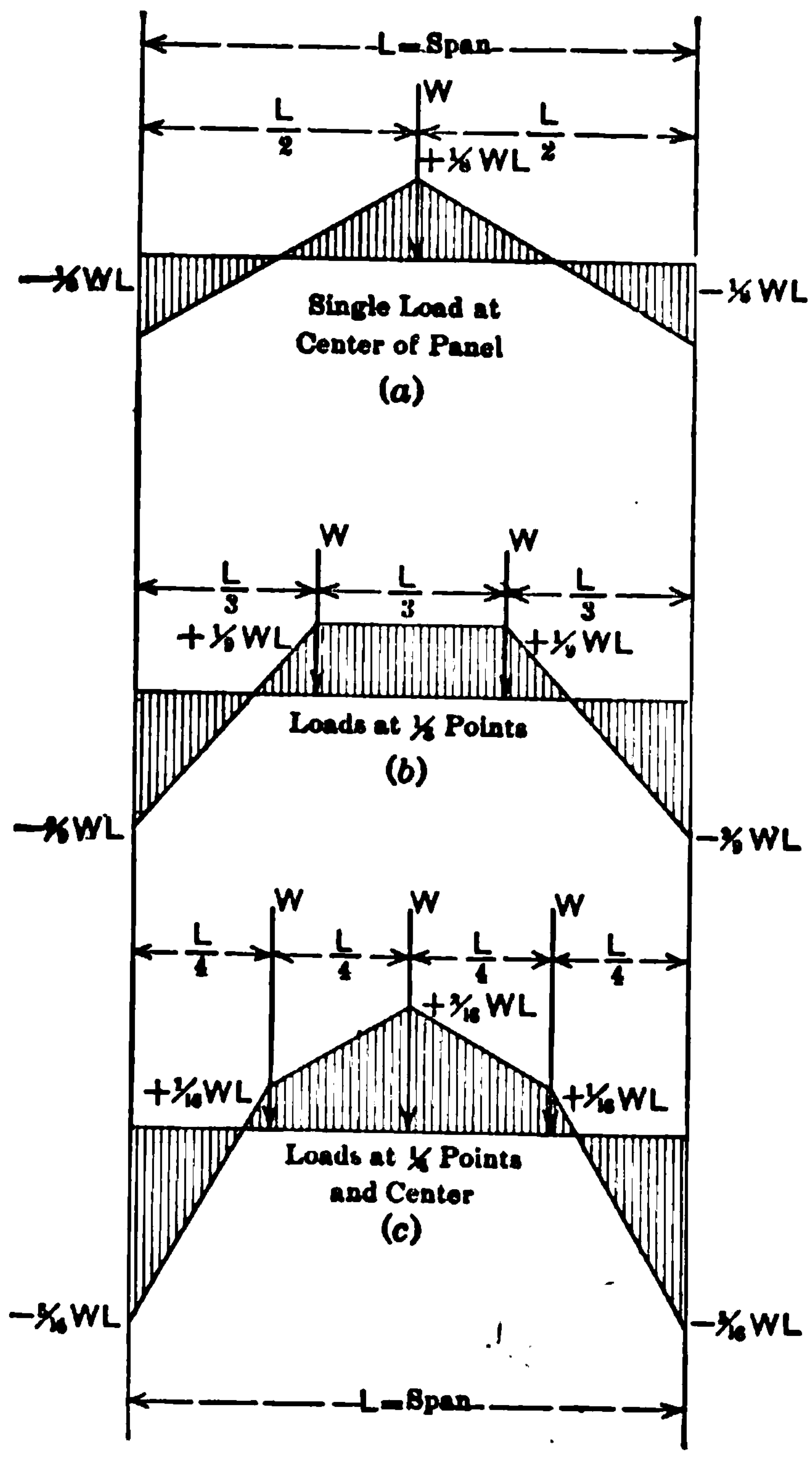


FIG. 5.

A chord section consisting of two $5 \times 3 \times \frac{3}{8}$-in. angles will be assumed, placed as shown in Fig. 6(*b*). The gross area of this section is 5.72 sq. in.; the neutral axis is located as shown in Fig. 6(*b*); and r for this axis is 1.61 ins. At the ends of the member, the moment is negative, and the bottom fibre is in compression, which gives

$c = 3.3$ ins. Positive moment occurs at the centre of the member, so that for this point $c = 1.7$ ins. With these values, the required section area as given by the above formula is:

$$\text{Area at end} = \frac{35{,}515}{16{,}000} + \frac{37{,}440 \times 3.30}{16{,}000 \times 1.61^2} = 2.22 + 2.98 = 5.20$$

sq. ins.

$$\text{Area at centre} = \frac{35{,}145}{11{,}620} + \frac{18{,}720 \times 1.7}{16{,}000 \times 1.61^2} = 3.02 + 0.77 = 3.79$$

sq. ins.

The assumed section is sufficient and will be adopted.

269. Design of Joints.—The main feature of joint design is the provision of sufficient rivets to insure adequate connection between

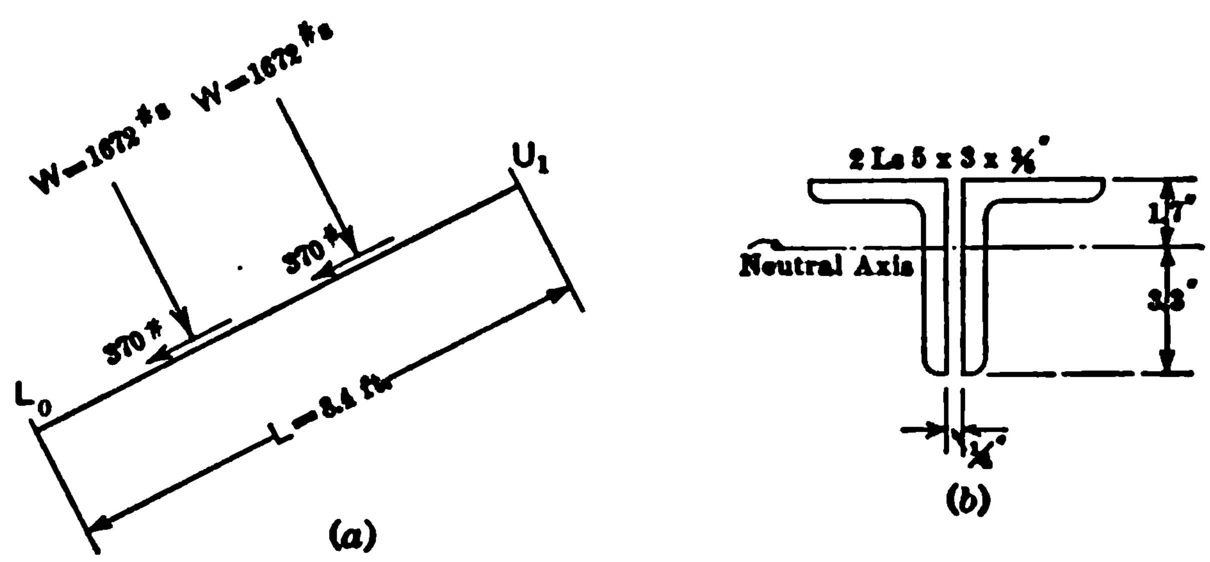


FIG. 6.

the several members meeting at a joint. Connection between the members is made by gusset plates which are placed between the angles of the top and bottom chord sections, as shown on the general drawing. The gusset plates are usually made of the minimum thickness of material, which is ¼ inch. At some of the more important joints, such as L_0, U_4, and L_2, where the stresses in members are large, such thin plates would result in large sizes because of the great number of rivets required. By using ⅜-in. plates at these joints, the number of rivets can be reduced, giving a more compact joint.

The size and shape of the gusset plates are determined by the number of rivets required in the several members. In general, not less than two rivets are used in the end of any member, as it is impossible to make a rigid connection with only one rivet. Chap. V

gives the general methods to be employed in the design of riveted connections.

The usual working stresses for shop-driven rivets are 12,000 and 24,000 lbs. per sq. in. respectively in shear and bearing. Corresponding values for field rivets are 10,000 and 20,000 lbs. respectively. Table D gives values for ⅝-in. rivets.

TABLE D

	Single Shear	Double Shear	Bearing Thickness of Plate				
			¼ In.	5/16 In.	⅜ In.	7/16 In.	½ In.
Shop Rivets.........	3,680	7,360	3,750	4,690	5,630	6,560	7,500
Field Rivets.........	3,070	6,140	3,130	3,910	4,690	5,470	6,250

Members $U_1\,L_1$ and $U_3\,M$ are single angles and the rivets are in single shear. The number required in the end of each member is 5320/3680 = 2 rivets, the number shown on the general drawing. Members $U_2\,L_1$ and $U_2\,M$ have rivets in bearing on a ¼-in. plate, and 5995/3750 = 2 rivets are required. At joint U_2 member $U_2\,L_2$ is in bearing on a ¼-in. plate, and at L_2 a ⅜-in. plate is provided. In order to maintain symmetrical conditions, the number required at U_2 is used at both ends of the member. Member $L_0\,L_2$ is continuous across joint L_1. Only enough rivets are needed in the gusset-plate between the angles and the plate to take up the difference in stress between members $L_0\,L_1$ and $L_1\,L_2$. Two rivets are sufficient; but in order to fasten the plate firmly to the angles, three will be used. Joint M is made exactly the same as L_1.

At joint U_2 only enough rivets are required between the gusset-plate and the top chord to take the total panel load to the gusset-plate. The number provided is in excess of those required, but a rigid joint can not be made with fewer rivets.

Joint L_0 must provide for the heavy bending stresses which exist in member $L_0\,U_1$ in addition to the direct stresses in the members. In order to keep down the size of the gusset-plate, it will be made ⅜-in. in thickness. The direct stress in $L_0\,U_1$ requires 34,775/5630 = 7 rivets. By increasing the number of rivets to 8, spaced as shown on the general drawing, it can be shown by the methods given in

Art. 91 that the combined effect of bending and direct stress will not exceed the allowable bearing on a rivet. In general, one or two extra rivets will be found sufficient to care for bending. If the bending moment in this member is not properly provided for, the end of member $L_0 U_1$ can not be assumed as fixed, as required by the design given in Art. 268.

The arrangement shown on the general drawing requires 35,565/5630 = 7 rivets between member $L_0 L_1$ and the gusset-plate. In some cases, the sole plate is placed on the horizontal legs of member $L_0 L_1$. The rivets must then be determined for the resultant of the stress in $L_0 L_1$ and the total end reaction.

In this design a shoe is provided which brings the end reaction directly to the gusset-plate. From the diagrams given in Fig. 4, it will be found that the resultant reaction for dead, wind, and one-half snow load is 18,800 lbs., which requires 18,800/5630 = 4 rivets between the shoe angles and the gusset-plate. The area of the bed-plate is determined from the vertical component of the reaction, which is 18,130 lbs. Using an allowable bearing of 300 lbs. per sq. in. on the masonry, the area required is 18,130/300 = 60.5 sq. ins. A 10½ × 10 × ⅝-in. plate is provided, connected to the gusset-plate by 2½ × 2½ × ⅜-in. angles. In determining the required thickness of the sole plate, it will be assumed that the portion of the plate projecting beyond the angles is a cantilever beam. If the full bearing value of the masonry, 300 lbs. per sq. in., is to be developed and the bending stresses in the plate are limited to 16,000 lbs. per sq. in., it can be shown that the thickness of the plate must be not less than ¼ of the overhang.* Since the overhang in this case is 2½ ins., the plate will be made ⅝ in. thick.

Anchor bolts ¾ in. in diameter will be provided. These bolts pass through slotted holes in order to provide for expansion due to temperature. It is usual to provide for a temperature change of 100 degrees F. With a coefficient of expansion of 0.0000065, a movement of $60 \times 12 \times 100 \times 0.0000065 = 0.468$ ins. must be provided for. The slotted holes shown on the general drawing provide ample room for expansion.

* Thickness of plate = 0.2375 overhang.

In general, the centre of the sole plate should be placed on a line through the intersection of the centre lines of members $L_0 U_1$ and $L_0 L_1$. Since in this case a large moment is brought to the joint by member $L_0 U_1$, the shoe will be placed off centre in order to set up a moment which will resist that due to $L_0 U_1$, thereby assuring fixed conditions at the joint, as assumed in the top chord design. The required offset is equal to the bending moment in $L_0 U_1$ divided by the vertical component of the reaction, or in this case 37,440/18,130 = 2.06 ins. The centre of the shoe will be placed 2 ins. to the right of the intersection of the gage lines, as shown on Plate VII.

A field splice is to be provided in the bottom chord at joint L_2. Field rivets will be placed in member $L_2 L_3$, and shop rivets in all other members. In order to keep down the size of the gusset plate it will be made 3/8-in. thick, and part of the bottom chord stress will be transferred across the joint by means of a 5½ × ¼-in. splice plate placed on the horizontal legs of the angles. In some of the trusses this splice plate will also be used as a lateral plate, as described in Art. 271.

As shown on the general drawing, four shop rivets in bearing on a 3/8-in. plate are provided in member $L_1 L_2$ and four field rivets in single shear are placed in the splice plate, giving a total strength of $4 \times 5630 + 4 \times 3070 = 34{,}800$ lbs. In member $L_2 L_3$ are provided two field rivets in bearing on the 3/8-in. gusset plate and four field rivets in single shear in the splice plate, giving a total strength of $2 \times 4690 + 4 \times 3070 = 21{,}660$ lbs. As the stress in L_1L_2 is 29,620 lbs., and that in $L_2 L_3$ is 17,580 lbs., the connection provided is adequate. The net area of the splice plate is determined by the amount of stress transferred across the joint by the plate. In this case the stress is due to four field rivets in single shear, or $4 \times 3070 = 12{,}280$ lbs. This requires a net area of $12{,}280/16{,}000 = 0.77$ sq. in. A 5½ × ¼-in. plate furnishes a net of 1.0 sq. in., after deducting two rivet holes.

It is to be noted that the spacing used for rivets in the vertical and horizontal legs of the angles of members $L_1 L_2$ and $L_2 L_3$ at joint L_2 is such that two rivet holes must be deducted from the area of the section in obtaining net area, as shown by the discussion in Art. 93. On this basis, the net area furnished by L_1L_2

is 2.02 sq. ins., and that for $L_2 L_3$ is 1.62 sq. ins. In each case the area furnished is greater than that required in Table C, Art. 264.

Another field splice is to be made at joint U_4. The required rivets will be determined for field rivets in bearing on a ⅜-in. gusset plate, and the same number will be used on the shop-riveted side of the joint. In member $U_4 M$, 17,985/4690 = 4 rivets are required; and in $U_4 U_3$, 31025/4690 = 7 rivets are required. Eight rivets are shown in place in member $U_4 U_3$. The extra rivet is placed in position in order to take care of the bending moment brought to the joint by the top chord, for reasons given in the design of joint L_0.

270. Minor Details.—Compression members made up of two angles must be riveted together at certain intervals in order to make the two angles act as a unit. To secure the full assumed strength of the column such connecting rivets must be so spaced as to make the value of l/r for each element not greater than about 40 or 50 (see Art. 57). The least radius of gyration of a 3 × 2½ × ¼-in. angle is 0.53 in. The spacing of connecting rivets should therefore not exceed about 50 × 0.53, or 26.5 ins.

The usual practice is to space such rivets about 2½ ft. apart. Tension members composed of two angles are also connected by similar rivets, spaced three or four feet apart. In order to keep the angles the same distance apart, ring fills, of the same thickness as the gusset plates, are placed between the angles.

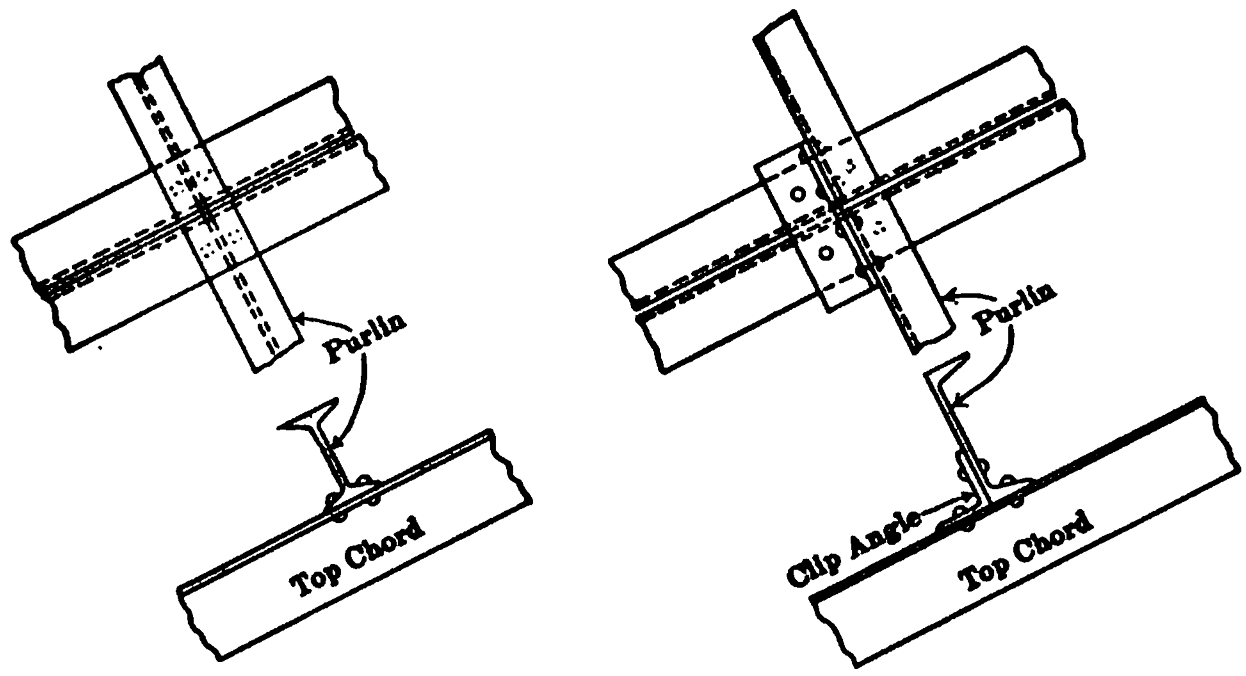
Purlins are usually riveted directly to the top chord. I-beams are fastened by two rivets in each beam, as shown in Fig. 7(*a*). The end purlin in the truss under consideration is fastened to an extension of the end gusset plate by means of a standard beam connection. This detail is used because the top chord does not provide sufficient support due to the arrangement of members.

Channel purlins are provided with a short piece of angle called a "clip angle." The purlin is riveted to the top chord and to the clip, as shown in Fig. 7(*b*).

271. Lateral Bracing.—Where the trusses rest on masonry walls, and where the end walls are also of masonry, it is not usually considered necessary to provide lateral bracing for roof trusses. The purlins and roofing provide the required lateral support for the top chord, and the anchor bolts in the walls securely fix the ends of the

truss. This arrangement is satisfactory in buildings used for storage purposes, where vibration is not present.

In shop buildings of this type, containing heavy machinery or line shafting attached directly to the trusses, the above arrangement is not rigid enough to take care of the vibrations set up by the machinery. The trusses must be given additional support, which is pro-



IBEAM CONNECTION
(a)

CHANNEL CONNECTION
(b)

FIG. 7.

vided by lateral bracing in the plane of the top and bottom chords. This bracing consists of single angles arranged as shown in Fig. 13, Art. 281. The top laterals are fastened to the horizontal legs of the top chord angles by means of lateral plates. Where the lateral angles cross they are fastened to the purlins in order to prevent vibration and excessive deflection. At the ends of the truss the lower laterals are fastened to gusset-plates riveted to the horizontal legs of the lower chord angles. The splice plates at joint L_2 are increased in size and made to serve as lateral plates, as shown on Plate VIII.

As shown in Fig. 13, this bracing is placed in every third or fourth panel. A strut is run the whole length of the building at lower chord joints L_2 and L_3. The section of this strut is determined for l/r conditions as in Art. 281. All details are similar to those shown in Plate VIII for the structures designed to rest on columns.

272. The General Drawing.—Plate VII is a shop or working drawing of the truss designed in the preceding articles. In general, two methods of making shop drawings are in use. The first method,

of which Plate VII is an example, gives complete information regarding sizes of members and plates, and the exact spacing of all rivets. A second method gives the sizes of members and plates, but does not give the spacing of rivets, the number and position of rivets being merely indicated. The method to be used depends upon the template shop practice in use where the truss is to be fabricated. In the first case, where complete information is given, the template maker can lay out the template for any plate or angle without reference to any other part of the truss. In the second case, the truss must be laid out to full size on the floor of the template shop, and the required clearances and rivet spacing determined with all members in place.

In making a shop drawing such as Plate VII, the centre lines of the truss members are first laid out to scale. The members are then drawn in about these centre lines. Two methods of placing the angles with respect to the centre lines of the truss are in use. In one method, which is used in Plate VII, the rivet gauge lines coincide with the centre lines of the truss. Where an angle has two rows of rivets, the gauge line nearest the back of the angle is used. The other method places the gravity axes of the angles on the center lines of the truss, while the rivets are placed on the gauge lines for the angle in question. In either case there will be moments set up at the joints due to the eccentric application of stresses at the joints. The stresses do not act along the centre lines of the members in the first case, and in the second case the connecting rivets are eccentric with respect to the stresses. As the shop drawings are somewhat simpler for the first case described above, it is the one in general use. Gauge line and gravity axis distances are given in all rolling mill handbooks.

The members meeting at a joint are separated by small spaces called "clearance." This clearance is provided to allow for overrun in width of angles, and for any small errors that may be made in cutting members to the required length. For trusses of the size given here, the allowance for clearance is usually 1/4 in. Edge distances for plates and ends of angles are taken as 1 1/4 in.

The distances between intersections of centre lines and adjacent rivets and the sizes of all plates are usually determined by a large

scale drawing of each joint called a "Layout." By making this layout at least one-quarter size of the truss details, all distances can be scaled, thus saving an immense amount of tedious trigonometric calculation. Plate VII follows the usual methods of dimensioning and line notation.

273. Estimate of Weight.—After the general drawing has been made, a detailed estimate of the weight of the truss should be made. The dead weight assumed in the design should then be checked up and revised if necessary. From Table B it can be seen that in most members, the dead-load stress is less than 25 per cent of the maximum stress in the member. Any great refinement in the determination of dead-load stresses is therefore unnecessary. Revision of dead-load stresses need be made only when an investigation shows that use of true dead-load stresses makes necessary the change in the section of a member or the number of rivets required at any joint. In the truss under discussion, the sections of all members, except the top chord, were determined by the specifications regarding minimum sections, and not by the stresses. If the end section of the top chord does not require revision, no changes in the members of this truss need be expected. The rivets at the joints were made to fit closely the actual stresses. Any changes due to incorrectly assumed dead load may then be expected in the joint details rather than in the sections of the members.

Table E gives the estimated weight of the truss of this design. The total weight is found to be 2868.6 lbs. As the total covered area is $60 \times 15 = 900$ sq. ft., the true weight per sq. ft. of covered area is $2868.6/900 = 3.19$ lbs. The weight estimated by the formula of Art. 256 was 3.0 lbs. per sq. ft. As the error is small, and since an investigation showed that no changes in details or members were necessary, the preliminary values will be adopted as final.

274. Trusses Supported on Columns.—When roof trusses are supported on columns instead of masonry walls, as assumed in Art. 256, the truss and its supporting columns become a framed bent which is exactly similar to a portal frame. Such structures are used extensively in mill-building construction. In order to illustrate the principles involved in the design of such structures, the truss of Fig. 1

will be set on columns. The dimensions of the resulting framed bent are shown in Fig. 8.

TABLE E

ESTIMATED WEIGHT 60-FOOT STEEL ROOF TRUSS

Item	No. of Pieces	Section	Weight per Foot	Total Feet	Total Weight, Pounds
Top chord	4	5×3×3/8×33′–7½″	9.8	134′–6″	1318.1
Bottom chord	4	2½×2½×5/16×19′–4¼″	5.0	77′–5″	387.1
Bottom chord	2	2½×2½×¼×22′–3″	4.1	44′–6″	182.5
Struts	4	3×2½×¼×7′–11 ⅛″	4.5	31′–8½″	142.7
Sub struts	4	2½×2×¼×3′–8 13/16″	3.62	14′–11¼″	54.1
Ties	8	2×2×¼×8′–5″	3.19	67′–4″	214.8
Ties	4	2×2×¼×17′–9½″	3.19	71′–2″	227.0
Sag tie	1	2×2×¼×14′–0 ⅝″	3.19	14′–0 ⅝″	44.8
Gusset-plates	2	14½×3/8×2′–4½″	15.3	3.6 sq. ft.	55.1
Gusset-plates	4	7½×¼×0′–8½″	10.2	1.77 "	18.0
Gusset-plates	2	11×¼×2′–1″	10.2	3.36 "	34.3
Gusset-plates	1	15×3/8×2′–5½″	15.3	2.26 "	34.6
Gusset-plates	4	6×¼×0′–9″	10.2	1.28 "	13.1
Gusset-plates	2	9¼×3/8×1′–2½″	15.3	1.65 "	25.2
Gusset-plates	1	5×¼×0′–7″	10.2	0.20 "	2.0
Splice-plates	2	5½×¼×1′–0″	10.2	0.92 "	9.4
Shoe-angles	4	2½×2½×3/8×0′–10½″	5.9	3′–6″	20.7
Sole plates	2	10×⅝×0′–10½″	21.25	1′–9″	37.2
Ring fills	48	2×¼ at 0.222 lb.	....	......	10.7
					2831.4
496 rivet heads at 0.075 lb. per head					37.2
Total weight					2868.6

The same general data regarding material, roof covering, wind and snow loads, and working stresses will be assumed as given in Art. 256. In addition, the sides and ends of the building will be assumed as covered with 22-gauge corrugated steel. A wind pressure of 20 lbs. per sq. ft. will be assumed for the sides and ends of the building.

Since the external forces supported by the roof are the same as assumed in Art. 256, the design of the purlins is the same as given in Art. 257, and will therefore not be repeated here.

275. Determination of Stresses.—*Dead and Snow Load Stresses.*—The methods of stress analysis for structures of the type shown in Fig. 8 are given in Art. 200, Part I. As the kneebraces come into action cnly under horizontal forces, these members will have zero

stress for dead and snow load. The dead-load stress diagram is then similar to that given in Fig. 4(*b*), page 405.

The formula for weight of the truss is not the same as that given in Art. 256. On page 80, Part I, is given a formula for dead weight of mill building trusses. This formula is:

$$w = \frac{P}{45}\left(1 + \frac{l}{5\sqrt{A}}\right), \text{ where}$$

w = weight per sq. ft. horizontal area, P = capacity of truss in lbs. per sq. ft., l = span, and A = distance between trusses. Assuming

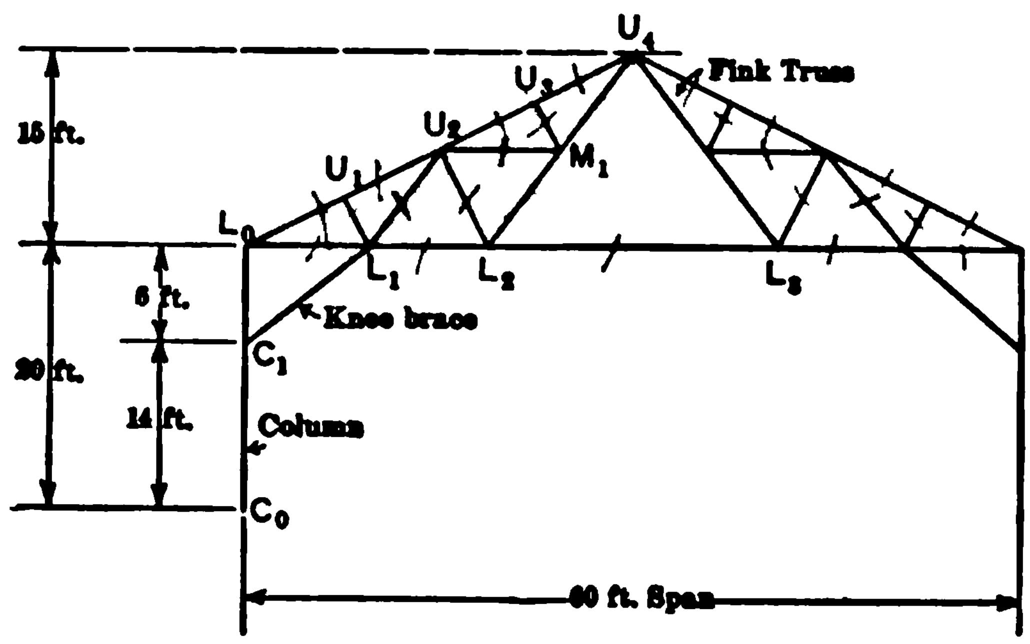


FIG. 8.

a capacity of 30 lbs. per sq. ft., with l = 60 and A = 15, the above formula gives w = 2.74 lbs. per sq. ft. Since this weight is but slightly less than that found for the truss of Fig. 1, the dead-load stresses will be taken as given in Table B of Art. 263. The snow-load stresses will also be the same as given in Table B. In Table F the dead- and snow-load stresses are given for the mill-building truss under consideration.

276. *Wind Stresses.*—The wind stresses in the truss of Fig. 8 depend upon the condition of the bases of the columns. Two cases will be considered: Case A, columns hinged at the base; and Case B, columns fixed at the base, points of inflection half-way between the foot of the kneebrace and the base of the column.

The wind panel loads on the roof surface due to the normal component of the wind pressure will be the same as given in Art. 261, or 2,820 lbs. per panel. On the sides of the building, wind panel

loads due to a pressure of 20 lbs. per sq. ft. must be considered. These panel loads will be assumed as concentrated at the top of the column, the foot of the kneebrace, and the base of the column. Each load will be assumed to be in proportion to the surface tributary to these points.

Case A.—Since the columns are assumed to be hinged at their bases, the entire framed bent must be taken into consideration. At the top of the column, the horizontal wind load is $3 \times 15 \times 20 = 900$ lbs.; at the kneebrace, the load is $10 \times 15 \times 20 = 3{,}000$ lbs.; and at the foot of the column, the load is $7 \times 15 \times 20 = 2{,}100$ lbs. These loads are shown on Fig. 9(*a*) and are labeled as for Case A.

The reactions at the foot of the columns, calculated by the method outlined in Art. 200, Part I, are shown in Fig. 9(*a*), and the stress diagram is shown in Fig. 9(*b*). Table F gives the values of the resulting stresses. Where two values are given for a member, the upper value is for the member on the left-hand side of the truss and the lower value is for the corresponding member on the right-hand side.

The values given in Table F for the columns are the true stresses in these members. Since auxiliary frames, shown by dotted lines in Fig. 9(*a*), were placed on the sides of the columns to aid in the graphical determination of stresses, the stresses given directly by Fig. 9(*b*) for the columns are not the true stresses. In order to determine the true stresses, these auxiliary frames must be removed and the stresses determined for the actual conditions.

The direct stress in the lower part of the windward column is equal to the vertical component of the reaction at the foot of the column, which is 4,230 lbs. (Fig. 9(*a*).) For the section of the column above the kneebrace, the stress can be determined by passing around joint C_1, using the stress in the kneebrace, and the true stress in the lower part of the column. The line h–3_1 of Fig. 9(*b*) shows the required stress. It must be remembered that when the auxiliary frame is removed, a bending moment exists at the foot of the kneebrace. The stress diagram for joint C_1 will therefore not close. A similar construction gives the stresses in the other columns.

Case B.—Since the columns are assumed to be hinged at a point half-way between the kneebrace and the foot of the column, only

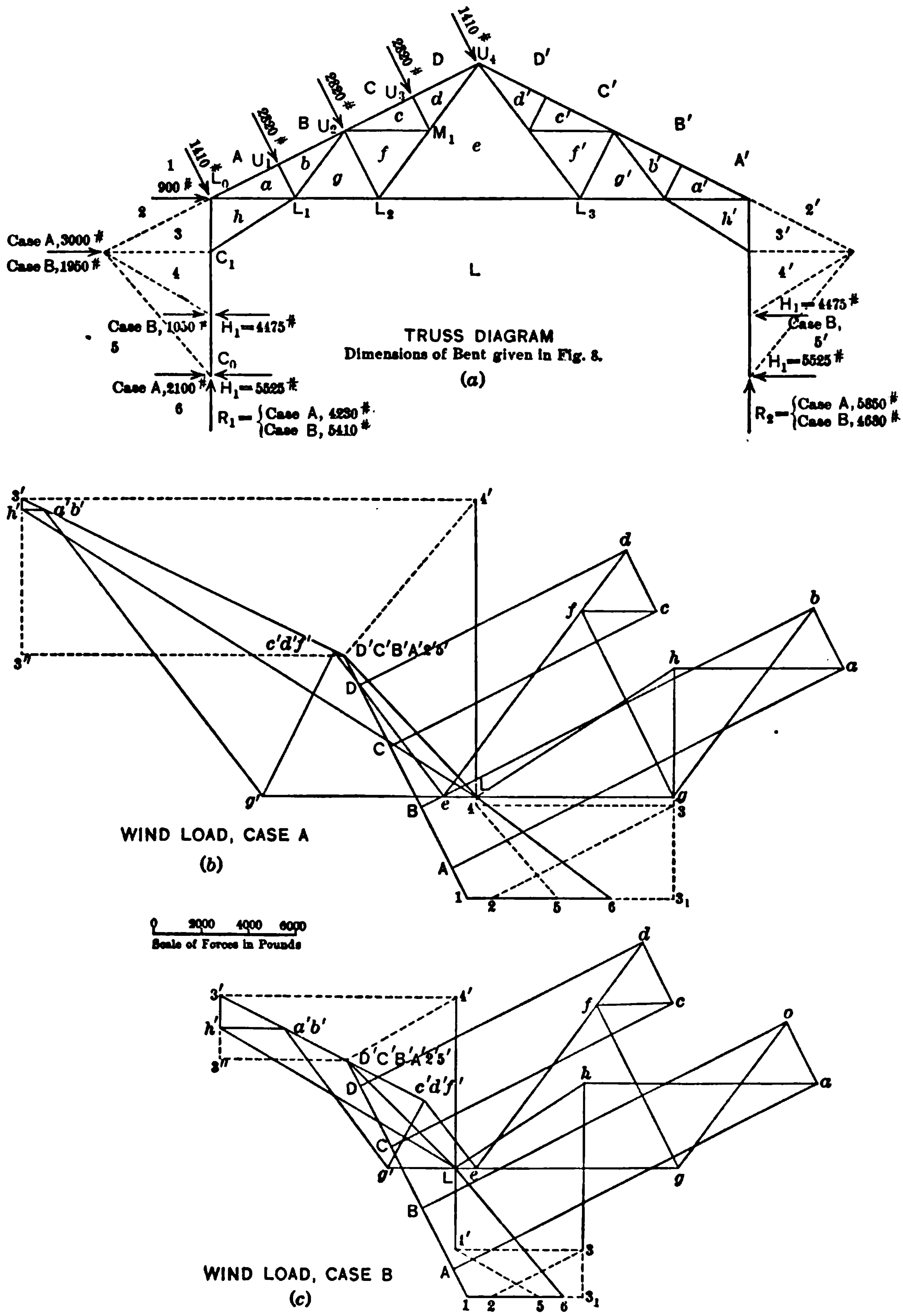

1410 #
2330 #
2330 #
2330 #
1410 #
900 #
Case A, 3000 #
Case B, 1950 #
Case B, 1050 #
H₁ = 4475 #
Case A, 2100 #
H₁ = 5525 #
R₁ = {Case A, 4230 # / Case B, 5410 #
H₁ = 4475 #
Case B, 5′
H₁ = 5525 #
R₂ = {Case A, 5850 # / Case B, 4680 #
TRUSS DIAGRAM
Dimensions of Bent given in Fig. 8.
(a)
WIND LOAD, CASE A
(b)
0 2000 4000 6000
Scale of Forces in Pounds
WIND LOAD, CASE B
(c)

FIG. 9.

the portion of the structure above these points need be considered. All loads and the reactions for this case are shown on Fig. 9(*a*), labeled as for Case B. Loads which are not labeled are common to both cases A and B.

Fig. 9(*c*) is the stress diagram for Case B, and Table F gives the resulting stresses.

277. *Maximum Stresses.*—The maximum stresses are to be made up for the same combinations of dead, snow, and wind load as outlined in Art. 258. Table F gives the stresses for the several combinations. It is to be noted that all values have not been filled in, for it can be seen by inspection in some cases that the resulting value is less than that for combinations already given.

The stresses to be used in designing are the greatest stresses in tension or compression given by the several combinations. These values are given in the last column of the table. The member notation is as shown on Fig. 9(*a*).

TABLE F

STRESSES IN MEMBERS

Member	Dead Load	Snow Load (Max.)	Snow Load (Min.)	Wind Load, Case A.	Wind Load, Case B.	D.L. + S.L. (Max.)	D.L. +W.L. Case A or B.	D.L. + W.L. + S.L. (Min.)	Maximum Stress
L_0U_1..	−9,550	−24,650	−12,325	−18,550 +13,700	−17,050 + 3,600	−34,200	−28,100 + 4,150	−40,425	−40,425 + 4,150
U_1U_2..	−9,000	−23,250	−11,625	−18,550 +13,700	−17,050 + 3,600	−32,250	−27,550 + 4,700	−39,175	−39,175 + 4,700
U_2U_3..	−8,450	−21,850	−10,925	−12,600 + 320	−13,200 − 3,500	−30,300	−21,650	−32,575	−32,575
U_3U_4..	−7,900	−20,450	−10,225	−12,600 + 320	−13,200 − 3,500	−28,350	−21,100	−31,325	−31,325
L_0L_1..	+8,540	+22,050	+11,025	+ 7,100 + 750	+ 9,720 + 2,100	+30,590	+18,260	+29,285	+29,285
L_1L_2..	+7,320	+18,900	+ 9,450	+ 8,360 − 8,800	+ 9,500 − 2,920	+26,220	+16,820 − 1,480	+26,270	+26,270 − 1,480
L_2L_3...	+4,880	+12,600	+ 6,300	− 1,300	+ 1,000	+17,480	+ 5,880	+12,180	+17,480
U_1L_1.. U_3M..	−1,090	− 2,820	− 1,410	− 2,820	− 2,820	− 3,910	− 3,910	− 5,320	− 5,320
U_2L_2..	−2,180	− 5,640	− 2,820	− 8,600 + 6,650	− 7,600 + 3,500	− 7,820	−10,780 + 4,470	−13,600	−13,600 + 4,475
U_2L_1..	+1,220	+ 3,150	+ 1,575	+ 9,750 −14,920	+ 7,500 − 7,900	+ 4,370	+10,970 −13,700	+12,545	+12,545 −13,700
U_2M..	+1,220	+ 3,150	+ 1,575	+ 3,150	+ 3,150	+ 4,370	+ 4,370	+ 5,945	+ 5,945
L_2M..	+2,440	+ 6,300	+ 3,150	+ 9,600 − 7,500	+ 8,500 − 3,950	+ 8,740	+12,040 − 5,060	+15,190	+15,190 − 5,060
MU_4..	+3,660	+ 9,450	+ 4,725	+12,750 − 7,500	+11,600 − 3,950	+13,110	+16,410 − 3,840	+21,135	+21,135 − 3,840
C_1L_1..	0	0	0	+ 9,920 −22,050	+ 6,550 −11,600	0	+ 9,920 −22,050	+ 9,920 −22,050	+ 9,920 −22,050
C_0C_1..	−4,880	−12,600	− 6,300	− 4,230 − 5,850	− 5,410 − 4,680	−17,480	−10,730	−17,030	−17,480
C_1L_0..	−4,880	−12,600	− 6,300	− 9,600 + 6,120	− 8,900 + 1,600	−17,480	−14,480 + 1,240	−20,780	−20,780 + 1,240

+ Denotes tension; − denotes compression. All stresses given in pounds.

TABLE G

DESIGN OF MEMBERS

Member	Stress	Length (Ins.)	r (Ins.)	l/r	Unit Stress (Lb. per Sq. In.)	Area Reqd. (Sq. Ins.)	Section	Area Provided (Sq. Ins.) Gross	Net
Top Chord		Special	Design				2∠s 5×3×3/8 ins.	....	....
L_0L_1....	+29,285	.....	....	.....	16,000	1.83	2∠s 3×3×1/4 ins.	2.88	2.50
L_1L_2....	+26,270 − 1,480	112	0.93	120.5	16,000 7,570	1.64 0.20	2∠s 3×3×1/4 ins.	2.88	2.50
L_2L_3....	+17,480	.....	....	.....	16,000	1.09	2∠s 3×3×1/4 ins.	2.88	2.50
U_1L_1....	− 5,320	50.2	0.42	119.5	7,630	0.70	1∠ 2 1/2×2×1/4 ins.	1.07	....
U_2L_2....	−13,600 + 4,470	100.4	0.95	105.8	8,600 16,000	1.58 0.28	2∠s 3×2 1/2×1/4 ins.	2.64	....
U_3M....	− 5,320	50.2	0.42	119.5	7,630	0.70	1∠ 2 1/2×2×1/4 ins.	1.07	....
U_2L_1....	+12,545 −13,700	112	0.95	118	16,000 7,750	0.78 1.77	2∠s 3×2 1/2×1/4 ins.	2.64	2.26
U_2M ...	+ 5,945	.....	....	.....	16,000	0.37	2∠s 2×2×1/4 ins.	1.88	1.50
L_2M....	+15,190 − 5,060	112	0.95	118	16,000 7,750	0.95 0.65	2∠s 3×2 1/2×1/4 ins.	2.64	2.26
U_4M ...	+21,135 − 3,840	112	0.95	118	16,000 7,750	1.32 0.495	2∠s 3×2 1/2×1/4 ins.	2.64	2.26
Knee-Brace..	+ 9,920 −22,050	132	1.09	121	16,000 7,530	0.62 2.93	2∠s 3 1/2×2 1/2×5/16 ins.	3.56	3.10
Column.		Special	Design				4∠s 4×3×5/16 ins.		

+ denotes tension; − denotes compression.

278. Design of Members.—The general considerations governing the form and arrangement of members and unit stresses are the same as given in Art. 264. Table G gives the data required for the design of members. It will be noted that certain truss members are subjected to a reversal of stress. Since this reversal of stress is due to a change in the direction of the wind, it is evident that the change in character of stress takes place gradually. Therefore the effect of rapid changes in stress is not in question, as in the case of a railway bridge, as considered in Art. 240, Chap. IX. Under the existing conditions, the truss members are to be designed to take each kind of stress considered separately. The section of the member will then be determined by the stress which requires the greater area, and the end connection will be determined by the greater stress. It will be found that in the truss under consideration, the design of all members subjected to compression is determined by the condition that l/r must not exceed 125.

The design of the top chord is similar to that given in Art. 268. In this case the direct stress is slightly larger than that in Art. 268, but it will be found that the same chord section will answer for both cases.

Bottom chord member $L_1 L_2$ has a tension of 26,270 lbs. and a compression of 1,480 lbs. Since this member must take a small amount of compression, it must be designed subject to the conditions that its l/r must not exceed 125, and that the net area must take the required tension.

The columns which support the trusses are subjected to direct stress and bending moment. It will be found that the design will

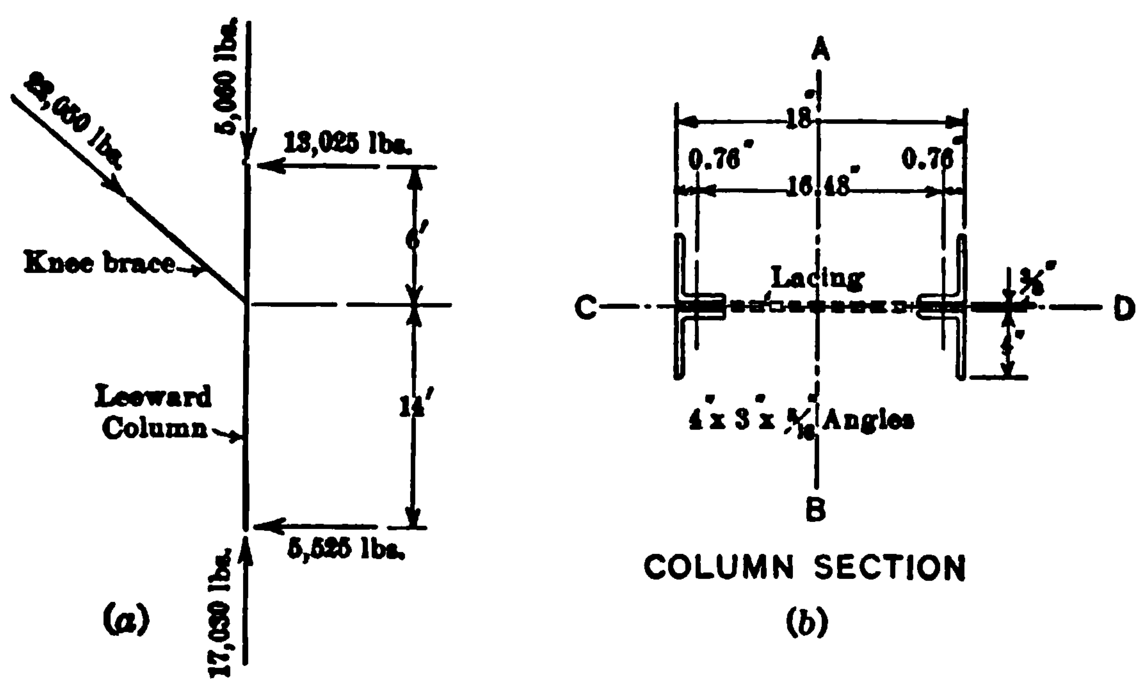


FIG. 10.

be governed by the conditions in the leeward column under dead load, minimum snow load, and wind load, Case A. Fig. 10(*a*) shows the forces acting on the column. The bending moment at the foot of the kneebrace is $5,525 \times 14 \times 12 = 928,000$ in.-lbs. A column section consisting of $4 \times 3 \times 5/16$ in. angles placed as shown in Fig. 10(*b*) will be assumed. This section has an area of 8.36 sq. ins., a moment of inertia about axis $A B$ of 574.3 ins.^{4}, and a corresponding radius of gyration of 8.3 ins. Since the bending moment is due to wind forces, the usual practice is to allow a 25 per cent increase in the working stresses. The allowable compressive stress is then $1.25\ (16,000 - 70 \times 240/8.3) = 17,450$ lbs. per sq. in. From the equation given in Art. 268 for top chord design, the area required is

$$A = \frac{17,030}{17,450} + \frac{928,000 \times 9}{20,000 \times 8.3^2} = 0.98 + 6.08 = 7.06 \text{ sq. ins.}$$

The assumed section is therefore ample. This section must now be investigated for column action about the axis $C D$ of Fig. 10(*b*). From Table F, the maximum direct load on the column is 17,480

lbs. The radius of gyration of two $4 \times 3 \times \frac{5}{16}$ in. separated by a $\frac{3}{8}$-in. space is 1.93 ins. for axis $C\,D$, and the value of l/r is $240/1.93 = 125$. Since wind forces do not enter in the above load, the allowable column stress is $16,000 - 70 \times 125 = 7,250$ lbs. per sq. in., and the required area is $17,480/7,250 = 2.41$ sq. ins. As the assumed section answers all requirements, it will be adopted as final.

The angles are to be connected by lacing, which must be designed to take the shear in the column. From Fig. 10(a), the shear below the kneebrace is 5,525 lbs., and that above the kneebrace is 13,025 lbs. Assuming single lacing inclined at an angle of 30° to the horizontal, the stress in each bar below the kneebrace is $5,525 \times \sec 30^\circ = 6,380$ lbs. It will be found that the size of lacing-bars will be determined by the rivets required to connect the lacing-bars to the angles. As the rivets are $\frac{5}{8}$ in. in diameter, and are in single shear, they have a value of $1.25 \times 3,680 = 4,600$ lbs. per rivet. Therefore two rivets are required in each end of a lacing-bar. This requires a $4 \times \frac{3}{8}$ in. bar. Assuming the lacing-bars to be fixed at the ends, the unsupported length can be taken as one-half the total length. On this assumption it will be found that the assumed bar is ample for compression. It will also provide sufficient net area for tension. If lacing-bars are used above the kneebrace, they will have a stress of $13,025 \times \sec 30^\circ = 15,100$ lbs. per bar. This requires four rivets in the end of each bar. As this is not a practical detail, it will be best to substitute a $\frac{3}{8}$-in. web-plate for lacing above the kneebrace, as shown on the general drawing of Plate VIII.

The base of the column must be designed for bearing due to the maximum vertical load, and for overturning due to wind load. From Table F, the maximum vertical load is 17,480 lbs. With an allowable bearing pressure on the masonry of 300 lbs. per sq. in., the base area required is 58.3 sq. ins. The base detail shown on the general drawing provides the required area.

The condition of loading causing maximum tendency to overturn the column is dead load and wind load, Case B, for the leeward column. From Table F, the vertical load on the column is 9,560 lbs.; and from Fig. 9(a), the horizontal force acting at the point of inflection is 4,475 lbs. Fig. 11 shows the column removed from the footing. As shown on the general drawing, the column is held in place on the

footing by means of anchor bolts spaced 22 ins. apart. Taking moments about the right-hand bolt for the forces shown in Fig. 11, the stress in an anchor bolt is $(4{,}475 \times 84 - 9{,}560 \times 11)/22 = 12{,}300$ lbs. For an allowable unit stress of 20,000 lbs. per sq. in., the required area of anchor bolt is 0.62 sq. ins. A $1\frac{1}{8}$ in. round rod will provide ample area. The column detail shown on the general drawing answers all requirements.

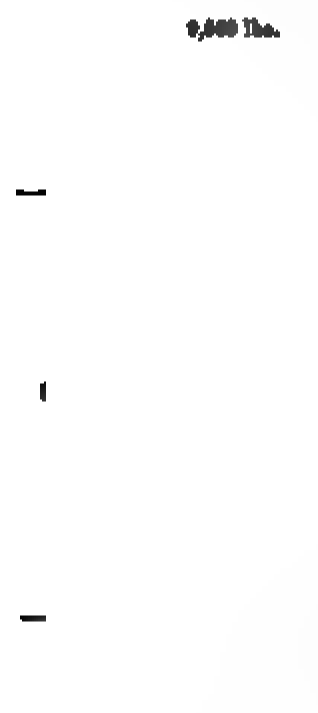


Fig. 11.

The design of the masonry footing is beyond the scope of this book. For methods of design the student is referred to standard books on Foundations.

279. Design of Girts.—The siding, which is 22 gauge corrugated steel with anti-condensation lining, is carried by girts which are fastened to the columns. These girts are similar to the purlins, which carry the roof, and they are designed by the same general methods.

A spacing of 4 ft. between girts will be used, the maximum allowed by Table A, Art. 256, for 22 gauge corrugated steel. The load to be carried by each girt is a vertical load due to the weight of siding and girt, and a horizontal load due to a wind pressure of 20 lbs. per sq. ft. From Table A, Art. 256, corrugated steel of 22 gauge weighs 1.6 lbs. per sq. ft. and anti-condensation lining described in Art. 256 weighs 1.3 lbs. per sq. ft. Since the girts are 4 ft. apart and the columns 15 ft. centres, each girt carries an area of 60 sq. ft. The weight of siding per girt is then $60(1.6 + 1.3) = 174$ lbs. Assuming a 6-in., 8-lb. channel as a girt, the total vertical load to be carried is $174 + 8 \times 15 = 294$ lbs. At 20 lbs. per sq. ft., the horizontal load due to wind is $60 \times 20 = 1{,}200$ lbs. Fig. 12(*a*) shows the graphical determination of the resultant load on a girt. The bending moment is $M = \frac{1}{8} \times 1{,}240 \times 15 \times 12 = 27{,}900$ in.-lbs. Since this bending is due principally to the action of wind force, the working stress can be taken as 20,000 lbs. per sq. in. The required section modulus is then $27{,}900/20{,}000 = 1.39$ ins.[3] Fig. 12(*b*) shows the *S* Polygon for the assumed channel, from which it can be seen a 6-in., 8-lb.

channel is sufficient. The method of attaching the girts to the column is shown on the general drawing.

280. Details of End Walls.—The arrangement used for closing in the ends of a mill building depends somewhat upon the plans for future additions to the building. In case the length of the build-

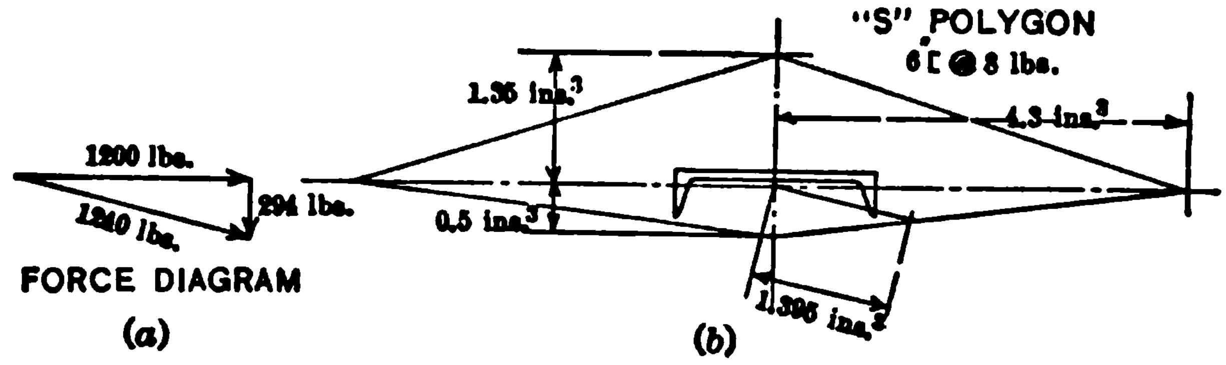


FIG. 12.

ing is to be increased, trusses similar to the interior trusses are used at the ends of building and the end is closed by a temporary wall. If no increase in length is planned, the end is closed in by means of a wall built up of beams which carry the roof and siding, as shown in Fig. 13. This latter arrangement will be adopted for the structure under consideration.

As shown in Fig. 13, the siding for the ends of the building is carried by vertical beams spaced 15 ft. apart. This allows the use of the same size of girts as on the sides of the building. The vertical beams are supported at the floor level and at the plane of the bottom chords of the trusses. In designing these beams, their length can be taken as 20 ft., and the load to be carried can be considered as a uniform load of 20 lbs. per sq. ft. As the beams are 15 ft. apart, the area carried by each is $15 \times 20 = 300$ sq. ft.; the total load is $300 \times 20 = 6{,}000$ lbs.; and the bending moment is $\frac{1}{8} \times 6{,}000 \times 20 \times 12 = 180{,}000$ in.-lbs. With a working stress of 20,000 lbs. per sq. in., the section modulus required is 9.0 ins.^{3}, which is furnished by a 7-in., 15-lb. I-beam. The portion of the beam between the plane of the lower chord of the trusses and the roof will be made of the same section as the part below the lower chord.

As shown in Fig. 13, the rafter which carries the purlin is supported by the vertical beams. The points of support are $15 \times \sec 26° 34' = 16.8$ ft. apart. Considering the load due to the purlins to be a uniform load of the amount given by the normal component

of Fig. 3(*b*), Art. 257, the load carried by the rafter between points of support is $7.5 \times 16.8 \times 39.8 = 5{,}020$ lbs. Assuming an 8-in., $11\frac{1}{4}$-lb. channel as a rafter section, the total load to be carried is $5{,}020 + 16.8 \times 11\frac{1}{4} \times \cos 26° 34' = 5{,}190$ lbs., and the bending

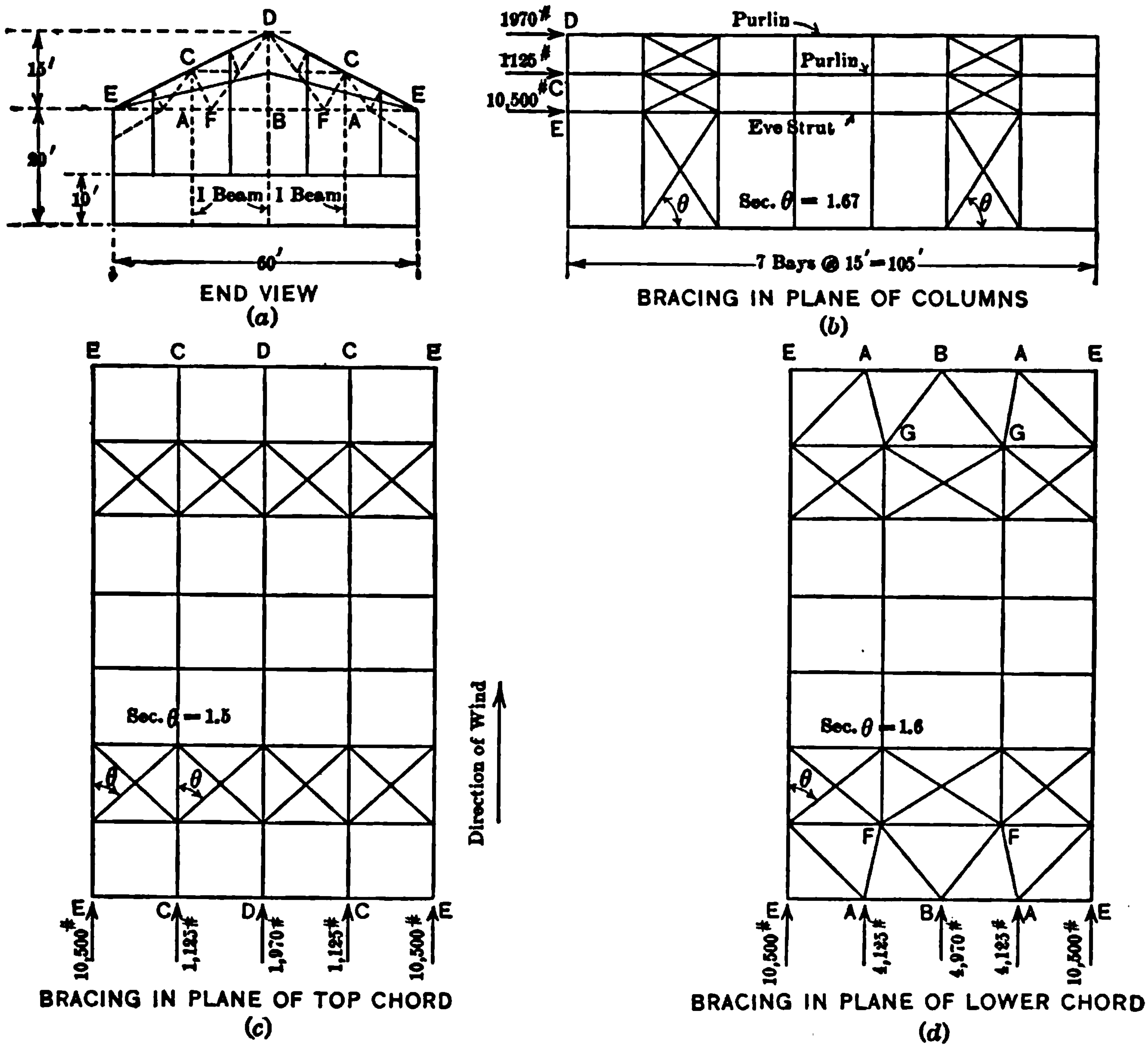


FIG. 13.

moment is $\frac{1}{8} \times 5{,}190 \times 16.8 \times 12 = 130{,}000$ in.-lbs. As this moment is due mainly to vertical loading, the working stress is 16,000 lbs. per sq. in., and the required section modulus is $130{,}000/16{,}000 = 8.15$ ins.[3] As the assumed beam has a section modulus of 8.1 ins.[3], it will be adopted.

281. Design of Lateral Bracing.—In order to brace thoroughly a building in which the trusses are set upon columns, and where masonry walls are not present, rigid lateral bracing must be provided

for wind pressures acting parallel or perpendicular to the longitudinal axis of the building. The kneebraced bents provide the bracing required for wind pressure in a direction perpendicular to the axis of the building. Fig. 9 gives the resulting stress diagrams. Wind pressures acting along the axis of the building will be taken by bracing in the plane of the top chord of the truss, the plane of the lower chord, and the planes of the columns. Fig. 13 shows the general arrangement of the bracing for a building 7 bays in length. As shown in this figure, bracing is provided in the second bay from each end of the building. In the plane of the lower chord of the truss, bracing is also provided in the end bays. The object of this bracing is to form supports for the vertical I-beams which carry the covering for the ends of the building. In general, bracing is usually provided in every third or fourth bay.

In calculating the stresses in the bracing, it will be assumed that each system takes care of the loads brought to the panel points by the area of the end of the building which is tributary to the point in question. Fig. 13(*a*) shows the assumed distribution of the area.

The bracing in the plane of the columns, shown in Fig. 13(*b*), must take care of all wind forces above a line 10 ft. above the floor level, it being assumed that wind forces below this line are carried to the foundations. On this assumption the area tributary to points E is $\frac{1}{2}(10 \times 60 + \frac{1}{2} \times 60 \times 15) = 525$ sq. ft., and the panel load due to a wind pressure of 20 lbs. per sq. ft. is 10,500 lbs. Assuming the bracing to be rods capable of taking tension only, and that the bracing in each bay takes half the total load, the stress in each diagonal is $\frac{1}{2} \times 10{,}500 \times \sec\theta = 8{,}750$ lbs. As considerable initial tension will be induced by tightening up the rods, this will be allowed for by adding, say 5,000 lbs. to the stress in the diagonal, giving a total stress of 13,750 lbs. At 20,000 lbs. per sq. in., the area required is 0.6875 sq. in. This area is provided by $^{15}/_{16}$-in. round rod, which will be upset to $1\frac{1}{4}$ ins. at the ends in order to provide sufficient area at the root of the threads.

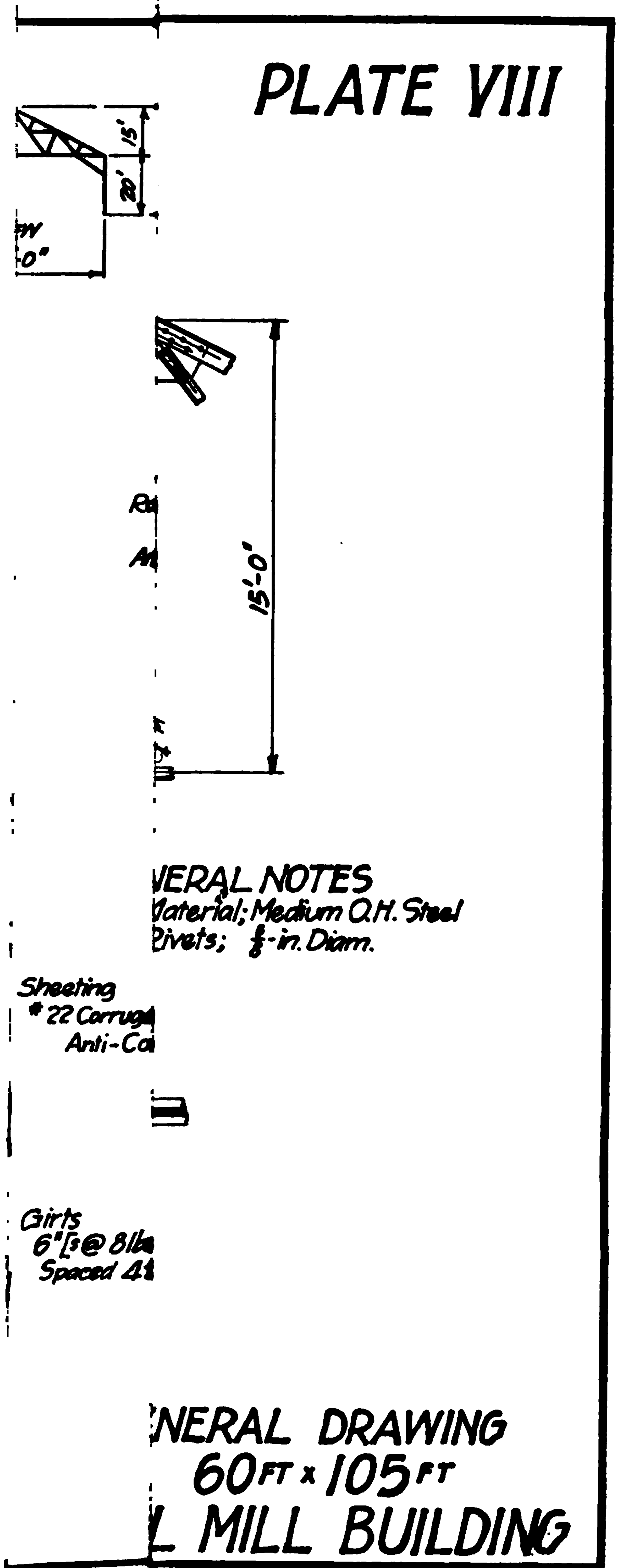
The lateral bracing in the plane of the top chord is to be designed for the wind panel loads brought to points C and D of Fig. 13(*a*). For the distribution of area shown, the load at C is 1,125 lbs. and that at D is 1,970 lbs. Fig. 13(*c*) shows the loads in position, and

the general arrangement of members. Calculating the shears as for a cantilever truss, the shear in the panels $C D$ is $\frac{1}{2} \times 1{,}970 = 985$ lbs., and that in panels $E C$ is $985 + 1{,}125 = 2{,}110$ lbs. Assuming that the members take tension only, that each bay takes one-half the shear, and that an initial tension of 5,000 lbs. is provided, it will be found that a $2 \times 2 \times \frac{1}{4}$-in. angle is required. The purlins aid in carrying the loads from one bay to the next in line. As these stresses are small, they can be neglected.

The loads brought to the lateral bracing in the plane of the lower chords of the trusses due to the distribution of area shown in Fig. 13(*a*) are 4,125 lbs. at *A* and 4,970 lbs. at *B*. Fig. 13(*d*) shows the arrangement of laterals and the applied loads. The members in the end bays support the I-beams, which carry the end covering, and carry the loads to the lateral trusses. As the stresses in these braces are small, l/r conditions govern the design. With l/r not to exceed 150, it will be found that two $6 \times 4 \times \frac{3}{8}$-in. angles must be used, with the 6-in. legs separated by a $\frac{1}{2}$-in. space.

The load brought to joint *F* is $\frac{1}{2}(4{,}125 + 4{,}970) = 4{,}548$ lbs. For these loads, the shear in centre panel is zero and that in the outside panels is 4,548 lbs. Making the same assumptions as for the top chord bracing, the stress in each diagonal, including 5,000 lbs. initial tension, is 8,640 lbs. A $2 \times 2 \times \frac{1}{4}$-in. angle will be found ample. The members $F G$ will be governed by l/r conditions. Two $4 \times 3 \times \frac{5}{16}$-in. angles, 4-in. legs vertical and separated by a $\frac{3}{8}$-in. space will be sufficient.

At the line of the tops of the columns, and at the same level across the ends of the building, a member called an eave strut is provided. Four angles, $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in., arranged as shown on the general drawing, will be found to provide a section of ample rigidity. Plate VIII shows all details of the lateral bracing.



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APPENDIX A

GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

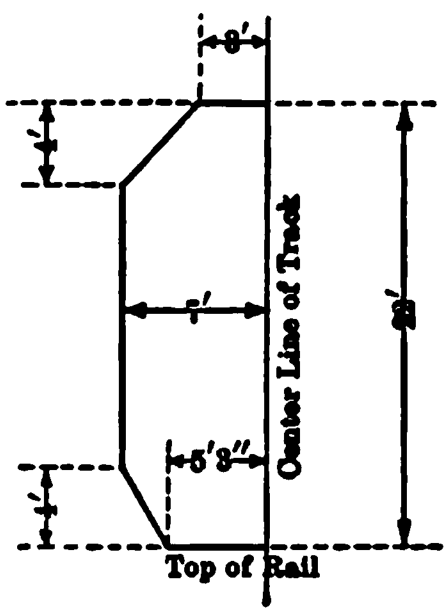
AMERICAN RAILWAY ENGINEERING ASSOCIATION, 1910*

PART FIRST—DESIGN

I. General

1. The material in the superstructure shall be structural steel, except rivets, and as may be otherwise specified. **Materials.**

2. When alignment is on tangent, clearances shall not be less than shown on the diagram; the height of rail shall, in all cases, be assumed at 6 ins. The width shall be increased so as to provide the same minimum clearances on curves for a car 80 ft. long, 14 ft. high, and 60 ft. centre to centre of trucks, allowance being made for curvature and superelevation of rails. **Clearances.**



3. The width centre to centre of girders and trusses shall in no case be less than one-twentieth of the effective span, nor less than is necessary to prevent overturning under the assumed lateral loading. **Spacing Trusses.**

4. Ends of deck plate girders and track stringers of skew bridges at abutments shall be square to the track, unless a ballasted floor is used. **Skew Bridges.**

5. Wooden tie floors shall be secured to the stringers and shall be proportioned to carry the maximum wheel load, with 100 per cent impact, distributed over three ties, with fibre stress not to exceed 2,000 lbs. per sq. in. Ties shall not be less than 10 ft. in length. They shall be spaced with not more than 6-in. openings; and shall be secured against bunching. **Floors.**

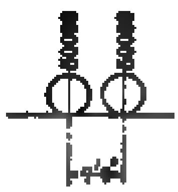
II. Loads

6. The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weigh $4\frac{1}{2}$ lbs. per ft. B.M., ballast 100 lbs. per cu. ft., reinforced concrete 150 lbs. per cu. ft., and rails and fastenings 150 lbs. per linear ft. of track. **Dead Load.**

*As printed in the 1915 Manual of the A. R. E. A. these specifications show numerous unimportant verbal changes without changes in substance.

Live Load.

7.* The live load, for each track, shall consist of two typical engines followed by a uniform load, according to Cooper's series, or a system of loading giving practically equivalent stresses. The minimum loading to be Cooper's E–40, as shown in the following diagrams:

and  the diagram that gives the larger stresses to be used.

Heavier Loading.

8. Heavier loadings shall be proportional to the above diagrams on the same spacing.

Impact.

9. The dynamic increment of the live load shall be added to the maximum computed live-load stresses and shall be determined by the formula $I = L \frac{300}{l + 300}$,

where I = impact or dynamic increment to be added to live-load stress,
L = computed maximum live-load stress.
l = loaded length of track in feet producing the maximum stress in the member. For bridges carrying more than one track, the aggregate length of all tracks producing the stress shall be used.

Impact shall not be added to stresses produced by longitudinal, centrifugal, and lateral or wind forces.

Lateral Forces.

10. All spans shall be designed for a lateral force on the loaded chord of 200 lbs. per linear foot plus 10 per cent of the specified train load on one track, and 200 lbs. per linear foot on the unloaded chord; these forces being considered as moving.

Wind Force.

11. Viaduct towers shall be designed for a force of 50 lbs. per sq. ft. on one and one-half times the vertical projection of the structure unloaded; or 30 lbs. per sq. ft. on the same surface plus 400 lbs. per linear ft. of structure applied 7 ft. above the rail for assumed wind force on train, when the structure is either fully loaded or loaded on either track with empty cars assumed to weigh 1,200 lbs. per linear ft., whichever gives the larger stress.

* See Addendum, clause (a).

12. Viaduct towers and similar structures shall be designed for a longitudinal force of 20 per cent of the live load applied at the top of the rail. Longitudinal Force.

13. Structures located on curves shall be designed for the centrifugal force of the live load applied at the top of the high rail. The centrifugal force shall be considered as live load and be derived from the speed in miles per hour given by the expression $60 - 2\frac{1}{2}D$, where "D" = degree of curve.

III. Unit Stresses and Proportion of Parts

14. All parts of structures shall be so proportioned that the sum of the maximum stresses produced by the foregoing loads shall not exceed the following amounts in pounds per sq. in., except as modified in paragraphs 22 to 25: Unit Stresses.

15. Axial tension on net section..................16,000 Tension.

16. Axial compression on gross section of columns...$16{,}000 - 70\dfrac{l}{r}$ Compression.

with a maximum of..............................14,000
where "l" is the length of the member in inches, and "r" is the least radius of gyration in inches.
Direct compression on steel castings.................16,000

17. Bending: on extreme fibres of rolled shapes, built sections, girders, and steel castings; net section........16,000 Bending.
on extreme fibres of pins..........................24,000

18. Shearing: shop driven rivets and pins..........12,000 Shearing.
field driven rivets and turned bolts.................10,000
plate girder webs; gross section.....................10,000

19. Bearing: shop driven rivets and pins..........24,000 Bearing.
field driven rivets and turned bolts.................20,000
expansion rollers; per linear inch.................. $600d$
where "d" is the diameter of the roller in inches.
on masonry.. 600

20. The lengths of main compression members shall not exceed 100 times their least radius of gyration, and those for wind and sway bracing 120 times their least radius of gyration. Limiting Length of Members.

21. The lengths of riveted tension members in horizontal or inclined positions shall not exceed 200 times their radius of gyration about the horizontal axis. The horizontal projection of the unsupported portion of the member is to be considered as the effective length.

22. Members subject to alternate stresses of tension and compression shall be proportioned for the stresses giving the largest section. If the alternate stresses occur in succession during the passage of one Alternate Stresses.

train, as in stiff counters, each stress shall be increased by 50 per cent of the smaller. The connections shall in all cases be proportioned for the sum of the stresses.

23. Wherever the live- and dead-load stresses are of opposite character, only two-thirds of the dead-load stresses shall be considered as effective in counteracting the live-load stress.

Combined Stresses. 24. Members subject to both axial and bending stresses shall be proportioned so that the combined fibre stresses will not exceed the allowed axial stress.

25. For stresses produced by longitudinal and lateral or wind forces combined with those from live and dead loads and centrifugal force, the unit stress may be increased 25 per cent over those given above; but the section shall not be less than required for live and dead loads and centrifugal force.

Net Section at Rivets. 26. In proportioning tension members the diameter of the rivet holes shall be taken ⅛-in. larger than the nominal diameter of the rivet.

Rivets. 27. In proportioning rivets the nominal diameter of the rivet shall be used.

Net Section at Pins. 28. Pin-connected riveted tension members shall have a net section through the pin-hole at least 25 per cent in excess of the net section of the body of the member, and the net section back of the pin-hole, parallel with the axis of the member, shall be not less than the net section of the body of the member.

Plate Girders. 29. Plate girders shall be proportioned either by the moment of inertia of their net section, or by assuming that the flanges are concentrated at their centres of gravity, in which case one-eighth of the gross section of the web, if properly spliced, may be used as flange section. The thickness of web plates shall be not less than 1/160 of the unsupported distance between flange angles (see 38).

Compression Flange. 30. The gross section of the compression flanges of plate girders shall not be less than the gross section of the tension flanges; nor shall the stress per sq. in. in the compression flange of any beam or girder exceed $16{,}000 - 200\frac{l}{b}$, when flange consists of angles only or if cover consists of flat plates, or $16{,}000 - 150\frac{l}{b}$ if cover consists of a channel section, where l = unsupported distance and b = width of flange.

Flange Rivets. 31. The flanges of plate girders shall be connected to the web with a sufficient number of rivets to transfer the total shear at any point in a distance equal to the effective depth of the girder at that point combined with any load that is applied directly on the flange. The

wheel loads, where the ties rest on the flanges, shall be assumed to be distributed over three ties.

32. Trusses shall preferably have a depth of not less than one-tenth of the span. Plate girders and rolled beams, used as girders, shall preferably have a depth of not less than one-twelfth of the span. If shallower trusses, girders, or beams are used, the section shall be increased so that the maximum deflection will not be greater than if the above limiting ratios had not been exceeded. **Depth Ratios.**

IV. Details of Design

General Requirements

33. Structures shall be so designed that all parts will be accessible for inspection, cleaning, and painting. **Open Sections.**

34. Pockets or depressions which would hold water shall have drain holes, or be filled with waterproof material. **Pockets.**

35. Main members shall be so designed that the neutral axis will be as nearly as practicable in the centre of section, and the neutral axes of intersecting main members of trusses shall meet at a common point. **Symmetrical Sections.**

36. Rigid counters are preferred; and where subject to reversal of stress shall preferably have riveted connections to the chords. Adjustable counters shall have open turnbuckles. **Counters.**

37. The strength of connections shall be sufficient to develop the full strength of the member, even though the computed stress is less, the kind of stress to which the member is subjected being considered. **Strength of Connections.**

38. The minimum thickness of metal shall be ⅜-in., except for fillers. **Minimum Thickness.**

39. The minimum distance between centres of rivet holes shall be three diameters of the rivet; but the distance shall preferably be not less than 3 ins. for ⅞-in rivets and 2½ ins. for ¾-in. rivets. The maximum pitch in the line of stress for members composed of plates and shapes shall be 6 ins. for ⅞-in. rivets and 5 ins. for ¾-in. rivets. For angles with two gauge lines and rivets staggered the maximum shall be twice the above in each line. Where two or more plates are used in contact, rivets not more than 12 ins. apart in either direction shall be used to hold the plates well together. In tension members, composed of two angles in contact, a pitch of 12 ins. will be allowed for riveting the angles together. **Pitch of Rivets.**

40. The minimum distance from the centre of any rivet hole to a sheared edge shall be 1½ ins. for ⅞-in. rivets and 1¼ ins. for ¾-in. rivets, and to a rolled edge 1¼ ins. and 1⅛ ins., respectively. The maximum distance from any edge shall be eight times the thickness of the plate, but shall not exceed 6 ins. **Edge Distance.**

Maximum Diameter. 41. The diameter of the rivets in any angle carrying calculated stress shall not exceed one-quarter the width of the leg in which they are driven. In minor parts ⅞-in. rivets may be used in 3-in. angles, and ¾-in. rivets in 2½-in. angles.

Long Rivets. 42. Rivets carrying calculated stress and whose grip exceeds four diameters shall be increased in number at least one per cent for each additional $^1/_{16}$ in. of grip.

Pitch at Ends. 43. The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivets, for a length equal to one and one-half times the maximum width of member.

Compression Members. 44. In compression members the metal shall be concentrated as much as possible in webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between its connections to the flanges. Cover plates shall have a thickness not less than one-fortieth of the distance between rivet lines.

Minimum Angles. 45. Flanges of girders and built members without cover plates shall have a minimum thickness of one-twelfth of the width of the outstanding leg.

Tie-Plates. 46. The open sides of compression members shall be provided with lattice and shall have tie-plates as near each end as practicable. Tie-plates shall be provided at intermediate points where the lattice is interrupted. In main members the end tie-plates shall have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than one-half this distance. Their thickness shall not be less than one-fiftieth of the same distance.

Lattice. 47. The latticing of compression members shall be proportioned to resist the shearing stresses corresponding to the allowance for flexure for uniform load provided in the column formula in paragraph 16 by the term $70 \frac{l}{r}$. The minimum width of lattice bars shall be 2½ ins. for ⅞-in. rivets, 2¼ ins. for ¾-in. rivets, and 2 ins. if ⅝-in. rivets are used. The thickness shall not be less than one-fortieth of the distance between end rivets for single lattice and one-sixtieth for double lattice. Shapes of equivalent strength may be used.

48. Three-fourths-inch rivets shall be used for latticing flanges less than 2½ ins. wide, and ¾-in. for flanges from 2½ to 3½ ins. wide; ⅞-in. rivets shall be used in flanges 3½ ins. and over, and lattice bars with at least two rivets shall be used for flanges over 5 ins. wide.

49. The inclination of lattice bars with the axis of the member shall be not less than 45 degrees, and when the distance between rivet lines in the flanges is more than 15 ins., if single rivet bar is used, the lattice shall be double and riveted at the intersection.

50. Lattice bars shall be so spaced that the portion of the flange included between their connections shall be as strong as the member as a whole.

51. Abutting joints in compression members when faced for bearing shall be spliced on four sides sufficiently to hold the connecting members accurately in place. All other joints in riveted work, whether in tension or compression, shall be fully spliced. **Faced Joints.**

52. Pin-holes shall be reinforced by plates where necessary, and at least one plate shall be as wide as the flanges will allow and be on the same side as the angles. They shall contain sufficient rivets to distribute their portion of the pin pressure to the full cross-section of the member. **Pin-Plates.**

53. Forked ends on compression members will be permitted only where unavoidable; where used, a sufficient number of pin plates shall be provided to make the jaws of twice the sectional area of the member. At least one of these plates shall extend to the far edge of the farthest tie-plate, and the balance to the far edge of the nearest tie-plate, but not less han 6 ins. beyond the near edge of the farthest plate. **Forked Ends.**

54. Pins shall be long enough to insure a full bearing of all the parts connected upon the turned body of the pin. They shall be secured by chambered nuts or be provided with washers if solid nuts are used. The screw ends shall be long enough to admit of burring the threads. **Pins.**

55. Members packed on pins shall be held against lateral movement. **Bolts.**

56. Where members are connected by bolts, the turned body of these bolts shall be long enough to extend through the metal. A washer at least $\frac{1}{4}$ in. thick shall be used under the nut. Bolts shall not be used in place of rivets except by special permission. Heads and nuts shall be hexagonal.

57. Where splice plates are not in direct contact with the parts which they connect, rivets shall be used on each side of the joint in excess of the number theoretically required to the extent of one-third of the number for each intervening plate. **Indirect Splices.**

58. Rivets carrying stress and passing through fillers shall be increased 50 per cent in number; and the excess rivets, when possible, shall be outside of the connected member. **Fillers.**

59. Provision for expansion to the extent of $\frac{1}{8}$-in. for each 10 ft. shall be made for all bridge structures. Efficient means shall be provided to prevent excessive motion at any one point. **Expansion**

60. Spans of 80 ft. and over, resting on masonry, shall have turned rollers or rockers at one end; and those of less length shall be arranged to slide on smooth surfaces. These expansion bearings shall be designed to permit motion in one direction only. **Expansion Bearings.**

Fixed Bearings. 61. Fixed bearings shall be firmly anchored to the masonry.

Rollers. 62. Expansion rollers shall be not less than 6 ins. in diameter. They shall be coupled together with substantial side bars, which shall be so arranged that the rollers can be readily cleaned. Segmental rollers shall be geared to the upper and lower plates.

Bolsters. 63. Bolsters or shoes shall be so constructed that the load will be distributed over the entire bearing. Spans of 80 ft. or over shall have hinged bolsters at each end.

Wall Plates. 64. Wall plates may be cast or built up; and shall be so designed as to distribute the load uniformly over the entire bearing. They shall be secured against displacement.

Anchorage. 65. Anchor bolts for viaduct towers and similar structures shall be long enough to engage a mass of masonry the weight of which is at least one and one-half times the uplift.

Inclined Bearings. 66. Bridges on an inclined grade without pin-shoes shall have the sole plates beveled so that the masonry and expansion surfaces may be level.

Floor Systems

Floor-Beams. 67. Floor-beams shall preferably be square to the trusses or girders. They shall be riveted directly to the girders or trusses or may be placed on top of deck bridges.

Stringers. 68. Stringers shall preferably be riveted to the webs of all intermediate floor-beams by means of connection angles not less than ½ in. in thickness. Shelf angles or other supports provided to support the stringer during erection shall not be considered as carrying any of the reaction.

Stringer Frames. 69. Where end floor-beams cannot be used, stringers resting on masonry shall have cross frames near their ends. These frames shall be riveted to girders or truss shoes where practicable.

Bracing

Rigid Bracing. 70. Lateral, longitudinal, and transverse bracing in all structures shall be composed of rigid members.

Portals. 71. Through truss spans shall have riveted portal braces rigidly connected to the end posts and top chords. They shall be as deep as the clearance will allow.

Transverse Bracing. 72. Intermediate transverse frames shall be used at each panel of through spans having vertical truss members where the clearance will permit.

End Bracing. 73. Deck spans shall have transverse bracing at each end proportioned to carry the lateral load to the support.

Laterals. 74. The minimum-sized angle to be used in lateral bracing shall be

$3\frac{1}{2}$ by 3 by $\frac{3}{8}$ ins. Not less than three rivets through the end of the angles shall be used at the connection.

75. Lateral bracing shall be far enough below the flange to clear the ties.

76. The struts at the foot of viaduct towers shall be strong enough to slide the movable shoes when the track is unloaded. **Tower Struts.**

Plate Girders

77. If desired, plate-girder spans over 50 ft. in length shall be built with camber at a rate of $^1/_{16}$-in. per 10 ft. of length. **Camber.**

78. Where flange plates are used, one cover plate of top flange shall extend the whole length of the girder. **Top Flange Cover.**

79. There shall be web stiffeners, generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than $^1/_{60}$ of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a maximum limit of six feet (and not greater than the clear depth of the web): **Web Stiffeners.**

$$d = \frac{t}{40}\,(12{,}000 - s),$$

Where d = clear distance, between stiffeners of flange angles.
t = thickness of web.
s = shear per sq. in.

The stiffeners at ends and at points of concentrated loads shall be proportioned by the formula of paragraph 16, the effective length being assumed as one-half the depth of girders. End stiffeners and those under concentrated loads shall be on fillers and have their outstanding legs as wide as the flange angles will allow and shall fit tightly against them. Intermediate stiffeners may be offset or on fillers, and their outstanding legs shall be not less than one-thirtieth of the depth of girder plus 2 ins.

80. Through plate girders shall have their top flanges stayed at each end of every floor beam, or, in case of solid floors, at distances not exceeding 12 ft., by knee braces or gusset plates. **Stays for Top Flanges.**

Trusses

81. Truss spans shall be given a camber by so proportioning the length of the members that the stringers will be straight when the bridge is fully loaded. **Camber.**

82. Hip verticals and similar members, and the two end panels of the bottom chords of single track pin-connected trusses, shall be rigid. **Rigid Members.**

83. The eye-bars composing a member shall be so arranged that adjacent bars shall not have their surfaces in contact; they shall be as **Eye-Bars.**

nearly parallel to the axis of the truss as possible, the maximum inclination of any bar being limited to one inch in 16 ft.

Pony Trusses. 84. Pony trusses shall be riveted structures, with double webbed chords, and shall have all web members latticed or otherwise effectively stiffened.

PART SECOND—MATERIALS AND WORKMANSHIP

V. Material

Steel. 85. Steel shall be made by the open-hearth process.

Properties. 86. The chemical and physical properties shall conform to the following limits:

Elements Considered	Structural Steel	Rivet Steel	Steel Castings
Phosphorus, max. { Basic..	0.04 per cent	0.04 per cent	0.05 per cent
Phosphorus, max. { Acid..	0.06 " "	0.04 " "	0.08 " "
Sulphur, maximum	0.05 " "	0.04 " "	0.05 " "
Ultimate tensile strength Pounds per square inch....	Desired 60,000	Desired 50,000	Not less than 65,000
Elong., min. %, in. 8", Fig. 1	1,500,000* / Ult. ten. str'gth	1,500,000 / Ult. ten. str'gth	15 per cent
Elong., min. %, in 2", Fig. 2	22		
Character of Fracture.....	Silky	Silky	Silky or fine granular
Cold Bends without Frac..	180° flat†	180° flat‡	90° $d=3t$

* See paragraph 96. † See paragraph 97, 98 and 99. ‡ See paragraph 100.

The yield point, as indicated by the drop of beam, shall be recorded in the test reports.

87. In order that the ultimate strength of full-sized annealed eye-bars may meet the requirements of paragraph 163, the ultimate strength in test specimens may be determined by the manufacturers; all other tests than those for ultimate strength shall conform to the above requirements.

Allowable Variations. 88. If the ultimate strength varies more than 4,000 lbs. from that desired, a retest shall be made on the same gauge, which, to be acceptable, shall be within 5,000 lbs. of the desired ultimate.

Chemical Analyses. 89. Chemical determinations of the percentages of carbon, phosphorus, sulphur, and manganese shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt of steel, and a correct copy of such analysis shall be furnished to the engineer or his inspector. Check analyses shall be made from finished material, if called for by the purchaser, in which case an excess of 25 per cent above the required limits will be permitted.

90. Plate, shape, and bar specimens for tensile and bending tests shall be made by cutting coupons from the finished product, which shall have both faces rolled and both edges milled to the form shown by Fig. 1; or with both edges parallel; or they may be turned to a diameter of ¾ in. for a length of at least 9 ins., with enlarged ends. Specimens.

91. Rivet rods shall be tested as rolled.

92. Pin and roller specimens shall be cut from the finished rolled or forged bar, in such manner that the centre of the specimen shall be 1 in. from the surface of the bar. The specimen for tensile test shall be turned to the form shown by Fig. 2. The specimen for bending test shall be 1 in. by ½-in. in section.

93. For steel castings the number of tests will depend on the character and importance of the castings. Specimens shall be cut cold from coupons moulded and cast on some portion of one or more castings from each melt or from the sink heads, if the heads are of sufficient size. The coupon or sink head, so used, shall be annealed with the casting before it is cut off. Test specimens to be of the form prescribed for pins and rollers.

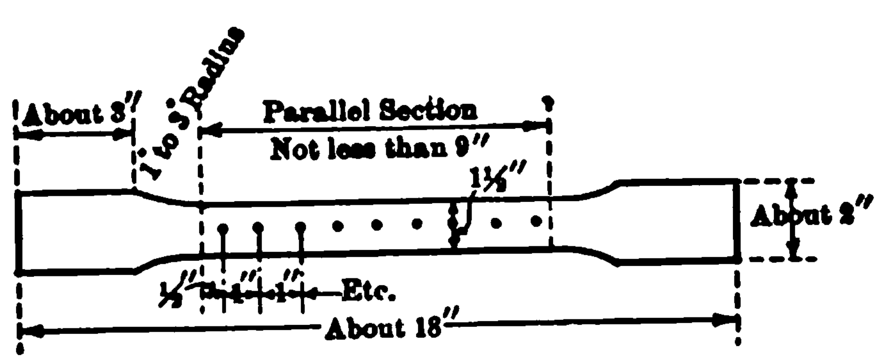


FIG. 1.

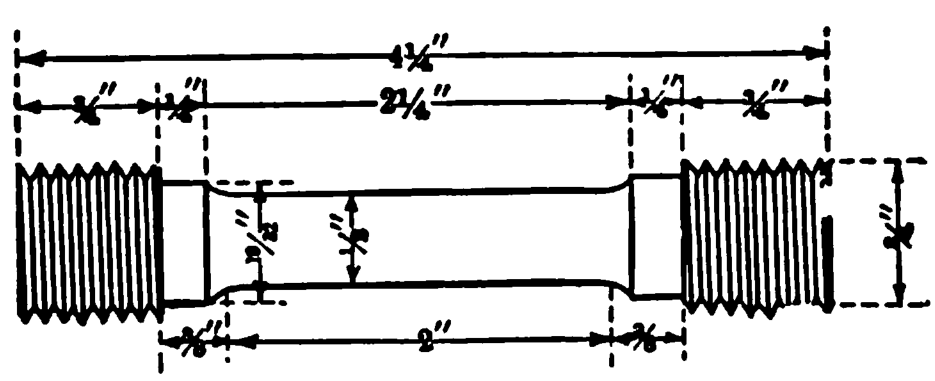


FIG. 2.

94. Rolled steel shall be tested in the condition in which it comes from the rolls. Specimens of Rolled Steel.

95. At least one tensile and one bending test shall be made from each melt of steel as rolled. In case steel differing ⅜ in. and more in thickness is rolled from one melt, a test shall be made from the thickest and thinnest material rolled. Number of Tests.

96. A deduction of 1 per cent. will be allowed from the specified percentage for elongation, for each ⅛ in. in thickness above ¾ in. Modification in Elongation.

97. Bending tests may be made by pressure or by blows. Plates, Bending Tests.

shapes, and bars less than 1 in. thick shall bend as called for in paragraph 86.

Thick Material. 98. Full-sized material for eye-bars and other steel 1 in. thick and over, tested as rolled, shall bend cold 180 degrees around a pin, the diameter of which is equal to twice the thickness of the bar, without fracture on the outside of bend.

Bending Angles. 99. Angles 3/4 in. and less in thickness shall open flat, and angles 1/2 in. and less in thickness shall bend shut, cold, under blows of a hammer, without sign of fracture. This test shall be made only when required by the inspector.

Nicked Bends. 100. Rivet steel, when nicked and bent around a bar of the same diameter as the rivet rod, shall give a gradual break and a fine silky uniform fracture.

Finish. 101. Finished material shall be free from injurious seams, flaws, cracks, defective edges, or other defects, and have a smooth, uniform, and workmanlike finish. Plates 36 ins. in width and under shall have rolled edges.

Melt Numbers. 102. Every finished piece of steel shall have the melt number and the name of the manufacturer stamped or rolled upon it. Steel for pins and rollers shall be stamped on the end. Rivet and lattice steel and other small parts may be bundled with the above marks on an attached metal tag.

Defective Material. 103. Material which, subsequent to the above tests at the mills, and its acceptance there, develops weak spots, brittleness, cracks, or other imperfections, or is found to have injurious defects, will be rejected at the shop and shall be replaced by the manufacturer at his own cost.

Variation in Weight. 104. A variation in cross-section or weight of each piece of steel of more than 2½ per cent from that specified will be sufficient cause for rejection, except in case of sheared plates, which will be covered by the following permissible variations, which are to apply to single plates, when ordered to weight:

105. Plates 12½ lbs. per sq. ft. or heavier:

(*a*) Up to 100 ins. wide, 2½ per cent above or below the prescribed weight.

(*b*) One hundred inches wide and over, 5 per cent above or below.

106. Plates under 12½ lbs. per sq. ft.:

(*a*) Up to 75 ins. wide, 2½ per cent above or below.

(*b*) Seventy-five inches and up to 100 ins. wide, 5 per cent above or 3 per cent below.

(*c*) One hundred inches wide and over, 10 per cent above or 3 per cent below.

107. Plates when ordered to gauge will be accepted if they measure not more than 0.01 in. below the ordered thickness.

108. An excess over the nominal weight, corresponding to the dimensions on the order, will be allowed for each plate, if not more than that shown in the following table, 1 cu. in. of rolled steel being assumed to weigh 0.2833 lb.:

Thickness Ordered	Nominal Weights	Width of Plate			
		Up to 75"	75" and Up to 100"	100" and Up to 115"	Over 115"
Ins.	Lbs.	%	%	%	%
1/4	10.20	10	14	18	..
5/16	12.75	8	12	16	..
3/8	15.30	7	10	13	17
7/16	17.85	6	8	10	13
1/2	20.40	5	7	9	12
9/16	22.95	4½	6½	8½	11
5/8	25.50	4	6	8	10
Over 5/8	.....	3½	5	6½	9

109. Except where chilled iron is specified, castings shall be made of tough gray iron, with sulphur not over 0.10 per cent. They shall be true to pattern, out of wind and free from flaws and excessive shrinkage. If tests are demanded, they shall be made on the "Arbitration Bar" of the American Society for Testing Materials, which is a round bar 1¼ ins. in diameter and 15 ins. long. The transverse test shall be made on a supported length of 12 ins. with load at middle. The minimum breaking load so applied shall be 2,900 lbs., with a deflection of at least $^1/_{10}$ in. before rupture. **Cast-Iron.**

110. Wrought-iron shall be double-rolled, tough, fibrous, and uniform in character. It shall be thoroughly welded in rolling and be free from surface defects. When tested in specimens of the form of Fig. 1, or in full-sized pieces of the same length, it shall show an ultimate strength of at least 50,000 lbs. per sq. in., an elongation of at least 18 per cent. in 8 ins., with fracture wholly fibrous. Specimens shall bend cold, with the fibre, through 135 degrees, without sign of fracture, around a pin the diameter of which is not over twice the thickness of the piece tested. When nicked and bent, the fracture shall show at least 90 per cent fibrous. **Wrought-Iron.**

VI. Inspection and Testing at the Mills

111. The purchaser shall be furnished complete copies of mill orders, and no material shall be rolled nor work done before the purchaser has been notified where the orders have been placed, so that he may arrange for the inspection. **Mill Orders.**

Facilities for Inspection. 112. The manufacturer shall furnish all facilities for inspecting and testing the weight and quality of all material at the mill where it is manufactured. He shall furnish a suitable testing machine for testing the specimens, as well as prepare the pieces for the machine, free of cost.

Access to Mills. 113. When an inspector is furnished by the purchaser to inspect material at the mills, he shall have full access, at all times, to all parts of mills where material to be inspected by him is being manufactured.

VII. Workmanship

General. 114. All parts forming a structure shall be built in accordance with approved drawings. The workmanship and finish shall be equal to the best practice in modern bridge works. Material arriving from the mills shall be protected from the weather and shall have clean surfaces before being worked in the shops.

Straightening. 115. Material shall be thoroughly straightened in the shop, by methods that will not injure it, before being laid off or worked in any way.

Finish. 116. Shearing and chipping shall be neatly and accurately done and all portions of the work exposed to view neatly finished.

Size of Rivets. 117. The size of rivets called for on the plans shall be understood to mean the actual size of the cold rivet before heating.

Rivet Holes. 118. When general reaming is not required, the diameter of the punch shall not be more than $^1/_{16}$ in. greater than the diameter of the rivet; nor the diameter of the die more than $\frac{1}{8}$ in. greater than the diameter of the punch. Material more than $\frac{3}{4}$ in. thick shall be subpunched and reamed or drilled from the solid.

Punching. 119. Punching shall be accurately done. Drifting to enlarge unfair holes will not be allowed. If the holes must be enlarged to admit the rivet, they shall be reamed. Poor matching of holes will be cause for rejection.

Reaming. 120. Where subpunching and reaming are required, the punch used shall have a diameter not less than $^3/_{16}$ in. smaller than the nominal diameter of the rivet. Holes shall then be reamed to a diameter not more than $^1/_{16}$ in. larger than the nominal diameter of the rivet. (See 135.)

Reaming after Assembling. 121. *[When general reaming is required it shall be done after the pieces forming one built member are assembled and so firmly bolted together that the surfaces shall be in close contact. If necessary to take the pieces apart for shipping and handling, the respective pieces reamed together shall be so marked that they may be re-

* See Addendum, clause (*d*).

assembled in the same position in the final setting up. No interchange of reamed parts will be permitted.]

122. Reaming shall be done with twist drills and without using any lubricant.

123. The outside burrs on reamed holes shall be removed to the extent of making a $^1/_{16}$ in. fillet.

124. Riveted members shall have all parts well pinned up and firmly drawn together with bolts, before riveting is commenced. Contact surfaces to be painted. (See 152.) **Assembling.**

125. Lattice bars shall have neatly rounded ends, unless otherwise called for. **Lattice Bars.**

126. Stiffeners shall fit neatly between flanges of girders. Where tight fits are called for, the ends of the stiffeners shall be faced and shall be brought to a true contact bearing with the flange angles. **Web Stiffeners.**

127. Web splice plates and fillers under stiffeners shall be cut to fit within ⅛ in. of flange angles. **Splice Plate and Fillers.**

128. Web plates of girders, which have no cover plates, shall be flush with the backs of angles or project above the same not more than ⅛ in., unless otherwise called for. When web plates are spliced, not more than ¼-in. clearance between ends of plates will be allowed. **Web Plates.**

129. The main sections of floor-beams and stringers shall be milled to exact length after riveting and the connection angles accurately set flush and true to the milled ends* [or if required by the purchaser, the milling shall be done after the connection angles are riveted in place, milling to extend over the entire face of the member]. The removal of more than $^3/_{32}$ in. from the thickness of the connection angles will be cause for rejection. **Floor-Beams and Stringers.**

130. Rivets shall be uniformly heated to a light cherry red heat in a gas or oil furnace so constructed that it can be adjusted to the proper temperature. They shall be driven by pressure tools wherever possible. Pneumatic hammers shall be used in preference to hand driving. **Riveting.**

131. Rivets shall look neat and finished, with heads of approved shape, full, and of equal size. They shall be central on shank and grip the assembled pieces firmly. Recupping and calking will not be allowed. Loose, burned, or otherwise defective rivets shall be cut out and replaced. In cutting out rivets, great care shall be taken not to injure the adjacent metal. If necessary, they shall be drilled out.

132. Wherever bolts are used in place of rivets which transmit shear, the holes shall be reamed parallel and the bolts shall make a **Turned Bolts.**

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* See Addendum, clause (*f*).

driving fit, with the threads entirely outside of the holes. A washer not less than ¼ in. thick shall be used under nut.

Members to be Straight.

133. The several pieces forming one built member shall be straight and fit closely together, and finished members shall be free from twists, bends, or open joints.

Finish of Joints.

134. Abutting joints shall be cut or dressed true and straight and fitted close together, especially where open to view. In compression joints, depending on contact bearing, the surfaces shall be truly faced, so as to have even bearings after they are riveted up complete and when perfectly aligned.

Field Connections.

135. Holes for floor-beam and stringer connections shall be subpunched and reamed according to paragraph 120, to a steel templet not less than one inch thick.* [If required, all other field connections, except those for laterals and sway bracing, shall be assembled in the shop and the unfair holes reamed; and when so reamed the pieces shall be match-marked before being taken apart.]

Eye-Bars.

136. Eye-bars shall be straight and true to size, and shall be free from twists, folds in the neck or head, or any other defect. Heads shall be made by upsetting, rolling, or forging. Welding will not be allowed. The form of heads will be determined by the dies in use at the works where the eye-bars are made, if satisfactory to the engineer, but the manufacturer shall guarantee the bars to break in the body when tested to rupture. The thickness of head and neck shall not vary more than $^1/_{16}$ in. from that specified. (See 163.)

Boring Eye-Bars.

137. Before boring, each eye-bar shall be properly annealed and carefully straightened. Pin-holes shall be in the centre line of bars and in the centre of heads. Bars of the same length shall be bored so accurately that, when placed together, pins $^1/_{32}$ in. smaller in diameter than the pin-holes can be passed through the holes at both ends of the bars at the same time without forcing.

Pin-Holes.

138. Pin-holes shall be bored true to gauges, smooth and straight; at right angles to the axis of the member and parallel to each other, unless otherwise called for. The boring shall be done after the member is riveted up.

139. The distance centre to centre of pin-holes shall be correct within $^1/_{32}$ in., and the diameter of the holes not more than $^1/_{50}$ in. larger than that of the pin, for pins up to 5 in. diameter, and $^1/_{32}$ in. for larger pins.

Pins and Rollers.

140. Pins and rollers shall be accurately turned to gauges and shall be straight and smooth and entirely free from flaws.

Screw Threads.

141. Screw threads shall make tight fits in the nuts and shall be

* See Addendum, clause (*e*).

U. S. standard, except above the diameter of 1⅜ in., when they shall be made with six threads per inch.

142. Steel, except in minor details, which has been partially heated, shall be properly annealed. Annealing.

143. Steel castings shall be free from large or injurious blow-holes and shall be annealed. Steel Castings.

144. Welds in steel will not be allowed. Welds.

145. Expansion bed plates shall be planed true and smooth. Cast wall plates shall be planed top and bottom. The finishing cut of the planing tool shall be fine and correspond with the direction of expansion. Bed Plates.

146. Pilot and driving nuts shall be furnished for each size of pin, in such numbers as may be ordered. Pilot Nuts.

147. Field rivets shall be furnished to the amount of 15 per cent, plus ten rivets in excess of the nominal number required for each size. Field Rivets.

148. Pins, nuts, bolts, rivets and other small details shall be boxed or crated. Shipping Details.

149. The scale weight of every piece and box shall be marked on it in plain figures. Weight.

150. Payment for pound price contracts shall be by scale weight. No allowance over 2 per cent of the total weight of the structure as computed from the plans will be allowed for excess weight. Finished Weight.

VIII. Shop Painting

151.* Steel work, before leaving the shop, shall be thoroughly cleaned and given one good coating of pure linseed oil, or such paint as may be called for, well worked into all joints and open spaces. Cleaning.

152. In riveted work, the surfaces coming in contact shall each be painted before being riveted together. Contact Surfaces.

153. Pieces and parts which are not accessible for painting after erection, including tops of stringers, eye-bar heads, ends of posts and chords, etc., shall have an additional coat of paint before leaving the shop. Inaccessible Surfaces.

154. Painting shall be done only when the surface of the metal is perfectly dry. It shall not be done in wet or freezing weather, unless protected under cover. Condition of Surfaces.

155. Machine-finished surfaces shall be coated with white lead and tallow before shipment or before being put out into the open air. Machine-Finished Surfaces.

IX. Inspection and Testing at the Shops

156. The manufacturer shall furnish all facilities for inspecting and testing the weight and quality of workmanship at the shop Facilities for Inspection.

* See Addendum, clause (*b*).

where material is manufactured. He shall furnish a suitable testing machine for testing full-sized members, if required.

Starting Work. 157. The purchaser shall be notified well in advance of the start of the work in the shop, in order that he may have an inspector on hand to inspect material and workmanship.

Access to Shop. 158. When an inspector is furnished by the purchaser, he shall have full access, at all times, to all parts of the shop where material under his inspection is being manufactured.

Accepting Material. 159. The inspector shall stamp each piece accepted with a private mark. Any piece not so marked may be rejected at any time and at any stage of the work. If the inspector, through an oversight or otherwise, has accepted material or work which is defective or contrary to the specifications, this material, no matter in what stage of completion, may be rejected by the purchaser.

Shop Plans. 160. The purchaser shall be furnished complete shop plans.

Shipping Invoices. 161. Complete copies of shipping invoices shall be furnished to the purchaser with each shipment. These shall show the scale weights of individual pieces.

X. Full-Sized Tests

Eye-Bar Tests. 162. Full-sized tests on eye-bars and similar members, to prove the workmanship, shall be made at the manufacturer's expense, and shall be paid for by the purchaser at contract price, if the tests are satisfactory. If the tests are not satisfactory, the members represented by them will be rejected.

163. In eye-bar tests, the minimum ultimate strength shall be 55,000 lbs. per sq. in. The elongation in 10 ft., including fracture, shall be not less than 15 per cent. Bars shall generally break in the body and the fracture shall be silky or fine granular, and the elastic limit as indicated by the drop of the mercury shall be recorded. Should a bar break in the head and develop the specified elongation, ultimate strength, and character of fracture, it shall not be cause for rejection, provided not more than one-third of the total number of bars break in the head. (See 136.)

ADDENDUM TO GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

POINTS TO BE SPECIFICALLY DETERMINED BY BUYERS WHEN SOLICITING PROPOSALS FOR STEEL RAILWAY BRIDGES

When general detail drawings are not furnished for the use of bidders specific answers should be given to questions *a*, *b*, and *c*, below.

Specific answers should also be given to questions *d*, *e*, and *f* if the

class of work described in any of the paragraphs there referred to is desired. If these features are not specifically demanded, the unbracketed paragraphs will be construed to define the kind of work desired.

(*a*) What class of live load shall be used? (Pars. 7 and 8.)

(*b*) Shall linseed oil or paint be used? If paint, what kind? (Par. 151.)

(*c*) Shall contractor furnish floor bolts?

(*d*) Shall general reaming be done? (Par. 121.)

(*e*) Shall field connections be assembled at the shop? (Par. 135.)

(*f*) Shall floor connection angles be milled after riveting? (Par. 129.)

APPENDIX B

Tables and Standards

TABLE I.—Shearing and Bearing Values of Rivets

Values above or to the right of upper zigzag lines are greater than double shear.
Values below or to the left of lower zigzag lines are less than single shear.

Diam. of Rivet	Area in Square Inches	Single Shear at 10,000	Bearing Values for Different Thicknesses of Plate at 20,000 Lbs. per Square Inch												
			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	1,100	1,880	2,340	2,810	3,280									
1/2	.1963	1,960	2,500	3,130	3,750	4,380	5,000								
5/8	.3068	3,070	3,130	3,910	4,690	5,470	6,250	7,030	7,810						
3/4	.4418	4,420	3,750	4,690	5,630	6,560	7,500	8,440	9,380	10,310	11,250				
7/8	.6013	6,010	4,380	5,470	6,560	7,660	8,750	9,840	10,940	12,030	13,130	14,220	15,310	16,410	
1	.7854	7,850	5,000	6,250	7,500	8,750	10,000	11,250	12,500	13,750	15,000	16,250	17,500	18,750	20,000

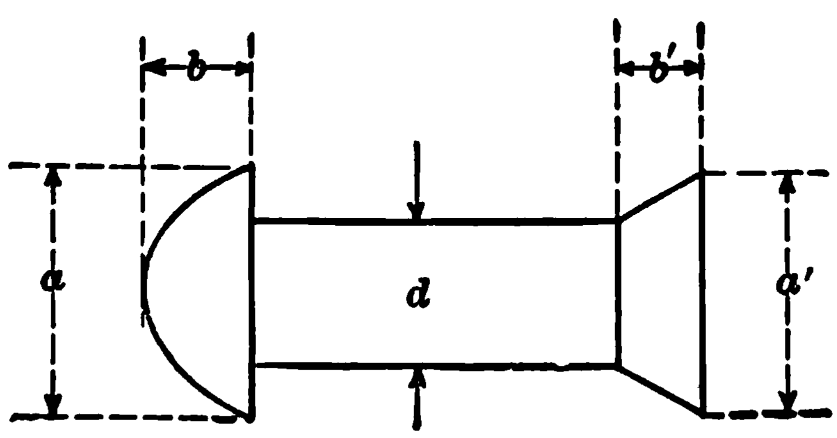
Diam. of Rivet	Area in Square Inches	Single Shear at 11,000	Bearing Values for Different Thicknesses of Plate at 22,000 Lbs. per Square Inch												
			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	1,210	2,070	2,570	3,090	3,610									
1/2	.1963	2,160	2,750	3,440	4,130	4,820	5,500								
5/8	.3068	3,370	3,440	4,300	5,160	6,020	6,880	7,740	8,600						
3/4	.4418	4,860	4,130	5,160	6,190	7,220	8,250	9,280	10,320	11,340	12,380				
7/8	.6013	6,610	4,810	6,020	7,220	8,430	9,630	10,840	12,040	13,240	14,440	15,640	16,840	18,050	
1	.7854	8,640	5,500	6,880	8,250	9,630	11,000	12,380	13,750	15,130	16,500	17,880	19,250	20,630	22,000

Diam. of Rivet	Area in Square Inches	Single Shear at 12,000	Bearing Values for Different Thicknesses of Plate at 24,000 Lbs. per Square Inch												
			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	1,320	2,250	2,810	3,380	3,940									
1/2	.1963	2,360	3,000	3,750	4,500	5,250	6,000								
5/8	.3068	3,680	3,750	4,690	5,630	6,560	7,500	8,440	9,380						
3/4	.4418	5,300	4,500	5,630	6,750	7,880	9,000	10,130	11,250	12,380	13,500				
7/8	.6013	7,220	5,250	6,560	7,880	9,190	10,500	11,810	13,130	14,440	15,750	17,060	18,380		
1	.7854	9,430	6,000	7,500	9,000	10,500	12,000	13,500	15,000	16,500	18,000	19,500	21,000	22,500	24,000

TABLE II

Rivet Heads and Clearances

Diameter of Rivet	Full Driven Head		Countersunk		Clearance for Riveting c	
	Diameter	Height	Diameter	Depth		
d	a	b	a'	b'	Standard	Minimum
$\frac{5}{8}$	$1\frac{1}{16}$	$\frac{29}{64}$	1	$\frac{5}{16}$	$1\frac{1}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{17}{32}$	$1\frac{3}{16}$	$\frac{3}{8}$	$1\frac{1}{4}$	1
$\frac{7}{8}$	$1\frac{7}{16}$	$\frac{39}{64}$	$1\frac{9}{16}$	$\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{8}$
1	$1\frac{5}{8}$	$\frac{11}{16}$	$1\frac{3}{4}$	$\frac{9}{16}$	$1\frac{1}{2}$	$1\frac{1}{4}$



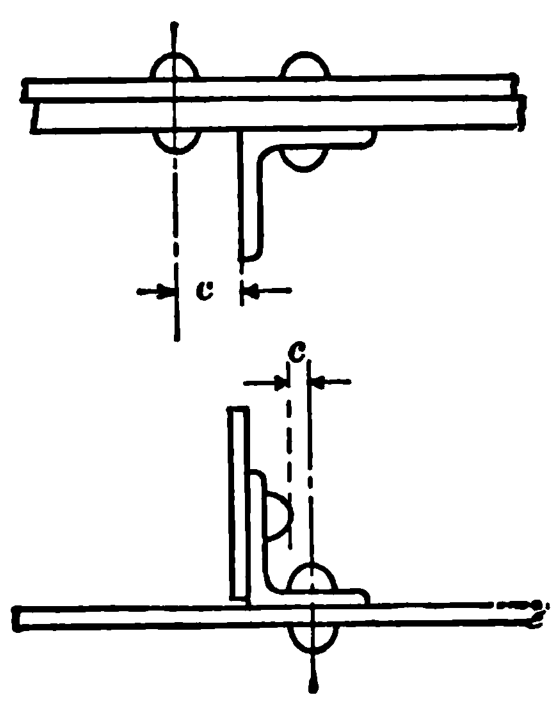
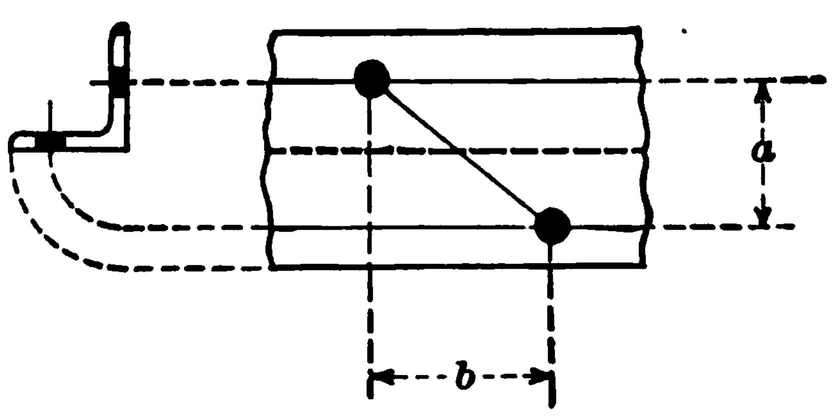


TABLE III

Rivet Stagger to Maintain Net Section

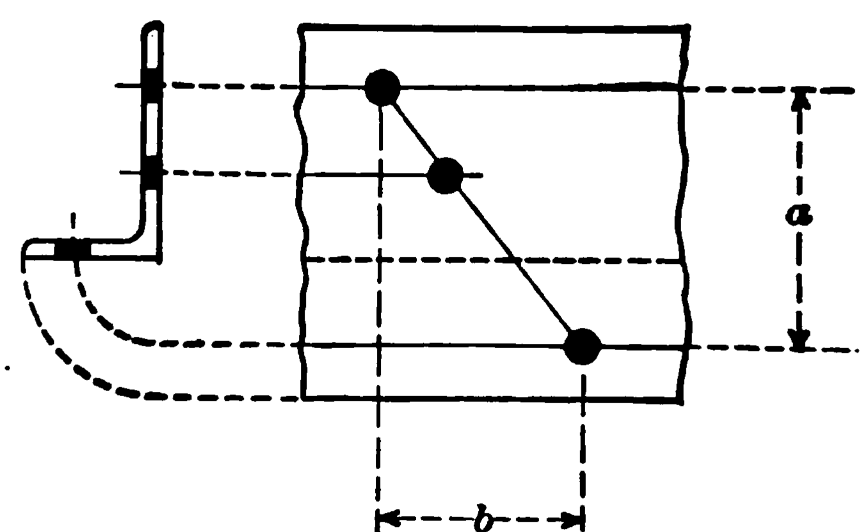
Sum of Gauges Less Thickness of Angle	¾″ Rivet	⅞″ Rivet	Sum of Gauges Less Thickness of Angle	¾″ Rivet	⅞″ Rivet
a	b	b	a	b	b
1	$1\frac{5}{8}$	$1\frac{3}{4}$	5	$3\frac{1}{16}$	$3\frac{5}{16}$
$1\frac{1}{2}$	$1\frac{7}{8}$	2	$5\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{2}$
2	$2\frac{1}{16}$	$2\frac{1}{4}$	6	$3\frac{3}{8}$	$3\frac{5}{8}$
$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{7}{16}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{3}{4}$
3	$2\frac{7}{16}$	$2\frac{5}{8}$	7	$3\frac{5}{8}$	$3\frac{7}{8}$
$3\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{13}{16}$	$7\frac{1}{2}$	$3\frac{3}{4}$	4
4	$2\frac{13}{16}$	3	8	$3\frac{7}{8}$	$4\frac{1}{8}$
$4\frac{1}{2}$	$2\frac{15}{16}$	$3\frac{3}{16}$	$8\frac{1}{2}$	4	$4\frac{1}{4}$



d = Diam. of rivet hole

One Hole Out

$b = \sqrt{2ad + d^2}$

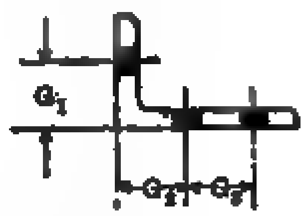


Two Holes Out

$b = \sqrt{2ad + d^2}$

TABLE IV

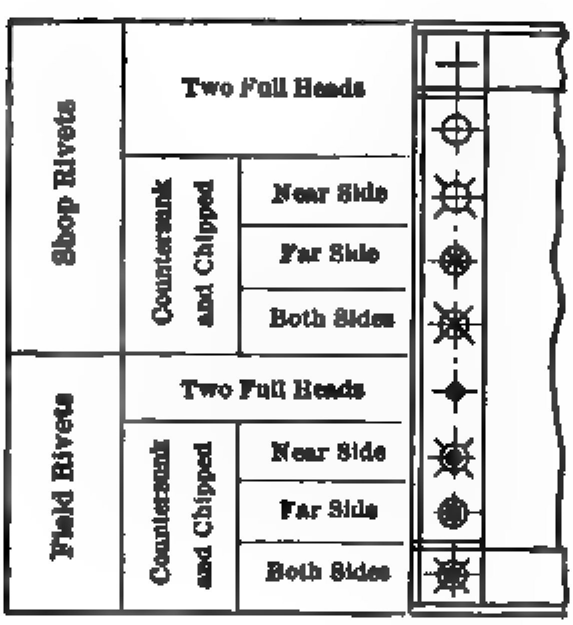
Standard Gauge Lines for Angles



Leg	8″	7″	6″	5″	4″	3½″	3″	2½″	2″
G_1	4½	4	3½	3	2½	2	1¾	1⅜	1⅛
G_2	[illegible]	2½	2¼	2	...	...	...	...	...
G_3	3	3	2½	1¾	...	...	...	...	...
Maximum Rivet	1⅛	1	⅞	⅞	[illegible]	⅞	⅞	[illegible]	⅝

TABLE V

Conventional Signs for Riveting



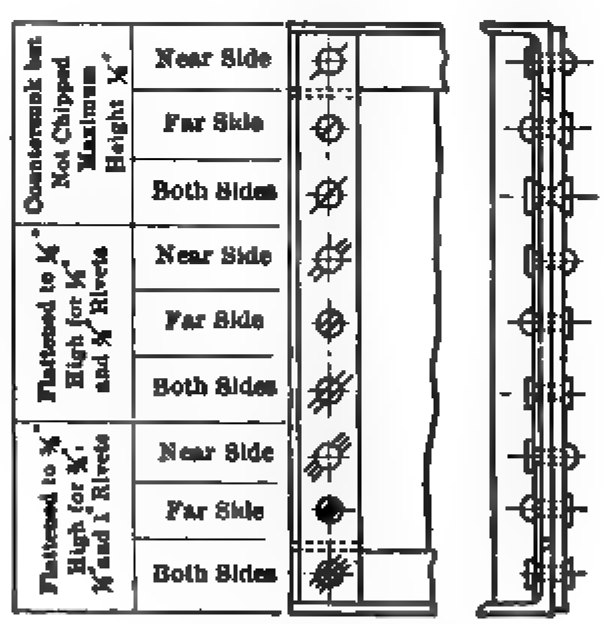
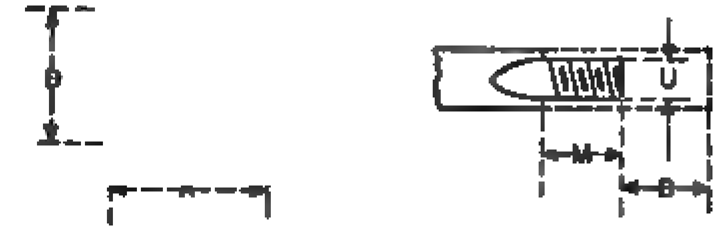


TABLE VI

Standard Eye-Bars

American Bridge Company

ORDINARY EYE-BAR | **ADJUSTABLE EYE-BAR**



Minimum length of short end from centre of pin to end of screw, 6′6″, preferably 7′0″; thread on short end to be left hand.

Bar: Width	Bar: Thickness Max.	Bar: Thickness Min.	Head: Diam. D	Head: Maximum Pin Diam.	Head: Maximum Pin Excess Head Over Bar Per Cent.	Head: Additional Material A For Ordering Bar	Head: Additional Material A For Figuring Weight	Bar: Width	Bar: Min. Thickness	Screw End: Diam. U	Screw End: Excess Upset Over Bar Per Cent.	Screw End: Length M	Screw End: Add. Material B For Ordering Bar	Screw End: Add. Material B For Figuring Weight
2 ″	1 ″	½″	4½″	1¾″	37.5	1′ 0″	0′ 7″	2 ″	* ⅝″	1¾″	47.8	4 ″	1′ 0′	8 ″
			5½″	2¼″		1′ 4″	0′11″		¾″	1⅞″	44.0	4½″	1′ 0″	7½″
			* 6½″	3¾″		1′ 9″	1′ 4″		⅞″	2 ″	42.8	4½″	0′11″	7½″
2½″	1 ″	⅝″	6 ″	2½″	40.0	1′ 3″	0′10″	2½″	* ¾″	2⅛″	52.6	4½″	1′ 0″	8 ″
			7 ″	3½″		1′ 7″	1′ 2″		⅞″	2¼″	48.4	5 ″	1′ 0″	8 ″
			* 8 ″	4½″		2′ 0″	1′ 7″		1 ″	2⅜″	46.4	5 ″	1′ 0″	7½″
3 ″	1½″	⅝″	7½″	3¼″	41.7	1′ 6″	1′ 1″	3 ″	* ¾″	2¼″	44.4	5 ″	1′ 0″	7½″
			8½″	4¼″		1′11″	1′ 5″		⅞″	2¼″	41 7	5 ″	0′10″	7 ″
			* 9½″	5¼″		2′ 4″	1′10″		1 ″	2½″	36.5	5½″	1′ 1″	8½″

4 "	1¾"	¾" ⅞" 1 "	10 " 11 " *12 "	4½" 5½" 6½"	37.5	1′11″ 2′ 3″ 2′ 8″	1′ 6″ 1′10″ 2′ 2″
5 "	2 "	¾" 1 " 1 "	12 " 13½" *15 "	5¼" 6¾" 8¼"	35.0	2′ 1″ 2′ 8″ 3′ 3″	1′ 8″ 2′ 2″ 2′ 9″
6 "	2 "	¾" 1 " 1 "	14 " 14¾" *16½"	5¾" 6½" 8¼"	37.5	2′ 4″ 2′ 6″ 3′ 2″	1′10″ 2′ 1″ 2′ 8″
7 "	2 "	1 " 1⅛" 1⅛"	16½" 17½" *18½"	7 " 8 " 9 "	35.7	2′ 7″ 2′11″ 3′ 4″	2′ 2″ 2′ 6″ 2′11″
8 "	2 "	1 " 1⅛" 1¼"	18 " 19 " *20 "	7 " 8 " 9 "	37.5	2′ 8″ 3′ 0″ 3′ 4″	2′ 3″ 2′ 6″ 2′11″
9 "	2 "	1⅛" 1¼"	20 " 22 "	7½" 9½"	38.9	2′11″ 3′ 7″	2′ 6″ 3′ 1″
10 "	2 "	1⅛" 1¼" 1⅜"	22½" 24 " *25 "	9 " 10½" 11½"	35.0	3′ 5″ 3′ 9″ 4′ 1″	2′10″ 3′ 3″ 3′ 7″
12 "	2 "	1¼" 1⅜" 1½"	26½" 28 " *29½"	10 " 11½" 13 "	37.5	3′ 8″ 4′ 2″ 4′ 8″	3′ 3″ 3′ 8″ 4′ 1″
14 "	2 "	1⅜" 1½" 1⅝"	31 " 33 " *34 "	12 " 14 " 15 "	35.7	4′ 3″ 4′10″ 5′ 5″	3′ 9″ 4′ 4″ 4′ 8″
16 "	2 "	1¾" 1⅞"	36 " *37½"	14 " 16 "	37.5 34.4	4′11″ 5′ 5″	4′ 5″ 4′10″

4 "	* ¾" ⅞" 1 " 1⅛"	2½" 2¾" 3 " 3¼"	36.5 44.0 52.1 60.6	5½" 5½" 6 " 6½"	1′ 1″ 0′11″ 1′ 1″ 1′ 2″	8½" 7½" 8½" 9½"
5 "	* ¾" ⅞" 1 " 1⅛" 1¼"	2⅞" 3 " 3¼" 3½" 3¾"	48.0 39.1 44.5 50.5 56.9	6 " 6 " 6½" 7 " 7 "	1′ 0″ 0′11″ 1′ 0″ 1′ 1″ 1′ 2″	8 " 7 " 8 " 8½" 9 "
6 "	*1 " 1⅛" 1¼" 1⅜"	3½" 3¾" 4 " 4¼"	41.1 45.3 50.0 54.9	7 " 7 " 7½" 8 "	1′ 0″ 1′ 0″ 1′ 1″ 1′ 2″	7½" 8 " 8½" 9½"
7 "	*1⅛" 1¼" 1⅜" 1½"	4 " 4¼" 4½" 4¾"	42.8 46.1 49.8 53.8	7½" 8 " 8½" 8½"	1′ 0″ 1′ 1″ 1′ 2″ 1′ 2″	8 " 8½" 9 " 9½"
8 "	*1⅛" 1¼" 1⅜" 1½" 1⅝"	4¼" 4½" 4¾" 5 " 5¼"	42.0 44.1 46.8 49.8 53.1	8 " 8½" 8½" 9 " 9½"	1′ 0″ 1′ 1″ 1′ 1″ 1′ 2″ 1′ 3″	8 " 8½" 8½" 9 " 10 "

Bars marked * should only be used when absolutely unavoidable.
Deduct pin-hole when figuring weight.

APPENDIX C

UNSYMMETRICAL BENDING

1. Unsymmetrical Bending.—The theory of bending, as treated in elementary works on the Mechanics of Materials, considers only cases for which the plane of bending moment coincides with one of the principal axes of the section and the neutral axis with the other. This results in the usual formula, $f = Mc/I$.

When rolled steel sections or wooden beams are used as purlins in roof construction, they are generally placed with their webs or sides perpendicular to the top chord of the roof truss. Fig. 1 shows several typical cases. Figs. (*a*), (*b*), and (*c*) show sections whose principal axes are parallel to the sides of the section. For the position shown, vertical loading causes a bending moment in a plane which is not parallel to the principal axes of the section. Fig. (*d*) shows an angle section, with one leg vertical, supporting a vertical load. As the principal axes are not parallel to the sides of the section, the plane of bending does not coincide with the principal axes of the section. Bending of the general nature shown in Fig. 1 is known as *Unsymmetrical Bending*. The usual formulas cannot

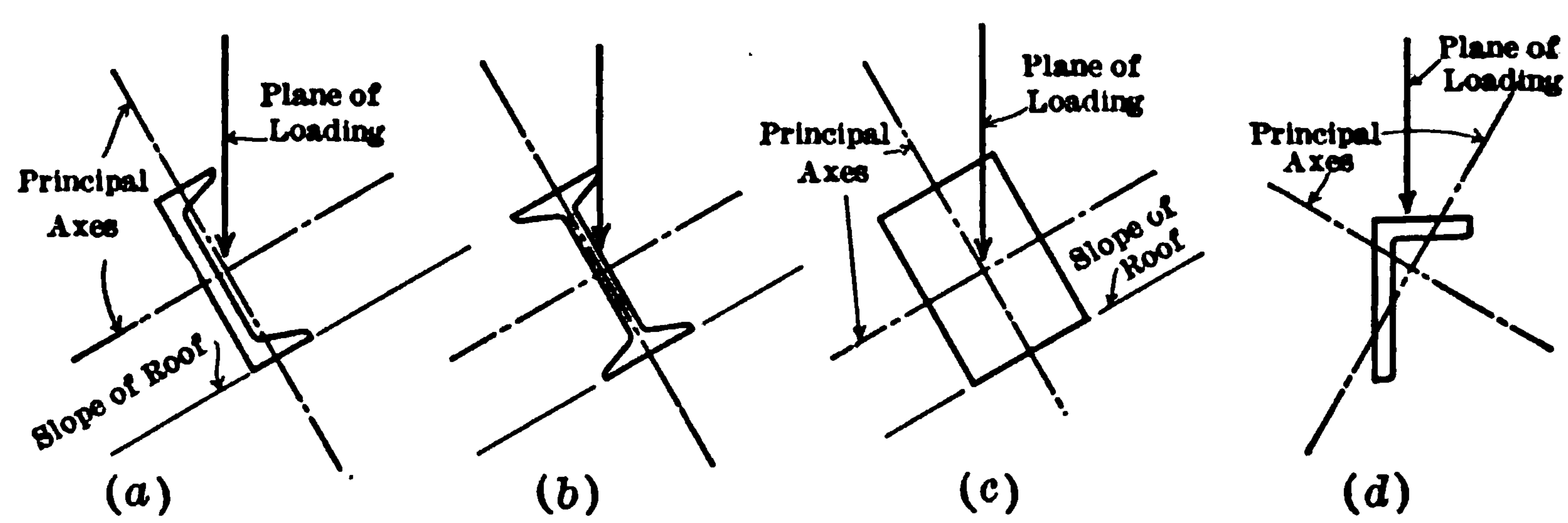


FIG. 1.

be applied to such cases; they require the use of formulas of a more general nature.

A very general and complete treatment of the subject of unsymmetrical bending has been given by Prof. L. J. Johnson, in Transactions American Society of Civil Engineers, Vol. 56, 1906.

In the following articles will be given a brief treatment of the subject of unsymmetrical bending. This treatment will be confined to cases of

pure bending only. In order to simplify the formulas for fibre stress and to reduce the work of calculation, all properties of the sections of beams will be referred to the principal axes, instead of the gravity axes, as in the case of the more general treatment mentioned above.

2. General Formulas for Fibre Stress and Position of Neutral Axis for Unsymmetrical Bending Moment.—Let the curved outline $B\,C$, of Fig. 2 (*a*), represent a right section of any straight beam of uniform cross-section, and let $O\,X$ and $O\,Y$ be the principal axes of this section. The beam is subjected to an unsymmetrical bending moment of amount M acting in a plane which passes through the longitudinal axis

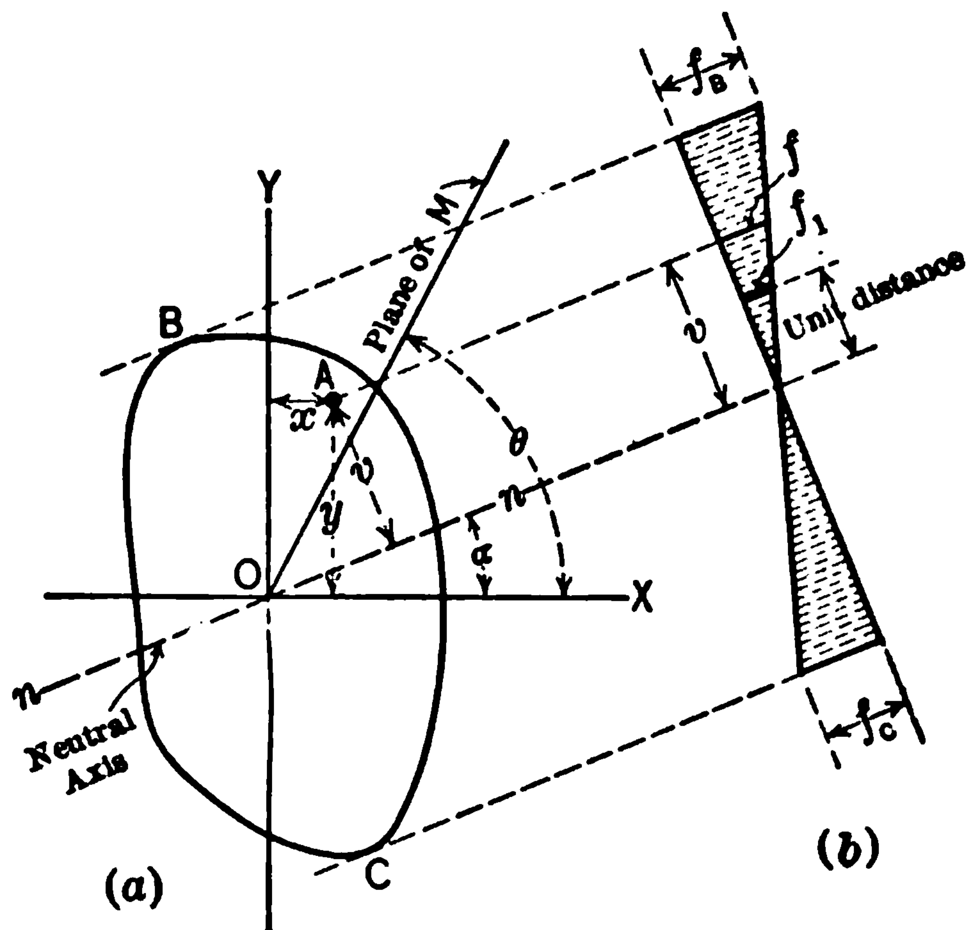


FIG. 2.

of the beam, and whose trace on the cross-section of the beam makes an angle θ with the axis $O\,X$. For this plane of bending moment, let $n\;n$ represent the position and direction of the neutral axis. Angles θ and α will be considered as positive when measured in a counter-clockwise direction from the axis $O\,X$.

Assuming linear variation of stress, Fig 2 (*b*) shows the stress conditions on a section perpendicular to the neutral axis. If f_1 be the intensity of fibre stress at unit distance from the neutral axis, the intensity of the fibre stress at point A at a distance v from the neutral axis is

$$f = -f_1 v \qquad (1)$$

Assuming positive bending moment, the fibre stresses above the neutral axis are compressive, and those below the neutral axis are tensile. Using a minus sign for compressive, and a plus sign for tensile stresses, the conditions shown for point A, of Fig. 2 (*a*), require a minus sign for eq. (1)

Referring point A to axes $O\,X$ and $O\,Y$ by substituting for v its value $(y \cos \alpha - x \sin \alpha)$, eq. (1) becomes

$$f = -f_1\,(y \cos \alpha - x \sin \alpha) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In eq. (2), values of x and y are considered positive when measured to the right and upward in Fig 2 (a).

The expression for fibre stress of eq. (2) contains two unknowns, f_1 and α. These unknowns can be determined from the principles of equilibrium, which require that the summation of moments of internal and external forces about any axis be equal to zero. As the external forces form a concurrent system, two independent equations can be written from which the unknowns can be determined. Convenient axes for summation of moments are offered by the principal axes of the section, as in the handbooks the properties of rolled sections in general use are given for these axes.

If $d\,a$ be the area of a fibre at point A, Fig. 2 (a), its fibre stress is $f\,d\,a$, and its moment about the axis $O\,X$ is $f\,y\,d\,a$. For the entire cross-section, the moment of internal forces is given by $\int_C^B f\,y\,d\,a$. The moment of external forces about axis $O\,X$ is given by $M \sin \theta$. Equating these values of internal and external moments, substituting for f its value from eq. (2) and expanding gives

$$M \sin \theta = f_1 \left(\int_C^B y^2 \cos \alpha \, d\,a - \int_C^B x\,y \sin \alpha \, d\,a \right).$$

In this expression $\int_C^B y^2\,d\,a = I_x$, the moment of inertia of the section about the axis $O\,X$, and $\int_C^B x\,y\,d\,a$ is the product of inertia of the section, which for principal axes is zero. The above expression then becomes,

$$M \sin \theta = +f_1\,I_x \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the same way, taking moments about the axis $O\,Y$ gives

$$M \cos \theta = -f_1\,I_y \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Solving eq. (3) for $\cos \alpha$ and eq. (4) for $\sin \alpha$, and substituting these values in eq. (2), gives

$$f = -\left(\frac{My \sin \theta}{I_x} + \frac{Mx \cos \theta}{I_y} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Eq. (5) can also be written,

$$f = -M \left(\frac{I_y\,y \sin \theta + I_x\,x \cos \theta}{I_x\,I_y} \right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is the desired value of fibre stress for unsymmetrical bending.

The direction of the neutral axis can be obtained by dividing eq. (4) by eq. (3) and solving for α, from which is obtained,

$$\tan \alpha = - \frac{I_x}{I_y} \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This expression can also be obtained by placing $f = 0$ in eq. (6), thus stating the conditions existing at the neutral axis. Solving the resulting expression for y gives,

$$y = - \frac{I_x}{I_y} x \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Eq. (8) can be reduced to the form of eq. (7) by noting that points of zero stress lie on the neutral axis, for which $y/x = \tan \alpha$.

Eq. (6) is a general expression for fibre stress for any fibre of a beam under an unsymmetrical bending moment, when the properties of the section are referred to the principal axes. Since for any point of a given section, x, y, I_x and I_y are constants, it can be seen from eq. (6) that the fibre stress varies with θ, the angle which the plane of bending makes with the axis $O X$. In the same way, eq. (7) shows that the direction of the neutral axis also depends upon θ. When $\theta = 0$, $f = - Mx / I_y$, and $\tan \alpha$ = infinite, or $\alpha = 90°$. This is the form of the equation for fibre stress as given in the elementary works on the Mechanics of Materials. For this case, the plane of bending coincides with the principal axis $O X$, and the neutral axis coincides with $O Y$, the other principal axis. Likewise, when $\theta = 90°$, $f = - My/I_x$ and $\alpha = 0$; that is, the plane of bending coincides with axis $O Y$, and the neutral axis coincides with axis $O X$.

In eq. (5), the expression for fibre stress is given as the sum of two terms, which, from the substitutions made above, can be seen to be equal to those obtained by resolving the moment M into its components parallel respectively to the axes $O X$ and $O Y$. Since these components act along the principal axis of the section, they can be treated separately by the usual formula, $f = Mc/I$, and the resulting fibre stresses added, thus giving an expression exactly the same as that of eq. (6). This provides a basis for a convenient and easily remembered method for the solution of occasional problems in unsymmetrical bending. It was necessary, however, to carry through the rather long derivation given above before the correctness of the shorter method could be established.

3. Solution of Problems in Unsymmetrical Bending. *Algebraic Method.* The complete solution of any problem in unsymmetrical bending requires the determination of the maximum intensity of fibre stress on the extreme fibres of the section under consideration. Two general methods of procedure are possible. By the first method, substitution is made in eq. (6) for each extreme fibre, or corner, of the section. On comparing the several values thus obtained, the fibre with the maximum stress intensity can be located. The second and better method is to determine,

first, from eq. (7) or (8) the position of the neutral axis. A true scale drawing is then made of the section and the neutral axis located. From this drawing can be determined by inspection, or by scaling if necessary, the fibre most remote from the neutral axis. Substituting in eq. (6) the co-ordinates of the extreme fibre thus determined gives at once the desired maximum stress intensity.

The application of eqs. (6) and (8) will be illustrated by the solution of two problems.

EXAMPLE 1. A 12-inch 40-pound channel section, placed as shown in Fig. 3, is used as a roof purlin to support a bending moment M acting in a vertical plane. Required the maximum intensity of fibre stress, and the extreme fibre on which it occurs.

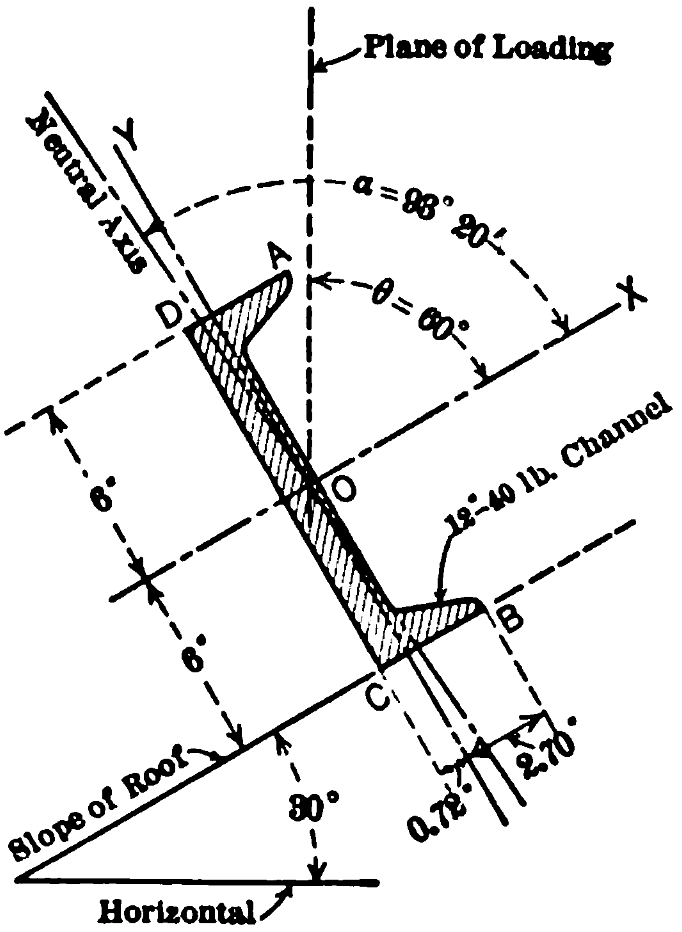


FIG. 3.

From the handbooks it will be found that the principal axes for this section are OX and OY, as shown in Fig. 3. For these axes, $I_x = 196.9$ inches^4, and $I_y = 6.63$ inches^4. The co-ordinates of the several apices of the section referred to the principal axes are as follows, the subscript corresponding to the apex in question: $x_A = +2.7$, $y_A = +6.0$; $x_B = +2.7$, $y_B = -6.0$; $x_C = -0.72$, $y_C = -6.0$; $x_D = -0.72$, $y_D = +6.0$. All values are given in inches. As stated in the preceding article, x and y are positive when measured to the right and upward with respect to OX and OY. For the conditions shown in Fig. 3, $\theta = 60°$. Substituting these values in eq. (6), using the co-ordinates for point A, gives

$$f_A = -M\left(\frac{+6.63 \times 6.0 \times 0.866 + 196.9 \times 2.7 \times 0.5}{196.9 \times 6.63}\right)$$

$$f_A = -\frac{+34.45 + 265.8}{1305}M = -0.230\,M.$$

The minus sign indicates that the fibre stress is compressive. A substitution for fibre B involves the same quantities, but, since y_B is negative, the first term in the numerator of eq. (6) becomes negative. Hence,

$$f_B = -\frac{-34.45 + 265.8}{1305}M = -0.177\,M.$$

A similar substitution for fibre C gives,

$$f_C = -M\left(\frac{6.63 \times (-6.0) \times 0.866 + 196.9 \times (-0.72) \times 0.5}{196.9 \times 6.63}\right)$$

$$f_C = +\frac{34.45 + 70.88}{1305} M = +0.0808\,M.$$

The sign of the result indicates a tensile stress for fibre C. In the same way

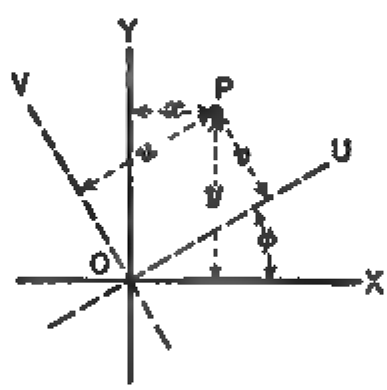
$$f_D = +\frac{-34.45 + 70.88}{1305} M = +0.0279\,M.$$

On comparing the fibre stresses as calculated, it will be found that fibre A has the maximum value, and the stress intensity at this point is $0.230\,M$ lbs. per sq. in.

The second method of calculation outlined above calls for the location of the neutral axis and the determination of the fibre most remote from this axis. Substituting in eq. (7) gives

$$\tan\alpha = -196.9 \times 0.5774 / 6.63 = -17.15$$

from which $\alpha = 93°20'$. Fig. 3 shows the neutral axis as located by this angle. From an inspection of Fig. 3 it can be seen at once that A is the



(b)

(a)

FIG. 4.

fibre most remote from the neutral axis. A single substitution in eq. (6) then gives the desired result. The calculations are as given above for point A.

EXAMPLE 2.—A 5 x 3½ x ½-inch angle placed as shown in Fig. 4 (a) carries a moment M which acts in a plane parallel to the longer leg of the angle. Required the intensity of maximum fibre stress and the fibre on which it occurs.

The handbooks give the properties of the angle under consideration with reference to the gravity axes of the section, shown by $O\,U$, $O\,V$ of Fig. 4 (a). As the handbooks also give the least radius of gyration for the section, and locate the axis for which it occurs, it is possible to determine the position of the principal axes, and the moments of inertia of the section for these axes.

From Mechanics, it can be shown that the least radius of gyration, and, therefore, also the least moment of inertia, occurs for one of the principal axes of a section. For the section under consideration, the handbooks, therefore, show that one of the principal axes makes an angle $\phi = \tan^{-1} 0.479$ with the axis $O V$ of Fig. 4 (*a*). The principal axes are, therefore, located as shown. From Mechanics $I_y = A r^2_y$, where I_y and r_y are respectively the moment of inertia and radius of gyration for axis $O Y$, and A is the area of the section. For this section, $I_y = 4.0 \times 0.75^2 = 2.25$ inches^4. To obtain the moment of inertia for the axis $O X$, the formula for the relation between moments of inertia for axes passing through the same point can be used, which is

$$I_x + I_y = I_u + I_v \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where I_x and I_y and the moments of inertia for the principal axes of the section, and I_u and I_v are the moments of inertia for any other set of axes. As the handbooks give the moments of inertia for the gravity axes, I_u and I_v are thus known. Also I_y has been calculated above. The value of I_x can therefore be obtained from eq. (9). From the handbooks it will be found that for the conditions shown in Fig. 4 (*a*), $I_u = 9.99$ inches^4, and $I_v = 4.05$ inches^4.

Then from eq. (9)

$$I_x = 9.99 + 4.05 - 2.25 = 11.79 \text{ inches}^4.$$

The principal moments of inertia are then $I_x = 11.79$ inches^4 and $I_y = 2.25$ inches^4.

To determine the fibre of maximum stress intensity, first locate the neutral axis, as in the preceding problem, by substitution in eq. (7). In this equation, the value of θ is to be taken as $115° - 36'$, as shown in Fig. 4 (*a*). From eq. (7)

$$\tan \alpha = - 11.79 \cot 115°{-}36' / 2.25 = + 2.50, \text{ or, } \alpha = 68° \; 12'.$$

The position of the neutral axis is shown in Fig. 4 (*a*). By scale from this figure, it will be found that fibre C is the most remote from the neutral axis, and is therefore the desired fibre of maximum stress intensity.

In substituting in eq. (6), the co-ordinates of point C must be referred to axes $O X$ and $O Y$. The handbooks do not give this information. It can be obtained by scaling from a large size layout of the angle, or by calculation from the values given in the handbooks for the gravity axes. The method for calculating the required values will be explained with the aid of Fig. 4 (*b*). In this figure, let P be any point, and let x and y be its co-ordinates with respect to the axes $O X$ and $O Y$, and u and v with respect to axes $O U$ and $O V$. Values of u and v can be obtained from the handbooks, as $O U$ and $O V$ represent the gravity axes of the section. If ϕ be the angle between the two sets of axes, then from Fig. 4 (*b*)

$$\left.\begin{aligned} y &= v \cos \phi + u \sin \phi \\ x &= u \cos \phi - v \sin \phi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In applying these formulas to the calculation of the co-ordinates of point C of Fig. 4 (a), attention must be paid to the signs of u and v, which are positive when measured upward and to the right with respect to point O. Then for point C, $u = -0.41$, $v = -3.34$, and $\phi = 25° - 36'$. From eq. (10), $y = -3.19$, $x = +1.07$. The calculated and scaled values were found to check.

With values of x and y as given above, and $\phi = 115° - 36'$, substitution in eq. (6) gives

$$f_c = -M\left[\frac{2.25 \times (-3.19)\sin 115° 36' + 11.79 \times 1.07 \cos 115° 36'}{11.79 \times 2.25}\right]$$

$$= +0.450\,M.$$

The sign of this result indicates that fibre C is under tensile stress.

If the neutral axis for this plane of bending be assumed to be horizontal, as is commonly done, the fibre stress at C is

$$f_C = Mc\,/\,I = 3.34\,M\,/\,9.99 = 0.334\,M,$$

a result only about 75 per cent of the true stress calculated above.

As a further application of the above methods, let the student calculate the stress intensity for fibre B of Fig. 4. The resulting stress is tensile, and its intensity is 0.187 M lbs. per sq. in.

4. The "S" Line.—From eq. (6) it can be seen that for any point of a given section the fibre stress can be found by dividing the bending moment M by the reciprocal of the expression in brackets. This divisor will hereafter be referred to as "S," and will be given a subscript corresponding to the fibre to which it refers. Thus for point A, Fig. 5,

$$f_A = M\,/\,S_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where

$$S_a = \frac{I_x\,I_y}{I_y\,y_A \sin\theta + I_x\,x_A \cos\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Eq. (12) is made up of terms such as I_x, I_y, x_A and y_A which are constants for any fibre of a given section, and an angle θ which is a variable depending for its value upon the direction of the plane of bending. If θ be taken as 0, or 90° in eq. (12), S_a takes the familiar form I/c, which is known as the *Section Modulus* of the section in question. As the expression of eq. (12) is a function of the direction of bending as well as of the properties of the section, it will be called the *Flexual Modulus* of the section for any given point.

For any given point of a given section, with θ variable, eq. (12) is the polar form of the equation of a straight line. This offers a convenient graphical determination of the variation in *flexual modulus* for any point due to changes in the position of the plane of the bending. In Fig. 5, the vector $O\,P$ gives the value of S_a for a given θ, and the line $C\,D$ is the locus of P for variable θ. The line $C\,D$ is known as the "*S Line*" of the section for point A

It will be convenient, in the work to follow, to have the equation of the S line given in rectangular co-ordinates. By placing $S_a \sin\theta = y$ and $S_a \cos\theta = x$ (see Fig. 5) in eq. (12), and solving for y, the equation of the S line in the slope form is,

$$y = -\frac{I_x}{I_y}\frac{x_A}{y_A}x + \frac{I_x}{y_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

When the plane of bending passes through point A, $x_A / y_A = \cot\theta$, and eq. (12) becomes

$$y = -\frac{I_x}{I_y}x\cot\theta + \frac{I_x}{y_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

By comparing eqs. (8) and (13a), it can be seen that for this particular set of conditions, the neutral axis and the S line are parallel.

5. The "S" Polygon.—For sections of irregular outline, it is evidently possible, and in fact necessary, to determine an S line for each extreme fibre

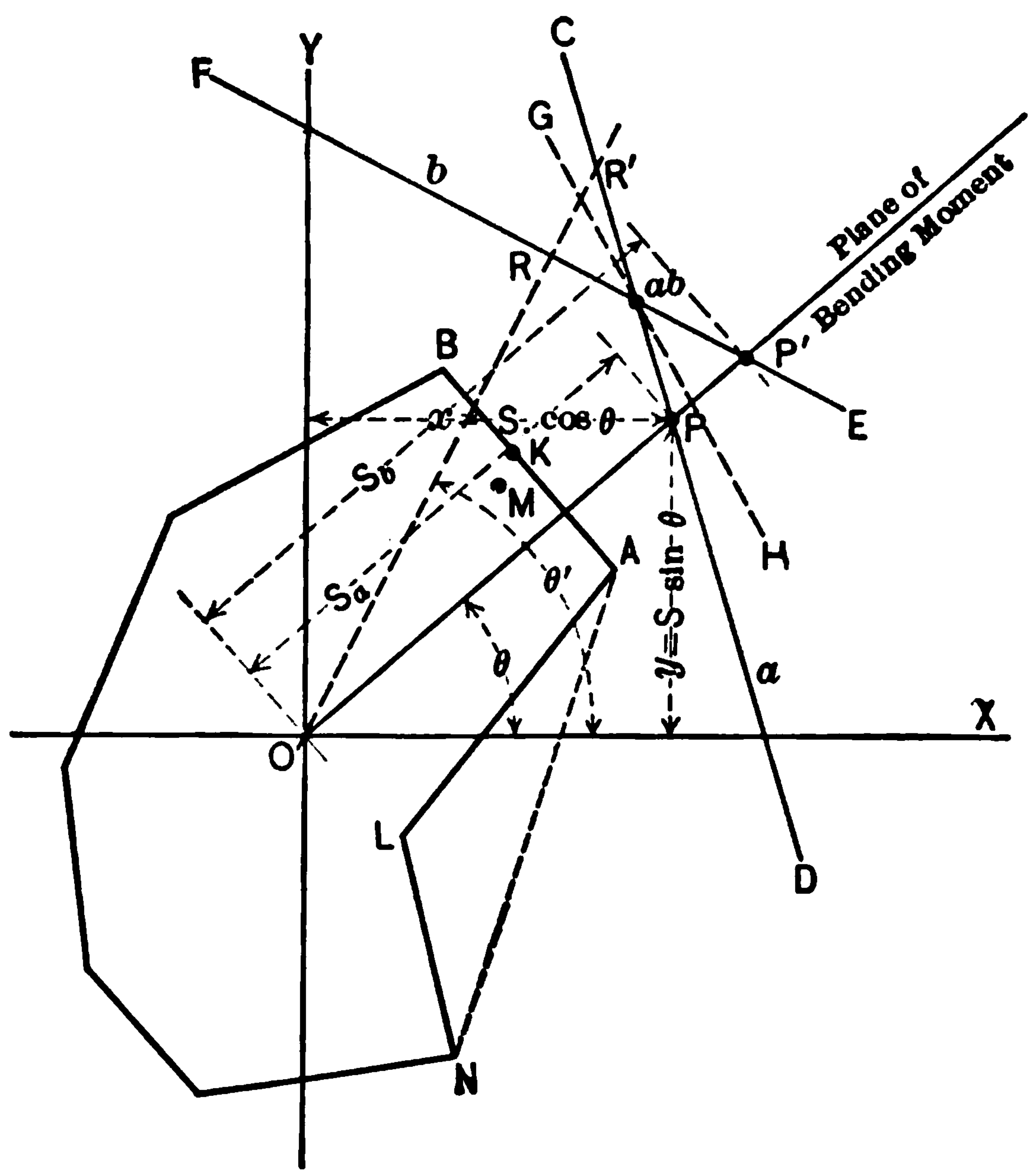


FIG. 5.

of the section. Thus in Fig. 5, let point B, with co-ordinates $x_B\, y_B$, be another extreme fibre. The equation of its S line is

$$y = -\frac{I_x}{I_y}\frac{x_B}{y_B}x + \frac{I_x}{y_B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This S line is shown by $F\,E$ in Fig. 5.

The value of S_b for bending in a plane at an angle θ to $O X$ is shown by the vector $O P'$. Since $O P$, the value of S_a for the same plane of bending, is less than $O P'$, it is evident from eq. (11) that for this particular plane of bending, fibre A has a greater stress intensity than fibre B. If, however, the plane of bending makes an angle θ' with $O X$ (Fig. 5), then S_b becomes less than S_a, and fibre B has the greater stress intensity. Therefore, to determine the fibre of maximum stress intensity, it is necessary to have S lines for all extreme fibres of the section. By comparing the values of S for these several points for a given plane of bending, it can be seen from the discussion given above for points A and B, that the fibre with the least S has the maximum stress intensity.

The extreme fibres, or corners of the rolled sections in general use, lie on a small number of straight lines. In Fig. 5, $A B$ is such a line. It can be shown by substituting particular values in eq. (13) that the S line for K, any point on $A B$, passes through the intersection of the S lines for A and B, as shown by the line $G H$, Fig. 5, and that it lies entirely outside the area enclosed by these S lines ($C D$ and $F E$ of Fig. 5). It can also be shown that for a point inside the section, such as M, Fig. 5, the S line lies entirely outside those for points A and B. This is due to the fact that point M is nearer the neutral axis than A or B, and therefore has a smaller stress intensity, and hence a greater S. For the same reason, it can be seen that an S line need not be drawn for a re-entrant corner, such as L, as S lines drawn for points A and N will take care of the perimeter of the section from A to N.

To determine the points for which S lines are to be drawn for any given section, consider a straight line to be rolled around the perimeter of the section. As the successive positions of this rolling line are parallel to successive positions of the neutral axis, it is evident from the above discussion that S lines need be drawn only for those points which this straight line touches, and in so doing does not cut across the section. Thus, for the irregular outline of Fig. 6, a straight line when rolled around the section can touch only points A, C, E, F, and G without cutting across the section. The lines $A C$, $C E$, $E F$, $F G$, and $G A$ show the consecutive positions of the rolling line. They form a polygon which is known as the *circumscribing polygon* of the section.

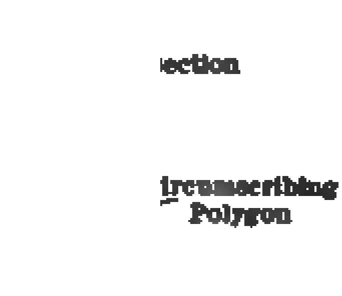


FIG. 6.

The S lines drawn for the apices of the circumscribing polygon of a given section will, if produced, form a closed figure, which is called the "*S Polygon*" for the section in question. This polygon will have as many sides as there are apices in the circumscribing polygon. In some cases, as sections with curved outlines, there will be an infinite number of apices in the circum-

scribing polygon. The S polygon will also have an infinite number of sides, thus forming a curved outline. For example, the S polygon for a circle is also a circle.

Two general methods can be used in constructing S polygons. By the first method, S lines can be drawn for each extreme point of the circumscribing polygon. The S lines for adjacent apices can then be produced to an intersection, thus determining the position of an apex of the S polygon. Proceeding in a like manner for each pair of adjacent extreme points, the entire S polygon is obtained. The second method locates the co-ordinates of the points of intersection of the S lines by the methods of analytical geometry. These points are then connected to form the required S polygon. Of the two methods, the latter is the better as it requires the location of only half as many points as the first method for the complete determination of an S polygon. It will therefore be used in the work to follow.

The co-ordinates of the intersection of the S lines for points A and B, of Fig. 5, can be determined by solving eqs. (13) and (14) for simultaneous values of x and y. Let these values for point $a\,b$, of Fig. 5, the intersection of S lines A and B, be denoted by x_{ab}, y_{ab}. Then from eqs. (13) and (14)

$$x_{ab} = \frac{I_y\,(y_B - y_A)}{x_A\,y_B - x_B\,y_A} \qquad (15)$$

and

$$y_{ab} = \frac{I_x\,(x_A - x_B)}{x_A\,y_B - x_B\,y_A} \qquad (16)$$

Similar values for pairs of adjacent extreme points of Fig. 5, as A and N, etc., when plotted and connected up will form the required S polygon.

In some sections, the sides of the circumscribing polygon are parallel to the axes $O\,X$ and $O\,Y$. For this condition, eqs. (15) and (16) take on a much simpler form. Thus, in Fig. 7, the side $A\,B$ is parallel to the axis $O\,Y$, and $x_A = x_B = a$. Substituting these values in eqs. (15) and (16) gives

$$\left.\begin{aligned} x_{ab} &= I_y/a \\ y_{ab} &= 0 \end{aligned}\right\} \qquad (17)$$

In eq. (17), I_y/a is the section modulus for fibres A and B for the axis $O\,Y$, a value which is given in the handbooks.

For a side $D\,C$ parallel to the axis $O\,X$, $y_C = y_D = c$, and from eqs. (15) and (16)

$$\left.\begin{aligned} x_{dc} &= 0 \\ y_{dc} &= I_x/c \end{aligned}\right\} \qquad (18)$$

Here $I_x\,/\,c$ is the section modulus for points D and C for the axis $O\,X$.

The sides $C\,B$ and $E\,D$ are not parallel to either principal axis. Values of the co-ordinates of the points of intersection of the S lines for these

points can be obtained by substitution in eqs. (15) and (16). It is possible, however, by rotating the axes of reference, to find a set of axes which will be parallel to any side of a given section. Then by referring the co-ordinates of the apices of the circumscribing polygon to these new axes, a set of formulas of the same nature as these of eqs. (17) and (18) can be obtained.

To obtain these simplified formulas, let $E D$ and $B C$, of Fig. 7, be two sides of a section which are oblique to axes $O X$ and $O Y$, and suppose, for the purpose of this discussion, that these sides are perpendicular to each other. Through point O locate a pair of rectangular axes $O U$ and

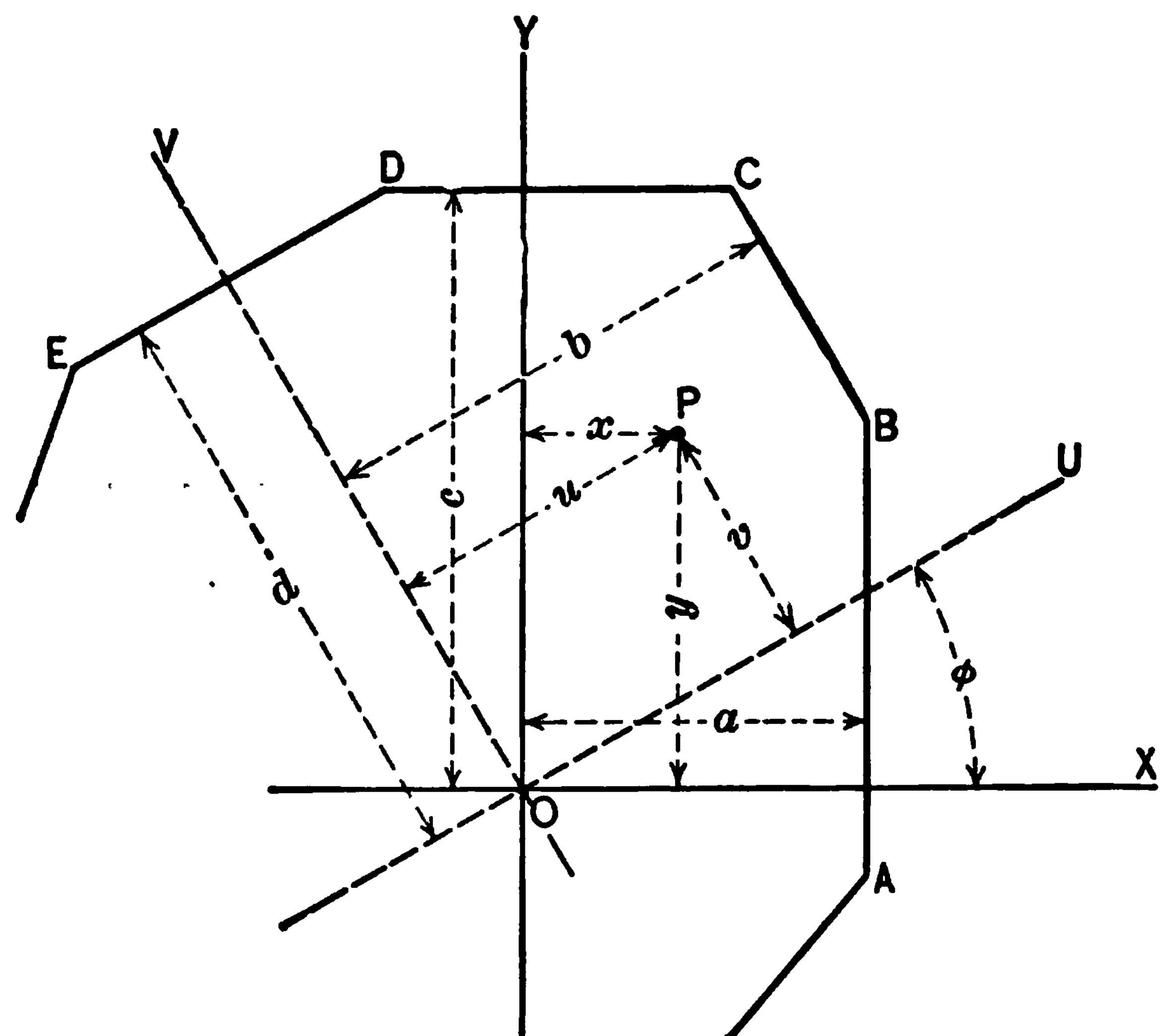


FIG. 7.

$O V$ which are parallel to $E D$ and $B C$. Let ϕ be the angle which $O U$ makes with $O X$, considered as positive when measured counter-clockwise. If P be any point with co-ordinates x and y with respect to axes $O X$ and $O Y$, and u and v with respect to axes $O U$ and $O V$, considered as positive when measured upward and to the right, it can be shown from Fig. 7 that

$$\left. \begin{aligned} y &= v \cos \phi + u \sin \phi \\ x &= u \cos \phi - v \sin \phi \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

By substituting these values of x and y in eq. (15) and (16), the co-ordinates of the extreme points of the section will be referred to the new axes $O U$ and $O V$. This substitution will not be made for the co-ordinates of the points of intersection of the S lines, for nothing is gained by changing the axes of reference for these points. In substituting values from eq.

(19) in eqs. (15) and (16), u and v will be given subscripts to correspond to those for x and y. This substitution gives

$$\left.\begin{aligned} x_{ab} &= \frac{I_y\,[(u_B - u_A)\sin\phi + (v_B - v_A)\cos\phi]}{(u_A v_B - u_B v_A)} \\ y_{ab} &= \frac{I_x\,[(v_B - v_A)\sin\phi + (u_A - u_B)\cos\phi]}{(u_A v_B - u_B v_A)} \end{aligned}\right\} \quad . \quad . \quad . \quad (20)$$

Since the angle ϕ is taken such that the axes of $O\,U$ and $O\,V$ are parallel to certain sides of the section in question, it follows that for side $C\,B$, which is parallel to the axis $O\,V$, the U co-ordinates for points B and C are equal, that is, $u_B = u_C = b$, as shown in Fig. 7. Substituting these values in eqs. (20) gives

$$\left.\begin{aligned} x_{bc} &= I_y \cos\phi/b \\ y_{bc} &= I_x \sin\phi/b \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In the same way for side $D\,E$, which is parallel to the axis $O\,U$, with V co-ordinates $v_D = v_E = d$, substitution in eqs. (20) gives

$$\left.\begin{aligned} x_{de} &= -\,I_y \sin\phi/d \\ y_{de} &= +\,I_x \cos\phi/d \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

For cases where $B\,C$ and $D\,E$ are not perpendicular, as assumed in this discussion, it will be necessary to use two sets of axes to obtain the desired results.

6. Construction of *S* Polygons.—To illustrate the principles outlined in the preceding article, the S polygons for a few standard sections in general use as beam sections will now be worked out.

S Polygon for a Rectangle.—The S polygon for a 6 × 10 inch rectangle will be computed and constructed. Fig. 8 shows the rectangle with the principal axes $O\,X$ and $O\,Y$ in position. The principal moments of inertia are $I_x = 500$ inches^4 and $I_y = 180$ inches^4, and the co-ordinates of the corners of the section, which in this case are apices of the circumscribing polygon, are $x_A = +\,3$, $y_A = +\,5$; $x_B = +\,3$, $y_B = -\,5$; $x_C = -\,3$, $y_C = -\,5$; and $x_D = -\,3$, $y_D = +\,5$. Since the sides of the rectangle are all parallel to the principal axes of the section, eqs. (17) and (18) can be used. As the side $A\,B$ is parallel to the axis $O\,Y$, the co-ordinates of the point of intersection of S lines for points A and B are given by eq. (17). With $I_y = 180$ inches^4, and $a = x_A = x_B = 3$ inches, eq. (17) gives $x_{ab} = 180/3 = +\,60$ inches^3, and $y_{ab} = 0$. This apex of the S polygon is therefore located on the axis $O\,X$, as shown in Fig. 8. For the side $A\,D$, which is parallel to $O\,X$, eq. (18), with $I_x = 500$ inches^4, and $c = y_A = y_D = 5$ inches, gives $x_{ad} = 0$, $y_{ad} = +\,100$ inches^3, a point on the axis $O\,Y$, as shown in Fig. 8.

In the same way it will be found that for sides $C\,B$ and $C\,D$, $x_{cb} = 0$, $y_{cb} = -\,100$ inches^3, $x_{dc} = -\,60$ inches^3, $y_{dc} = 0$.

Plotting these points and connecting the points with a common letter by straight lines as, for example, points $d\ a$ and $a\ b$ with a line denoted by a, in Fig. 8; also points $a\ b$ and $c\ b$ with a line b, the complete S polygon is obtained. In this figure, the several S lines are indicated by a letter which shows the corner of the section to which the line refers.

It is to be noted that the co-ordinates of the apices of the S polygon, as x_{ab}, y_{ad}, etc., are equal to the section moduli for the rectangle. This

12 in.40 lb.

lb. I Beam

S Polygon
6" x 10" Rectangle

12 in.40 lb.I Beam
(a)

12 in.40 lb.Channel
(b)

FIG. 8.

FIG. 9.

offers a convenient method for constructing this S polygon without the use of eqs. (17) and (18). Since these values are given directly in handbooks, they can be plotted in the proper positions on the X and Y axes, and the S polygon drawn as before.

S Polygon for a 12-In. 40-Lb. I-Beam.—Fig. 9 (*a*) shows the S polygon for a 12-in. 40-lb. I-beam. As the circumscribing polygon for the I-beam is a rectangle, the methods of calculation are exactly the same as given above for the rectangular figure. The detail calculations will not be given. All data are shown on Fig. 9 (*a*).

S Polygon for a 12-in. 40-lb. Channel.—The circumscribing polygon for a channel section is also a rectangle, but since the axis $O\ Y$ is not an axis

of symmetry, the resulting S polygon will not be symmetrical, as in the case of those for the rectangle and the I-beam.

For the channel section under consideration, the handbooks give $I_x = 196.9$ inches^4, $I_y = 6.63$ inches^4, $x_A = +2.7$ inches, $y_A = +6.0$ inches; $x_B = +2.7$ inches, $y_B = -6.0$ inches; $x_C = -0.72$ inches, $y_C = -6.0$ inches; and $x_D = -0.72$ inches, $y_D = +6.0$ inches. Substituting these values in eqs. (17) and (18), the co-ordinates of the apices of the S polygon are found to be:

$$\begin{aligned} x_{ab} &= +2.46 \text{ inches}^3 & y_{ab} &= 0 \\ x_{bc} &= 0 & y_{bc} &= -32.8 \text{ inches}^3 \\ x_{cd} &= -9.21 \text{ inches}^3 & y_{cd} &= 0 \\ x_{da} &= 0 & y_{da} &= +32.8 \text{ inches}^3 \end{aligned}$$

Plotting these values gives the S polygon shown in Fig. 9 (*b*).

S Polygon for an Angle Section.—The S polygon for a $5 \times 3\frac{1}{2} \times \frac{1}{2}$ inch angle will be computed and constructed. This is the same angle section as used in Prob. 2, in Art. 3. Fig. 4 shows the location of the principal axes of the section, for which the moments of inertia were found to be $I_x = 11.79$ inches^4, and $I_y = 2.25$ inches^4. (See p. 464.) As shown in Fig. 10, the sides of the circumscribing polygon, which is $A\,B\,C\,D\,E$, are not parallel to either of the principal axes. The co-ordinates of the apices of the S polygon are to be calculated by eqs. (15) and (16), or by rotating the co-ordinate axes, as explained on p. 469, eqs. (21) and (22) can be used. As the latter method is the simpler, it will be used here.

Fig. 10.

From the handbooks it will be found that the gravity axes of the angle section shown by $O\,U$ and $O\,V$ in Fig. 10, make an angle of $25° - 36'$ with the principal axes. Sides $A\,B$, $A\,E$, $E\,D$ and $D\,C$ are parallel to the gravity axes. Referred to the axes $O\,U$ and $O\,V$, the co-ordinates of the extreme points of the section are as shown on Fig. 10.

The co-ordinates of the apices of the S polygon are given by eqs. (21) and (22), with $\phi = 25° - 36'$. For side $A\,B$, which is parallel to the $O\,V$ axis, eq. (21) is to be used, with $u_B = u_A = 2.59$ inches. Then from eq. (21)

$$x_{ab} = + \ 2.25 \times 0.902/2.59 = + 0.783 \text{ inches}^3$$
$$y_{ab} = + 11.79 \times 0.432/2.59 = + 1.965 \text{ inches}^3.$$

In plotting the position of this point, it is to be remembered that x_{ab} and y_{ab} are referred to axes OX and OY, the rotation of axes of reference having been made only with respect to the extreme points of the section. Side DE is also parallel to the OV axis: eq. (21) is to be used, which gives:

$$x_{de} = + \frac{2.25}{-0.91} \times 0.902 = - 2.23 \text{ inches}^3$$

$$y_{de} = + \frac{11.79}{-0.91} \times 0.432 = - 5.60 \text{ inches}^3$$

Sides AE and DC are parallel to axis OU. Substitution in eq. (22) gives,

$$x_{ae} = - \frac{2.25}{1.66} \times 0.432 = - 0.585 \text{ inches}^3$$

$$y_{ae} = + \frac{11.79}{1.66} \times 0.902 = + 6.42 \text{ inches}^3$$

and

$$x_{dc} = - \frac{2.25}{-3.34} \times 0.432 = + 0.291 \text{ inches}^3$$

$$y_{dc} = + \frac{11.79}{-3.34} \times 0.902 = - 3.18 \text{ inches}^3.$$

The side BC of the circumscribing polygon is parallel to a pair of rectangular axes shown by OR and OT in Fig. 10. These axes make an angle of $8° - 05'$ with OX and OY, and are so located that $\phi = (360° - 8°05') = 351° - 55'$. The angle between axes OX and OR, and the perpendicular distance from point O to side BC were obtained from a full size drawing of the section by means of a protractor and scale. These values can be calculated, if desired, but the method used here is accurate enough for problems of this nature, and considerable time can be saved in this way.

As side BC is parallel to axis OT, which is similar to axis OV of Fig. 7, eq. (21) is to be used. With $b = 1.51$ inches, $\cos \phi = \cos 351° - 55' = + \cos 8° - 05' = + 0.990$; and $\sin 351° - 55' = - \sin 8° - 05' = - 0.141$; substitution in eq. (21) gives,

$$x_{bc} = + 2.25 \times 0.990/1.51 = + 1.48 \text{ inches}^3$$
$$y_{bc} = + 11.79 \times (- 0.141)/1.51 = - 1.10 \text{ inches}^3.$$

Plotting these co-ordinates with respect to the OX and OY axes, and connecting the proper points, the complete S polygon is obtained, as shown in Fig. 10.

7. Solution of Problems in Unsymmetrical Bending by Means of S Polygons.—The S polygon offers a convenient semi-graphical method for the solution of problems in the investigation and design of beams subjected to unsymmetrical bending. It is particularly useful where a beam, such as a roof purlin, is to be designed for combinations of loads acting in several planes. To illustrate the general methods, several problems will be worked out in detail. These problems will be divided into two groups. One group will consist of problems in the investigation of beams, and the other will consist of design problems.

Investigation of Beams.—An important problem in the investigation of beams is the determination of the relative moment carrying capacity of the various forms of rolled sections. This can be done by comparing the S polygons for the sections. Thus, Fig. 9 shows the S polygons for 12-in. channels and I-beams, each section weighing 40 lbs. per ft. As these S polygons are drawn to the same scale, they give a graphical comparison of the relative moment carrying capacity of an I-beam and a channel of the same weight. It is evident by an inspection of Fig. 9 that the advantage is in favor of the I-beam. Any other sections can be compared by this method.

Another problem of importance is the determination of the planes of greatest and least strength for a given section. This will show the best position for a certain section in order to take advantage of its greatest strength as a beam, and also to avoid its weakest position. From eq. (11), Art. 4, it can be seen that the fibre stress varies inversely with S_a for the fibre in question. Therefore, a beam has its greatest strength for the plane of loading with the greatest S_a, measured as shown by $O\ P$ of Fig. 5, and its least strength where S_a has its least value.

For the rectangle, I-beam, and channel section, shown in Figs. 8 and 9, the sections have their greatest strength as beams for bending moments in the plane of the axis $O\ Y$. The least strength of these sections is developed on a plane which passes through the point O and is perpendicular to the S lines. For the rectangle and the I-beam there will be four such planes, one for each S line, while for the channel there are two such planes, one for the S line a and the other S line b.

The angle section of Fig. 10 has a very unsymmetrical S polygon, and the determination of its greatest and least strength planes is not as simple as for those of rectangular form. It will be found that the plane of greatest strength lies between axes $O\ Y$ and $O\ T$ in a position such that the intercepts on this plane from point O to S lines a and d are equal. The plane of the least strength is perpendicular to the S line b.

The S polygon can also be used for the determination of the fibre stress for any point of a given section due to a given bending moment. This is a problem similar to those considered in Examples 1 and 2 of Art. 3. These will now be solved by the S polygon method.

EXAMPLE I.—A 12-in. 40-lb. channel placed with its web at an angle of 30° to the vertical is used as a roof purlin to support a bending moment M acting in a vertical plane. Required, the maximum intensity of fibre stress and the extreme fibre on which it occurs.

Fig. 11 shows the S polygon for the channel section in question. The co-ordinates of the apices of the S polygon were calculated in Art. 6, and

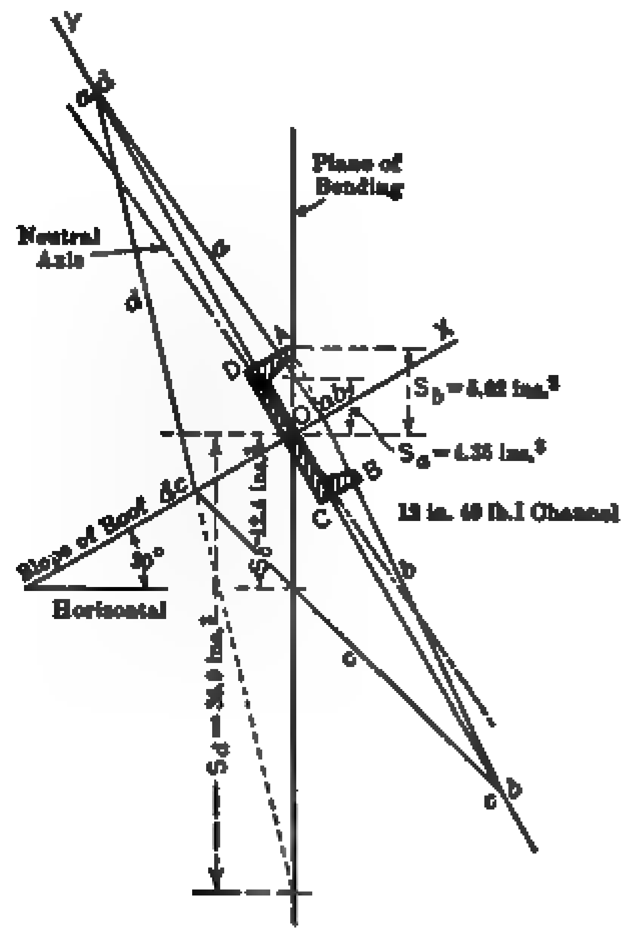


FIG. 11.

FIG. 12.

are given on Fig. 9 (a). Drawing through point O a vertical line to represent the position of the plane of bending, and scaling the intercepts of this line from point O to the several S lines, it will be found that the values of S are as shown on Fig. 11. Then from eq. (11), Art. 4, $f_A = M/4.35 = 0.23\ M$; $f_D = M/36.0 = 0.0278\ M$; $f_C = M/12.4 = 0.0808\ M$; and $f_B = M/5.62 = 0.178\ M$. These values check those calculated algebraically in Art. 3. As before, it will be found that fibre A has the maximum stress intensity.

The character of stress on a fibre can be determined by locating the neutral axis from eq. (7), Art. 2, and plotting it on Fig. 11, as shown. As fibres A and B are above the neutral axis, they will be in compression (positive moment) and fibres D and C will be in tension.

If the neutral axis be drawn in as the first step in the solution, it is possible to determine by inspection that A is the fibre of maximum stress,

as in the solution of Art. 3. The value of S for this point only is required for a complete solution of the problem.

EXAMPLE 2.—A $5 \times 3\frac{1}{2} \times \frac{1}{2}$-in. angle placed with the longer leg vertical carries a bending moment M which acts in a vertical plane. Required, the intensity of maximum fibre stress and the fibre on which it occurs.

Fig. 12 shows the S polygon for the section in question as plotted from the calculations given in the preceding article. By an inspection of Fig. 12 it can be seen that the S line for point C gives the least intercept on the trace of the plane of bending. Point C, therefore, has the maximum fibre stress, which is $f_C = M/2.22 = 0.450 M$. This checks the work of Art. 3. As fibre C is below the neutral axis, the stress is tensile.

Design of Beams.—In designing beams by S polygon methods, the first step is the determination of the flexual modulus required for the given plane of bending. From eq. (11), Art. 4, $S_A = M/f_A$. After this value has been plotted to scale, the S polygon of the sections to be tried are also plotted to the same scale on the co-ordinate axes. In order to answer the requirements of the design, the required flexual modulus must be inside the S polygon of the section.

FIG. 13.

EXAMPLE 3.—A roof purlin is to be placed with its web at an angle of 30° to the vertical, and is to be subjected to a gross bending moment of 32,000 in.-lbs. Determine the beam section of least weight which can carry this moment with a maximum extreme fibre stress not to exceed 16,000 lbs. per sq. in.

The flexual modulus required in the plane of bending is $S = M/f = 32{,}000/16{,}000 = 2.0$ inches^3. This is shown in position in Fig. 13. From the S polygons of I-beam and channel sections given in Fig. 9, it can be seen that for the plane of bending under consideration, fibres A and C of the I-beam section, and fibre A of the channel section will have the maximum stress intensity. It will therefore be necessary to draw only the S line for fibre A, as shown in Fig. 13. The S lines for several rolled

sections are shown plotted to scale. Of the sections tried, the 10-in. 15-lb. channel answers the requirements exactly, as the required and furnished values of S are equal. The 7-in. 15-lb. I-beam also answers the requirements, and the extreme fibre stress is somewhat less than the allowable maximum, as shown by the fact that the S provided is greater than that required.

In Art. 257, Chap. XI, the design of a roof purlin for several combinations of dead, snow, and wind load is given. The solution is based on the principles used in the above problem.

8. Deflection of Beams Under Unsymmetrical Bending.—In many cases the amount and direction of the deflection of beams under unsymmetrical bending is desired. This can be determined by methods similar to those used in Art. 2. The bending moment is resolved into its components parallel to the principal axes of the section. From these moments can be determined the components of the deflection parallel to the principal axes, and also the amount and direction of the resultant deflection.

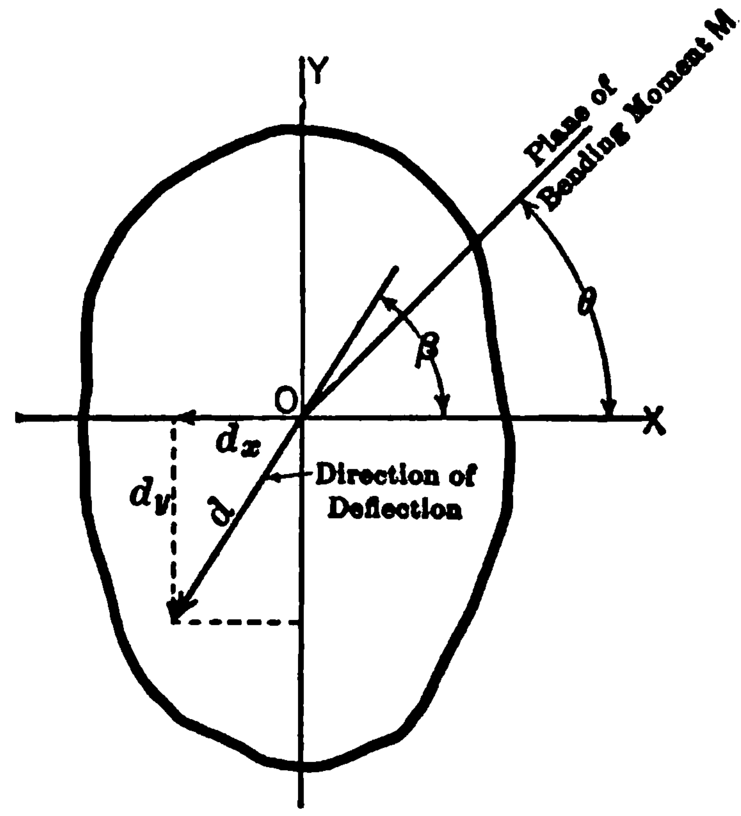


FIG. 14.

Let Fig. 14 show a beam section acted upon by a moment M, with components $M \cos\theta$ and $M \sin\theta$ parallel respectively to the axes OX and OY. From eq. 22, of Art. 214, Part I, the deflection of a beam under moment M is given by the formula,

$$d = \int_0^l \frac{Mm}{EI} dx$$

where m is the moment due to a 1-lb. load acting in the direction of the desired deflection.

Since the bending moment for axis OY is $M \sin\theta$, the vertical component of the deflection, which will be denoted by d_y, is

$$d_y = \int^l \frac{M \sin\theta}{EI_x} m_y \, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where m_y is the moment due to a 1-lb. load acting along the axis OY. Likewise,

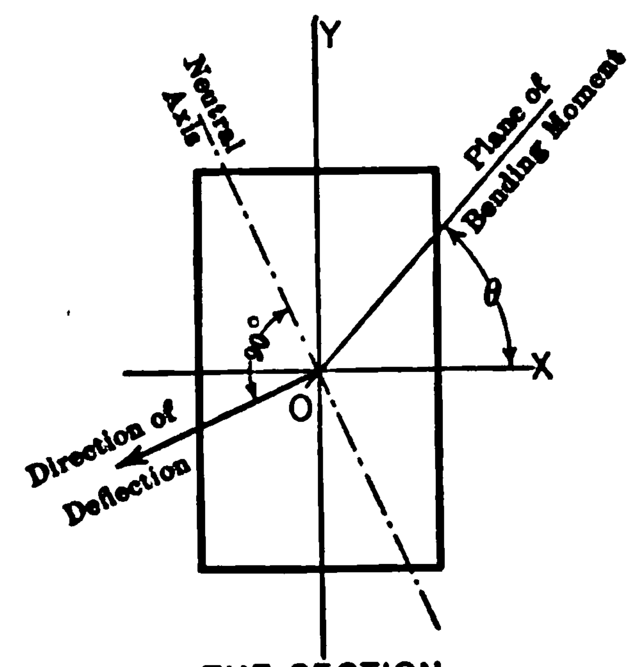
$$d_x = \int_0^l \frac{M \cos\theta}{EI_y} m_x \, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The resultant deflection is the vector sum of these terms. Since $m_x = m_y = m$, this gives

$$d = (d_y^2 + d_x^2)^{\frac{1}{2}} = \int_o^l \frac{M\,m\,dx}{E}\left(\frac{I_x^2 \cos^2\theta + I_y^2 \sin^2\theta}{I_x^2\, I_y^2}\right)^{\frac{1}{2}} \quad . \quad . \quad (25)$$

From Fig. 14, the angle β which the resultant deflection makes with the axis $O\,X$ is

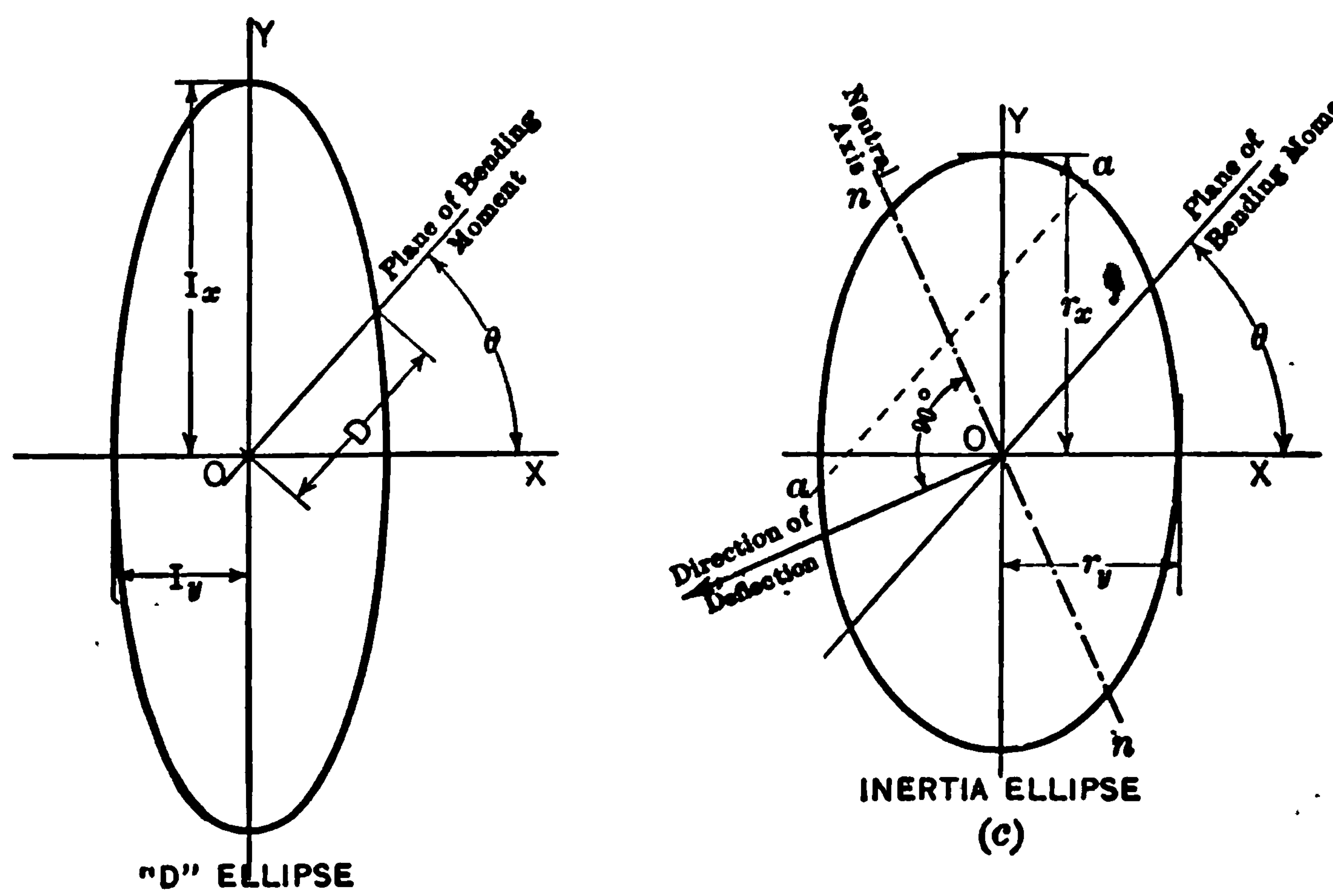
$$\tan\beta = d_y/d_x = \frac{\int_o \frac{Mm}{EI_x}\,dx \sin\theta}{\int_o^l \frac{Mm}{EI_y}\,dx \cos\theta} = \frac{I_y}{I_x}\tan\theta \quad . \quad . \quad . \quad (26)$$



THE SECTION
(a)

As this expression is the negative reciprocal of that given in eq. (7), the resultant deflection is perpendicular to the neutral axis for any given plane of bending.

Graphical methods can also be used for the determination of the amount and direction of deflection. As these methods are based on certain properties of the ellipse, it is probable that unless the same section is to be investigated for deflection in several planes, the time con-



"D" ELLIPSE
(b)

INERTIA ELLIPSE
(c)

FIG. 15.

sumed in making a graphical solution will be greater than that for the algebraic method.

From eq. (25), the deflection can be written in the form

$$d = \frac{1}{D}\int_o^l \frac{M\,m}{E\,I}\,dx$$

where

$$D = \left(\frac{I_x^2\,I_y^2}{I_x^2\cos^2\theta + I_y^2\sin^2\theta}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (27)$$

This can be shown to be the equation of an ellipse with major and minor axes I_x and I_y. Fig. 15 (*b*) shows this ellipse for a 6×10-in. rectangle placed as shown in Fig. 15 (*a*). For any plane of bending, the value of D is measured as shown in Fig. 15 (*b*).

The direction of bending can be determined from the inertia ellipse for the section. For a discussion of the ellipse of inertia, the student is referred to any of the more advanced works on the Mechanics of Materials. This ellipse is constructed for major and minor axes equal to the radii of gyration of the section for axes $O\,X$ and $O\,Y$. Fig. 15 (*c*) gives the inertia ellipse for the rectangle of Fig. 15 (*a*). After the inertia ellipse has been drawn, the neutral axis is located for bending in any plane, and a line drawn perpendicular to this axis locates the plane of bending.

To locate the neutral axis, draw through point O a line parallel to the plane of bending. Draw *a-a*, any chord of the ellipse parallel to the plane of bending. Bisect this chord, and through its center draw *n-n*, which is parallel to the neutral axis for bending at an angle θ to $O\,X$. This construction is based on the fact that eq. (7) expresses the relation which exists between conjugate diameters of an ellipse. The proof can be found in works on analytical geometry. A line drawn perpendicular to *n-n* gives the direction of deflection, as shown in Fig. 15 (*c*).

The relative deflection of the beam sections determined in Example 3, of Art. 7, will now be calculated, using eq. (25). From Fig. 13, the value of $\bar{\theta}$ can be seen to be 60°. For the 7-in. 15-lb. I-beam, with $I_x = 36.2$ ins.^{4} and $I_y = 2.67$ ins.^{4}, eq. (25) gives

$$d = 0.324\int_o^l \frac{M\,m}{E}\,dx$$

In the same way, for the 10-in. 15-lb. channel, with $I_x = 66.9$ ins.^{4} and $I_y = 2.30$ ins.^{4}

$$d = 0.377\int^l \frac{M\,m}{E}\,dx$$

The I-beam is therefore the stiffer section, which in some cases might result in the use of this section rather than the channel for the conditions of Example 3, Art. 7.

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